The Family Firm Ownership Puzzle*

Ronald Anderson, Temple University
Masud Karim, Temple University
Nan Li†, Shanghai Jiao Tong University
David M. Reeb, National University of Singapore

December 21, 2021

[Abstract]

Conventional wisdom suggests that family shareholders should exit their large, concentrated equity stakes in publicly traded firms and seek benefits arising from diversification. However, founding families maintain a substantive and undiversified stake in many publicly traded U.S. firms. The classical models without ambiguity cannot quantitatively explain the decision of these family owners to hold a large portion of their wealth in the family firm. We propose a robust portfolio-choice model with ambiguity about the return volatility, where family owners can exploit their information advantage about their firm to reduce the ambiguity of their firm relative to other firms in a diversified portfolio. Our model rationalizes family owners' decision to concentrate their wealth in the family firm and predicts that the *less wealthy, less risk averse, and younger families* are more likely to exit the firm. The empirical results based on more than 500 U.S. family firms' cross-section data support these novel predictions. Based on family ownership and exit decisions, we find that information advantage and ambiguity about return volatility are critical to understanding the family owners' decision to maintain substantive control in countries with well-developed financial markets and legal regimes.

JEL Codes: G11; D81; L22

* We benefited from discussions with Gurdip Bakshi, Xiaohui Gao Bakshi, Sugato Bhattacharyya, Sudipto Dasgupta, Laura Field, Qiang Fu, Lars Peter Hansen, Mark Huson, Kose John, Bob Kimmel, Jinfeng Lu, Chenghu Ma, Randall Morck, Shige Peng, Nagpurnanand Prabhala, Oleg Rytchkov, Tan Wang, Bernard Yeung, Frank Yu, Luigi Zingales, and seminar participants at the National University of Singapore, the University of Alberta, Shanghai Advanced Institute of Finance (SAIF) and Temple University. Nan Li acknowledges financial support from National Natural Science Foundation of China (No. 72073097) and Shanghai Pujiang Talent Program (16PJC054).

† Corresponding author. Antai College of Economics and Management, Shanghai Jiao Tong University, nanli@sjtu.edu.cn.

1. Introduction

Conventional wisdom and classical finance theory suggest investors in countries with well-developed equity markets benefit from holding a diversified portfolio rather than concentrated stakes in a single or small number of firms. Yet, founding families maintain a substantive and undiversified stake in a substantial number of publicly traded U.S. firms[1] (Shleifer and Vishny, 1986). Anderson and Reeb (2003) find that "families that appear in both Forbes' Wealthiest Americans Survey and the S&P 500 have over 69 percent of their wealth invested in their firm". Anderson et al. (2009) report that the U.S. family owners maintain large, concentrated stakes in almost one-half of the Russell 3000 companies, frequently holding the vast majority of their investable wealth in the family firm.

Leland and Pyle (1977) argue that family owners have private information about the firm's prospects and hold large stakes to signal firm quality. Extending this notion, Zingales (1995) suggests that family owners should slowly decrease their position in the firm after the initial IPO. DeMarzo and Urusevic (2006) highlight the trade-off between agency considerations and diversification in the family exit decision. Alternatively, family owners could act as a commitment device to mitigate risk-shifting activities (Anderson et al., 2003), relying on private payoffs to justify foregoing the benefits of diversification[2] (Shleifer and Vishny, 1986; Morck and Yeung, 2003).

[1] Family block holders are not the only type of investor with large shareholdings in the financial markets. Robinson and Sensoy (2013) find private equity funds and venture capitalists hold less well-diversified portfolios relative to mutual funds. Choi et al. (2017) finds that concentrated investment strategies of institutional investors in international markets can be optimal and enhance risk-adjusted returns, which is consistent with the predictions of our model that investors optimally choose to hold concentrated portfolios in which they have an information advantage and amplify their advantage through learning.

[2] Family owners may receive several potential non-monetary benefits from controlling the firm (Gomez-Mejia et al., 2007). Yet, family control does not appear to require large and substantive stakes in the firm to achieve these ends. For instance, Anderson and Reeb (2004) highlight how family owners with stakes ranging from 5% to 10% maintain control of the firm through the director nomination process and thus limit outside influence or interference. Others emphasize family use of dual-class shares as a control enhancing mechanism (Almeida and Wolfenzon, 2006).

Burkart et al. (2003) show that family ownership may be an optimal ownership structure to protect shareholder interests in economies offering weak shareholder protections and/or economies with undeveloped equity markets. The shareholder protection and private benefits perspectives provide substantial insights into the cross-country patterns of family ownership (Faccio and Lang, 2002).

However, the strong shareholder protections and well-developed financial markets in the U.S. should indicate that investors need not make the trade-off between holding concentrated equity stakes or a diversified portfolio, and family ownership should not persist in U.S. firms. A simple reference model in a classical mean-variance framework for the Russell 3000 indicates that family owners should only hold about three percent of their investable wealth in the family firm and the remainder in a broad portfolio of other assets to obtain diversification benefits[3]. Anecdotally, the model suggests that the Walton family (of Wal-Mart Stores) would need to privately capture over an additional $2 billion *per year* to justify their huge ownership position relative to holding a well-diversified portfolio. Presumably, even if the Walton family were to reduce their substantial equity holdings from 50% to 25% of the firm's outstanding equity, they could still achieve the same private benefits of control. Hence, a puzzling issue remains as to why family ownership remains prevalent among publicly traded firms in countries with well-developed equity markets and legal systems.

We propose to explain this seeming puzzle of family ownership in a robust portfolio-choice model where investors face risks and ambiguity or Knightian uncertainty[4]. Knight (1921) formalized a distinction between risk and uncertainty. Risk refers to the situation where we do not know the outcome but can accurately measure the odds, while uncertainty refers to the situation where we

[3] To justify the average stakes of about 70% of the wealth in the family firm, the reference model indicates that the family would need to extract an additional $700 million to be indifferent between a concentrated stake relative to holding a diversified portfolio. These estimates are computed in a Markowitz (1952) framework using a constant-absolute-risk-aversion utility function (CARA), with absolute risk aversion of 0.2, risk free rate of 2%, expected return of 9%, single stock standard return deviation of 49.24%, a portfolio return standard deviation of 19.21% (see Elton and Gruber, 1977) and family ownership of $1.5 billion.

[4] In this paper, we interchangeably use the terms ambiguity and Kightian uncertainty.

do not know the outcome or the odds. The Ellsberg Paradox (Ellsberg 1961) shows that decision-makers, under ambiguity or uncertainty with unknown probability, choose to maximize the worst-case expected utility, and this decision is not compatible with the Savage axioms of expected utility maximization. Later on, Gilboa and Schmeidler (1989) establish the axioms of the maximin expected utility theory (MMEU) compatible with the decisions to maximize the worst-case expected utility under ambiguity.

In the classical portfolio-choice model of Markowitz (1952) and Merton (1971), it is assumed that stock returns follow a unique probability distribution, and all investors know exactly this distribution. Therefore, investors aim to maximize the (risk-adjusted) expected return of their portfolio by allocating their wealth to different assets. However, more and more researchers and practitioners realize that investors are operating in conditions of unknown probability distributions or Knightian uncertainty. In this situation, ambiguity-averse investors have to form multiple perceptions about the distribution or model with the information at hand and try to maximize the worst-case expected utility or make robust decisions. Hansen (2007) shows that ambiguity aversion induces extra caution in investment choices, and establishes conditions under which asset prices depend separately on risk aversion and model uncertainty aversion parameters.

Boyle et al. (2012) and many others find that a robust portfolio-choice model with ambiguity about expected returns can *qualitatively* explain various under diversification puzzles in the financial markets[5]. In these models, investors facing ambiguous asset returns choose to maximize expected

[5] French and Poterba (1991) document the home-bias puzzle, and the own-company stock puzzle is documented by Benartzi (2001), Meulbrook(2005), Mitchell and Utkus (2003). Bossaerts et al. (2010), Cao, Wang, and Zhang (2005), Dow and Werlang (1992), Easley and O'Hara (2009), Epstein and Schneider (2010), Garlappi, Uppal, and Wang (2007), and Peijnenburg (2014) show that ambiguity aversion may cause investors not to invest or to reduce the faction of investment in stocks, if investors find stock returns not only risky but also ambiguous.

return with respect to the worst-case probability distribution and concentrate their wealth on assets with relatively less ambiguity, albeit riskier[6].

Investors may have different degrees of ambiguity over different assets as they may have access to different information about different assets. Inside investors of a firm, such as CEOs or family owners, have access to more information about company fundamentals and can make more accurate inferences about the probability distributions of their firms. On the other hand, they do not possess an information advantage over outside firms and thus have more ambiguity for these investment choices.

However, models with ambiguity about expected return can not *quantitatively* explain the concentrated ownership of family owners. We find that ambiguity regarding return volatility plays a vital role in quantitatively explaining the concentrated ownership decision. Table A.1 reports the calibration of the share of wealth invested in the family firm and the diversified portfolio with different degrees of ambiguity about expected return and return volatility. When only expected returns are ambiguous, then even if the ambiguity about the expected return of the diversified portfolio is so high that the family owners think the worst-case expected return of a single stock is 150% of that of the diversified portfolio, they will only invest 6% of their wealth in the single stock. Thus, although the model with ambiguous expected return can *qualitatively* explain the under-

[6] Most models assume that stock returns follow normal distributions with ambiguous mean but unique and known return volatility, partly justified by the "mean blur" (Luenberger 2008). That is, it is much more difficult to estimate the mean than the volatility. However, recent studies highlight the difficulties in empirically estimating return volatility dynamics (Bollerslev et al., 2012; Drechsler, 2013). In addition, if the volatilities of Brownian motions are ambiguous, then the linear expectation and Ito's lemma do not hold anymore, so the conventional portfolio-choice and asset pricing models can not be easily extended in the case of ambiguous return volatility. Peng (2006) establishes the theoretical foundation of the nonlinear expectation (G-Expectation) associated with G-Brownian Motion (Brownian Motion with ambiguous volatility) and formulates the stochastic calculus of Itô's type for G-Brownian motion. Epstein and Ji (2013) apply G-expectation and stochastic calculus of Peng (2006) to study a continuous-time asset pricing model with ambiguity about volatility and drift.

diversification puzzles of investors, the model can hardly explain the observed family owners' to invest about 70% of their wealth in a single stock versus a well-diversified portfolio[7].

On the other hand, if there is ambiguity about return volatility, then with moderate ambiguity such that the family owners think the worst-case volatility of a single firm is the same as that of the diversified portfolio, they will invest 50% of their wealth in the single firm. Thus, the concern for return-volatility ambiguity significantly impacts the ownership level than concern for expected-returns ambiguity.

Our model builds on the MMEU framework and does not separate ambiguity and ambiguity aversion. Klibanoff et al. (2005) show that the maxmin preference model is a limiting case of the smooth recursive preferences when the degree of ambiguity aversion goes to infinity.[8] This framework helps study the economic equilibrium impact of changing ambiguity aversion while holding ambiguity constant or vice versa. However, due to data limitations, neither ambiguity aversion nor ambiguity is easily observed or measured, and variations in ambiguity aversion and ambiguity are often observationally equivalent. Hence, we adopt the MMEU framework, which allows us to solve the model explicitly and derive rich testable implications[9].

The most important insight of our model is that family owners optimally choose to invest more in the family firm or are likely to exit when the information they possess helps reduce the ambiguity about the family firm relative to other firms. Due to the very nature of ambiguity, it is challenging to measure ambiguity about the family firm relative to other firms. Motivated by Epstein and Schneider (2008), we model the ambiguity about returns to learnable and exogenous components.

[7] Prior empirical research indicates that family owners, on average, hold about 25% of the firm's outstanding equity, equating to more than 80% of their wealth invested in the firm.

[8] Both approaches have widely varying implications for optimal portfolio allocation, Abdellaoui et al. (2011), Bossaerts et al. (2010), Hayashi and Wada (2010) and Dimmock et al. (2016) examine the effect of uncertainty on portfolio choice via experiments. Peijnenburg (2014) shows that the maxmin approach better matches the data. Ahn et al. (2014) explicitly compare the maxmin preferences and smooth preference via a portfolio choice experiment to explore which describes actual behavior and find evidence in favor of a kinked specification.

[9] Our model implications are consistent with the numerical simulation results of Klibanoff et al. (2005).

Family owners can use their information about family firms to reduce ambiguity about the learnable but not the exogenous components. On the other hand, industries differ in the relative importance of two components of ambiguity. For example, industries with more innovation, more competitor entry, lower barriers to entry, more entrants, and more dispersed analyst-forecast errors are likely to have more important exogenous ambiguity. Thus, the private information possessed by family owners is less useful in these industries.

Using a large sample of U.S. family firms in Russell 3000 from 2001 through 2010, we examine the model's predictions via two testing procedures. First, in industries with less innovation, less competitor entry, higher barriers to entry, and less dispersed analyst-forecast errors, that is, less "exogenous" ambiguity, private information is more useful for family owners to reduce their ambiguity about family firm relative to other firms. Our empirical results support our model predictions that families hold larger equity stakes in industries with less exogeneous ambiguity.

Second, we test the model implications on family exit decisions using a hazard or survival model. Family-exit decisions describe the situation where the family consciously chooses to move from a single-firm portfolio to investing in a diversified portfolio of firms. Our model predicts that families should be less likely to sell-off their ownership stakes (exit the firm) with (i) greater tenure in the firm, (ii) greater absolute levels of wealth invested in the firm, and (iii) greater (absolute) risk-aversion, all relative to the average family owners. These predictions seem in sharp contrast to the adverse selection story in Leland and Pyle (1977), where the family incrementally sells down their stakes to obtain a diversified portfolio.

In addition, the model predicts that family exits should be positively related to the ambiguity about future earnings[10] instead of risk conventionally measured by the earnings variance. Again, this

[10] We propose to use kurtosis of analyst forecast errors

prediction provides a sharp distinction with the traditional adverse selection framework for evaluating family ownership, implying that earnings variance rather than ambiguity about earnings is positively related to family exits (e.g., Villalonga and Amit, 2010).

To investigate the relationship between the family's decision to exit the firm and ambiguity, we examine 100 family exits relative to a propensity-score-matched sample of not-exiting family firms. We test whether ambiguity surrounding the firm's information environment affects the family's decision to sell off their ownership stake. Strikingly, we find that families are less likely to exit their ownership stakes as family wealth increases, family experience in the firm increases, and family risk-aversion increases. The results are statistically and economically significant, supporting the implications of our model versus that of models without ambiguity.

We instrument for family risk aversion using representation by female family members on the board of directors and/or as large shareholders.[11] Our analysis suggests that families are significantly less likely to exit the firm when more female family members remain as board members and/or large shareholders. Further tests indicate that family owners are less likely to exit the firm when family members serve as CEO (relative to external CEOs), suggesting family members face fewer information constraints (less ambiguity) when actively running the firm. The analysis supports the model's predictions, suggesting that family owners are more likely to retain their ownership stakes when facing less ambiguity about the firm's prospects.

Our analysis provides an alternative explanation for why family owners defy conventional finance wisdom by maintaining large, concentrated stakes in a single firm. Yet, family blockholders

[11] Croson and Gneezy (2009) and Bertrand (2011) provide comprehensive surveys on the literature on differences in risk aversion between women and men. Levin et al. (1988) and Bruce and Johnson (1994) provide evidence based from betting that women exhibit greater risk aversion than men. Barsky et al. (1997) document that women actively report lower risk propensities than men. Jianakaoplos and Bernasek (1998) and Sundén and Surette (1998) document that women are significantly more risk-averse in their allocation of wealth than men of equal economic status. Borghans et al. (2009) provide experimental evidence of greater risk aversion in women relative to men, while any ambiguity aversion differences appear relatively inconsequential.

are not the only type of investor with large shareholdings. Financial intermediaries also often hold large equity stakes in firms. Although some financial intermediaries, such as mutual funds and hedge funds, typically hold well-diversified portfolios, others, such as private equity funds and venture capitalists, hold less well-diversified portfolios relative to mutual funds but still frequently hold 20 or more firms in their asset mix (Robinson and Sensoy, 2013). This model does not directly link to the holdings of financial intermediaries. However, one could conceivably use this type of analysis to investigate the limited diversification found in venture capital and private equity firms relative to mutual funds.

Our study makes two significant contributions. First, our analysis contributes to the growing family firm literature by explaining family owners' decisions to maintain ownership and control in countries with well-developed financial markets and legal regimes. Prior literature posits that family ownership arises as a response to markets or legal systems offering weak protection of shareholder rights, suggesting that family shareholders must make a trade-off between their monitoring of the firm (or signaling, commitment, asset protection, etc.) and diversifying their wealth across a broad basket of assets. Markets in the U.S., however, offer strong shareholder protections, indicating that family owners need not make the trade-off between concentrated equity stakes and holding a diversified portfolio (Burkart et al., 2003). Yet, we observe that family shareholders continue to hold concentrated stakes in nearly one-half of U.S. firms. Our analysis indicates that founding family control in economies with well-developed equity markets need not rely on expropriation arguments but instead can represent the benefit of exploiting information advantage to reduce uncertainty. Furthermore, our model predicts that family firms are more likely to persist in the industries where the information advantage is more valuable to reduce uncertainty about the family firm relative to other firms, and the family firms are more likely to persist if their information advantage is easier to pass on from one generation to the next, or can be amplified through managing experience, all of which are consistent with the empirical evidence in the prior literature on family firms in the U.S.

Second, our study joins the growing literature on the continued control by founding families of publicly traded firms in well-developed economies and the vast literature on the economic decisions under ambiguity. By incorporating ambiguity about return volatility and the information advantage of family owners, we generate substantial insights about how the decision-makers use the specialized knowledge. We find that the benefit of removing uncertainty by exploiting the information advantage the family owners possess justifies their concentrated ownership. Barillas, Hansen, and Sargent (2009) estimate the benefits of removing aggregate uncertainty based on aggregate consumption, while we provide evidence of the benefits of reducing relative uncertainty based on firm-level actual investment decisions.

The rest of this paper is organized as follows. Section 2 presents the optimal portfolio-choice model with ambiguity about return volatility and testable implications of the model. Section 3 discusses the data and construction of primary variables. Section 4 provides empirical results, and Section 5 summarizes and concludes the paper.

2. The model

Our model is motivated by the quantitative aspect of the puzzle: why family owners invest a large proportion of their wealth in a single firm and what drives their decision to exit from the family firm and invest in a diversified portfolio. We seek to answer these questions in a model of investment decision with ambiguity-averse investors (family owners) who choose between investing in two risky assets with different degrees of ambiguity about expected return and return volatility, namely, a single stock (family firm) and a diversified portfolio of other equities.

Without loss of generality, we assume the choice set of family owners contains two risky assets with different degrees of ambiguity and no risk-free asset[12]. We obtain similar implications on the relationship between the family ownership and characteristics of the family owner and the family firm in a model with two risky assets and a government bond.[13]

2.1 The portfolio choice with no risk-free asset and with uncertainty

We consider a static, discrete-time economy with two risky assets. Asset 1 refers to the stock of a single firm (family firm), and asset 2 is a diversified portfolio. The return on each risky asset is[14]

$$r_i = e_i + \varepsilon_i, \;\; for\; i = 1, 2 \tag{1}$$

where e_i stems from the earnings and other learnable fundamentals of firm operations by those with inside information, and ε_i stems from external forces that no one can understand or learn, that is, "exogenous ambiguity." We assume that the learnable fundamentals e_i and the noises ε_i are independent of each other, and all follow normal distributions so that the returns on both risky assets r_i, also follow normal distributions and are independent of each other.[15] The actual values of the mean and volatility of return on both assets are denoted as $(\bar{r}_{i,0}, \sigma_{i,0})$, for i =1, 2.

2.1.1 The portfolio choice without uncertainty

In the classic Markowitz (1952) mean-variance analysis, investors are assumed to make decisions with knowledge about the actual value of expected return and risk $(\bar{r}_{i,0}, \sigma_{i,0})$ for $i=1,2$. For investors with constant absolute risk aversion (CARA) utility for wealth and risk aversion coefficient

12 From a practical perspective, investors rarely find a risk-free asset. Table A.2 shows the average return and volatility of treasury bonds versus that of the CRSP market index from 1995 to 2017. The volatility of the 30-day treasury bills is 2.11%, significantly different from zero, and the volatility of 30-year bonds is 17.61%, close to that of the CRSP stock market index (20.19%).

13 See the Internet Appendix for details.

14 This setup is motivated by Epstein and Schneider (2008)

15 We maintain the normality assumption so that means and variances are sufficient statistics of the distributions. The assumption of independence is not essential to our results, with the main results continuing to hold if we allow for a correlation between returns on the family firm and the diversified portfolio.

$\gamma > 0$, they choose the share (α) invested in asset 1 (single firm) to maximize expected utility in each period t,

$$\max_{0\le\alpha\le 1} E_t[-exp(\gamma W_{t+1})] \tag{2}$$
$$s.t\ W_{t+1} = W_t\left[\alpha r_{1,t+1} + (1-\alpha) r_{2,t+1}\right]$$

where W_t is the wealth of the investor given at time t. It is straightforward to show that problem (2) is equivalent to the following optimization problem,

$$\max_{0\le\alpha\le 1}\left[\alpha\bar{r}_{1,0} + (1-\alpha)\bar{r}_{2,0} - \frac{\gamma W_t}{2}\left(\alpha^2\sigma_{1,0}^2 + (1-\alpha)^2\sigma_{2,0}^2\right)\right] \tag{3}$$

and the solution to this problem is,

$$\alpha^* = \begin{cases} 1 & if\ \bar{r}_{1,0} - \bar{r}_{2,0} \ge \gamma W_t \sigma_{1,0}^2 \\ \frac{\bar{r}_{1,0}-\bar{r}_{2,0}}{\gamma W_t(\sigma_{1,0}^2+\sigma_{2,0}^2)} + \frac{\sigma_{2,0}^2}{(\sigma_{1,0}^2+\sigma_{2,0}^2)} & if\ -\gamma W_t\sigma_{2,0}^2 < \bar{r}_{1,0} - \bar{r}_{2,0} < \gamma W_t\sigma_{1,0}^2 \\ 0 & if\ \bar{r}_{1,0} - \bar{r}_{2,0} \le -\gamma W_t\sigma_{2,0}^2 \end{cases} \tag{4}$$

We are interested in the scenario where the two risky assets have identical expected returns ($\bar{r}_{1,0} = \bar{r}_{2,0}$), but different risks ($\sigma_{1,0} > \sigma_{2,0}$), the volatility of the diversified portfolio is less than that of a single stock, then we obtain the diversification result of Markowitz (1952),

$$\alpha^* = \frac{\sigma_{2,0}^2}{(\sigma_{1,0}^2 + \sigma_{2,0}^2)}$$

The family chooses the minimum variance portfolio and invests more in the asset with relatively lower risk, i.e., the diversified portfolio. Thus, the family owners' decision to retain a large portion of their wealth in the higher-risk family firm raises a puzzling question. In Appendix 1, we calibrate our model to show the implied share of the wealth an individual should invest in a single firm when the single firm and the diversified portfolio exhibit the same risk – about 3% without ambiguity. However, even if the expected return of a single firm is 150% of that of the diversified portfolio, the investor without ambiguity aversion still chooses to invest only 6% of her wealth in a single stock. Thus, a family's decision to invest large stakes in a single firm becomes difficult to justify without uncertainty.

Previous literature on family firms rationalizes the concentrated ownership of family firms by introducing agency costs, private benefits of ownership, and other market frictions in a model without uncertainty aversion. We compare in detail the implications of these models with our model in Section 2.4.

2.1.2 The portfolio choice with uncertainty

In financial markets, investors do not observe the true value of expected return and volatility and rely on other data to estimate these values. In a general model with ambiguity, investors are uncertain about the distribution of the asset returns. With the normality assumption of the distribution, it is sufficient to assume that investors are uncertain about the mean and standard deviation of returns. We assume investors believe that the expected return of each asset *i* draws from a set of *N* possible values $R_i = \{\bar{r}_{i,n}: n = 1,2,\dots,N\}$, which contains the true value of the expected return ($\bar{r}_{i,0}$), for *i =1,2*. That is,

$$\bar{r}_{i,min} \leq \bar{r}_{i,0} \leq \bar{r}_{i,max},$$

where $\bar{r}_{i,min} \equiv \min_{n=1,2,\dots,N} \bar{r}_{i,n}$, and $\bar{r}_{i,max} \equiv \max_{n=1,2,\dots,N} \bar{r}_{i,n}$, for *i* = 1, 2. In addition, we assume the investors believe that the standard deviation of the returns on asset *i* come from a set of *M* possible values, $\Sigma_i = \{\sigma_{i,m}: m = 1,2,\dots,M\}$, which contains the true values of standard deviation ($\sigma_{i,0}$), for *i = 1,2*. That is,

$$\sigma_{i,min} \leq \sigma_{i,0} \leq \sigma_{i,max},$$

where $\sigma_{i,min} \equiv \min_{m=1,2,\dots,M} \sigma_{i,m}$, and $\sigma_{i,max} \equiv \max_{m=1,2,\dots,M} \sigma_{i,m}$, for *i* = 1, 2.

We model ambiguity aversion in the maxmin expected utility (MEU) framework proposed by Gilboa and Schmeidler (1989). Investors care about the uncertainty regarding the distribution of returns and choose the optimal portfolio to maximize their utility in the worst-case scenario. There is no differentiation between ambiguity and aversion to ambiguity in this framework. We allow the range of potential expected returns and volatility to capture both the ambiguity about the return and the ambiguity aversion of the investors. Naturally, investors who are more ambiguity averse or face more uncertainty will exhibit concerns about wider ranges of potential expected returns and volatility.[16] Klibanoff et al. (2005)'s "smooth ambiguity aversion" framework allows a separation between ambiguity (the level of uncertainty) and ambiguity aversion (attitude with respect to ambiguity), and they show that the maxmin-preference model is a limiting case of the smooth recursive preferences when the degree of ambiguity aversion trends to infinity.[17] This framework is useful in studying the economic equilibrium impact of changing ambiguity aversion while holding ambiguity constant or vice versa. However, due to data limitations, neither ambiguity aversion nor ambiguity is easily observed or measured, and variations in ambiguity aversion and ambiguity are often observationally equivalent. Hence, we adopt the MMEU framework, which allows us to solve the optimal portfolio choice of the family owners analytically and formally derive testable implications through comparative statics.

We further assume that the family owners possess private information regarding the fundamentals of the family firm that allows them to reduce the ambiguity about the fundamentals of their firm over time and through experience. However, like other outside investors, the family owners do not possess material private information about other firms in the diversified portfolio as they do not manage these other firms. In our setup, the reduction in the ambiguity regarding the family firm relative to the diversified portfolio is captured by the shrinkage in the distance between the worst-case and the true values of expected return and volatility. Hence, for the family owners who can exploit private information to reduce the ambiguity regarding the family firm, the

16 In this optimal portfolio choice problem within the maxmin framework, it is the worst-case, risk-return tradeoff that measures ambiguity and ambiguity aversion.

17 Both approaches have widely varying implications for optimal portfolio allocation, Abdellaoui et al. (2010), Bossaerts et al. (2010) Hyayshi and Wada (2010) and Dimmock et al. (2013) examine the effect of uncertainty on portfolio choice via experiments. Peijnenburg (2014) shows that the maxmin approach better matches the data. Ahn et al. (2014) explicitly compare the maxmin preferences and smooth preference via a portfolio choice experiment to explore which describes actual behavior and find evidence in favor of a kinked specification.

uncertainty range of the expected return and volatility of family firm is tighter than that of the diversified portfolio, i.e. $\bar{r}_{2,max} - \bar{r}_{2,min} > \bar{r}_{1,max} - \bar{r}_{1,min}$ and $\sigma_{2,max} - \sigma_{2,min} > \sigma_{1,max} - \sigma_{1,min}$.

For each period *t*, the portfolio choice problem of the ambiguity-averse family owners with CARA utility can be formulated as

$$\max_{\alpha} \min_{\bar{r}_{1,n_1},\bar{r}_{2,n_2},\sigma_{1,m_1},\sigma_{2,m_2}} \left\{\left[\alpha\bar{r}_{1,n_1} + (1-\alpha)\bar{r}_{2,n_2}\right] - \frac{\gamma W_t}{2}\left[\alpha^2\sigma_{1,m_1}^2 + (1-\alpha)^2\sigma_{2,m_2}^2\right]\right\} \quad (5)$$

Let's first examine the minimization problem of (5). Since (5) is monotonically decreasing in σ_1^2 and σ_2^2 the maximum possible standard deviation is always chosen, that is,

$$\sigma_1^* = \sigma_{1,max} \equiv \max_{m=1,2,..,M}\{\sigma_{1,m}\}$$

$$\sigma_2^* = \sigma_{2,max} \equiv \max_{m=1,2,..,M}\{\sigma_{2,m}\}$$

If the family is long in both the family firm and the diversified portfolio ($0 < \alpha < 1$), then the minimum possible mean returns are chosen for both assets. If the family shorts the diversified portfolio ($\alpha \geq 1$), then the maximum possible mean return of the diversified portfolio and minimum possible mean return of the family stock is chosen, and vice versa, that is,

$$(\bar{r}_1^*, \bar{r}_2^*) = \begin{cases} (\bar{r}_{1,min}, \bar{r}_{2,max}), & if\ \alpha \geq 1 \\ (\bar{r}_{1,min}, \bar{r}_{2,min}), & if\ 0 < \alpha < 1 \\ (\bar{r}_{1,max}, \bar{r}_{2,min}), & if\ \alpha \leq 0 \end{cases}$$

Given the solution to the minimization problem, the optimal share invested in the family business (α^*) can be then characterized as follows,

$$\alpha^* = \begin{cases} 1 & if\ \bar{r}_{1,min} - \bar{r}_{2,min} \geq \gamma W_t \sigma_{1,max}^2 \\ \frac{\bar{r}_{1,min} - \bar{r}_{2,min}}{\gamma W_t(\sigma_{1,max}^2 + \sigma_{2,max}^2)} + \frac{\sigma_{2,max}^2}{(\sigma_{1,max}^2 + \sigma_{2,max}^2)} & if\ -\gamma W_t \sigma_{2,max}^2 < \bar{r}_{1,min} - \bar{r}_{2,min} < \gamma W_t \sigma_{1,max}^2 \\ 0 & if\ \bar{r}_{1,min} - \bar{r}_{2,min} \leq -\gamma W_t \sigma_{2,max}^2 \end{cases} \quad (6)$$

Comparing the optimal portfolio in the model with uncertainty (6) and without uncertainty (4), it is straightforward that the ambiguity-averse investor makes the decision based on the worst-case expected return and variance; while the naïve investor without concern for uncertainty makes decisions based on the true value of the expected return and variance.

We focus on the scenario where the two risky assets have identical expected returns ($\bar{r}_{1,0} = \bar{r}_{2,0}$), but different risks ($\sigma_{1,0} > \sigma_{2,0}$) and the true value of the volatility of the diversified portfolio is less than that of a single stock. Hence, if ambiguity about the expected return of the family firm is less (more) than that of the diversified portfolio, then $\bar{r}_{1,min} > (<)\, \bar{r}_{2,min}$. In the model without uncertainty, the investor would choose to invest more in the diversified portfolio. In contrast, in the model with uncertainty, it depends on the relative ambiguity of the two risky assets. Note that due to "mean blur" (Luenberger, 2008), it is much more difficult to estimate mean than volatility (risk) and harder for family owners to use private information to reduce the uncertainty regarding the expected return than the uncertainty regarding the risk. Hence it is reasonable to assume that $\bar{r}_{2,min}$ is close to $\bar{r}_{1,min}$ if $\bar{r}_{1,0} = \bar{r}_{2,0}$. Even if the true value of the risk of the diversified portfolio is smaller than that of the family firm ($\sigma_{2,0} < \sigma_{1,0}$), the family owners may be able to exploit their private information to reduce the uncertainty regarding the risk of the family firm such that $\sigma_{2,max} > \sigma_{1,max}$. If the information possessed by the family owners allows them to reduce the ambiguity about the family firm, then the family owners would invest more in the single firm. That is, if

$$\bar{r}_{1,min} - \bar{r}_{2,min} > \gamma W_t(\sigma_{1,max}^2 - \sigma_{2,max}^2),$$

then the share of wealth invested in the family firm (α*) is larger than 1/2. If the relative ambiguity of the family firm is reduced even further, such that

$$\bar{r}_{1,min} - \bar{r}_{2,min} > \gamma W_t \sigma_{1,max}^2,$$

then the family owners would invest 100% of their wealth in the family firm.[18]

We compare the choice sets of investors with and without uncertainty in Figure 1. Note that if the family owners believe the minimum expected return of the family firm is larger than that of a

18 It is worth noting that our model offers a rationale for non-diversification of corporate insiders that have a choice of risky assets with different ambiguities regarding expected return and risk. But it may not apply for professional investors such as venture capitalists, hedge fund managers, and mutual fund managers, as they understand the diversified portfolio and a single firm equally well and there is no large variation in the ambiguities regarding the expected returns of different risky assets. Furthermore, professional investors do not choose to hold a concentrated portfolio whereas family owners make a choice for a concentrated portfolio.

diversified portfolio, that is, $\bar{r}_{1,min} > \bar{r}_{2,min}$, then the family owners would always hold a positive share of their wealth in the family firm unless the risk or ambiguity about the risk of the family firm gets large enough ($\sigma_{2,max} \rightarrow +\infty$), we refer to this case as the "non-exiting family". On the other hand, if $\bar{r}_{1,min} < \bar{r}_{2,min}$, then the family owners would consider exiting when their risk aversion, wealth, and uncertainty about the expected return or risk of the family firm changes. We refer to this case as the "exiting family." Panel A of Figure 1 illustrates the scenario for non-exiting family owners ($\bar{r}_{1,min} > \bar{r}_{2,min}$). For investors without ambiguity aversion, the choice set is represented by (F_0, D_0), where F_0 represents the family firm and D_0 represents the diversified portfolio. In our calibration, the family owners would hold merely 3% of their wealth in the family firm (F_0). For a family with ambiguity aversion, but only with respect to expected return, the choice set is represented by (D_B, F_B). In our calibration, the family owners would hold just 6% of their wealth in the family firm (F_B) if $\bar{r}_{1,min} = 1.5\bar{r}_{2,min}$. For the family with ambiguity aversion concerning expected return and risk (volatility of return), the choice set is represented by (D_A, F_A). In our calibration, the family owners would hold over 80% of their wealth in the family firm (F_A) if $\bar{r}_{1,min} = 1.5\bar{r}_{2,min}$ and $\sigma_{1,max} = 0.5\sigma_{2,max}$.

Panel A in Figure 1 shows that a more risk-averse investor would hold less in the family firm than a less risk-averse investor. A simple comparative statics analysis also implies that for the non-exiting family owners, the share of the wealth invested in the family business is a decreasing function of the risk or the relative ambiguity of the family firm, and a decreasing function of risk aversion and family wealth. Furthermore, the more experience the family has with the firm, the better they understand the family business, hence the less ambiguity about the expected return and the more investment in the family firm[19]. The above analysis thus leads to the first set of hypotheses in our empirical tests as follows:

[19] Klibanoff et al. (2005) obtain similar predictions with numerical comparative statics. Specifically, they find that increases in ambiguity aversion (relative ambiguity in our context) leads to decrease in the share of wealth invested in the ambiguous asset.

Hypothesis 1: The family invests more in their own business in industries where their private information is more useful to reduce ambiguity about their firms relative to other firms.

Due to the very nature of ambiguity, it is challenging to measure ambiguity about the family firm relative to other firms. As we model the ambiguity about returns to learnable and exogenous components. Family owners can use their information about family firms to reduce ambiguity about the learnable but not the exogenous components. On the other hand, industries differ in the relative importance of two components of ambiguity. For example, industries with more innovation, more competitor entry, lower barriers to entry, more entrants, and more dispersed analyst-forecast errors are likely to have more important exogenous ambiguity. Thus, the private information possessed by family owners is less useful in these industries.

Table 2 reports the percentage of family firms and the average family ownership of family firms in each of the Fama-French 48 industry groups. Consistent with our model prediction, in industries with less innovation or more branding, such as, Textiles, Beer & Liquor, Real Estate, Candy & Soda, Printing and Publishing, Communication, Apparel, and Entertainment industry groups, more than 50% are family firms, and the average family ownership is more than 20%[20]; in contrast, in industries with more innovation or less branding, such as Electrical Equipment, Computers Business Supplies, Electronic Equipment, Healthcare, Medical Equipment, Measuring and Control Equipment, Pharmaceutical Products industries groups, less than 30% are family firms, and the average family ownership is, around 6%.

Hence in our empirical tests, we develop proxies such as industry innovation, branding, and entry barriers to measure the exogenous ambiguity of industries in which family firms operate. Although our proxies for industry ambiguity likely capture both a component of industry risk and industry ambiguity, our model implications on the relationship between the industry (exogenous)

[20] Except for Texile industry group with family ownership of 13.9%.

ambiguity and family ownership are in sharp contrast to the alternative models without ambiguity. Hence our tests can effectively differentiate our model against models without ambiguity.

Hypothesis 1.a: The family concentrates more on the industries with less innovation

Hypothesis 1.b: The family concentrates more on the industries with more branding

Hypothesis 1.c: The family concentrates more on the industries with fewer entrants

Hypothesis 1.d: The family concentrates more on the industries with a higher entry barrier

Hypothesis 1.e: The family concentrates more on the industries with less dispersion of analyst forecast errors

In the following subsection, we examine the model implications on the exit decision of family owners.

2.2. The family's exit decision

One of the critical insights of our model is that the family exit decision depends on the ambiguity regarding family firm stock returns relative to the diversified portfolio. First, when the exogenous ambiguity increases so that private information possessed by the family no longer helps them reduce the ambiguity about their firm, the family will exit. More specifically, if the worst-case expected return of the family firm is smaller or the worst-case return volatility of the family firm becomes much larger than that of the diversified portfolio, then the family would consider exiting. Hence we have the following testable implication regarding the family-exit decision,

Hypothesis 2.1: The family is more likely to exit in industries where it is more difficult to use private information to reduce the ambiguity about family firm relative to other firms.

Epstein and Schneider (2007) show that investors can reduce the ambiguity through Bayesian updating, implying in our model that family owners can use their information about the firm's fundamentals to reduce the relative ambiguity about their firm. Moreover, the longer the family's experience with the firm, the less ambiguity about the firm, hence the larger is the family ownership stake. Thus, our model predicts that family owners with longer tenure in the firm are less likely to exit the firm[21].

Hypothesis 2.2: The family is more likely to exit when the family is younger.

Next, solution (6) implies that the necessary conditions for the family to exit are,

$$\bar{r}_{1,min} - \bar{r}_{2,min} \leq -\gamma W_t \sigma_{2,max}^2 \tag{7.a}$$

$$\text{or } \sigma_{1,max}/\sigma_{2,max} \to \infty \tag{7.b}$$

which implies two conditions under which family owners would reduce their holding in the family firm to zero. First, the uncertainty about the expected return of the family firm increases, such that family owners' perception of the worst-case expected return of the family firm is lower than that of a diversified portfolio. Second, the uncertainty about the return volatilities of the family firm increases dramatically, such that family owners' perception of the worst-case return volatility of the family firm is extremely larger than that of a diversified portfolio.

Under condition (7.a), the optimal investment share in the family firm is,

$$\alpha^* = \frac{\overbrace{\bar{r}_{1,min} - \bar{r}_{2,min}}^{<0}}{\gamma W_t (\sigma_{1,max}^2 + \sigma_{2,max}^2)} + \frac{\sigma_{2,max}^2}{(\sigma_{1,max}^2 + \sigma_{2,max}^2)}$$

A simple comparative statics analysis implies that the family owners are *more* likely to exit as their risk aversion and wealth *decrease*. Note that this prediction arises when unexpected shocks enlarge the uncertainty about the family firms, such that the worst-case expected return of the family firm is smaller than that of a diversified portfolio. In this case, the family firm appears to the family owner as an asset with a lower expected return and possibly lower risk. Hence the more risk-averse family owners invest more instead of less in the family firm. Thus, family owners are more likely to

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[21] Prior empirical studies find that family owners have longer horizons than non-family firms. Anderson and Reeb (2003) posit that the long horizon of family owners leads them to make better investment choices. However, the mechanism for this longer horizon is typically not addressed in the literature. Our model provides important insights on family owners horizons in this context.

exit if they are less risk averse. This prediction sharply contradicts the implications of models without ambiguity but based on agency costs or overconfidence that less risk averse families are less likely to exit.

Panel B in Figure 1 illustrates the investment decision under condition (7.a) where family owners may consider exiting. In the case of both expected returns and volatilities are ambiguous, the choice set is represented by (D_E, F_E). A risk-neutral family would invest 100% of their wealth in the diversified portfolio (D_E). The optimal portfolio of a risk-averse investor is represented by E, which is between D_E and F_E. By comparing the optimal portfolio choice of risk-neutral and risk-averse investors, we find that in this case, the ownership of the family firm increases with risk aversion, that is, the family is more likely to exit when they are *less* risk averse.[22]

Hence we have developed two more hypotheses for the exiting families,

Hypothesis 2.3: The family is more likely to exit when the family owners are less risk averse;

Hypothesis 2.4: The family is more likely to exit when the family owners are less wealthy;

It is worth noting that Hypothesis 2.3-2.4 regarding the risk aversion and family wealth are sensitive to the relative worst-case expected return of the family firm and diversified portfolio. On the one hand, if $\bar{r}_{1,min} > \bar{r}_{2,min}$ or the worst-case expected return of the family firm is larger than that of the diversified portfolio, then the family would not consider exiting, and would invest more in the family firm as their risk aversion or wealth decreases. The family firm offers a better risk-return tradeoff in the worst-case scenario, as family owners have more information to reduce the ambiguity of the family firm relative to the diversified portfolio. On the other hand, if the uncertainty regarding the family firms increases such that the worst-case expected return of the

[22] The standard agency model implies negative relation between (relative) risk aversion and ownership (Bitler et al. 2005). Faccio et al. (2011) emphasize how family owners may seek diversification through firm-level investments.

family firm is smaller than that of the diversified portfolio, that is, under condition (7.a),

$$\bar{r}_{1,min} < \bar{r}_{2,min}$$

then the family firm appears to be an asset with less expected return and possibly lower risk than a diversified portfolio. A decrease in risk aversion would make the family firm less attractive to the family owner. The family owner would invest less in the family firm until they exit completely when their risk aversion or wealth decreases. If the worst-case expected return of the family firm is the same as that of the diversified portfolio, then the share invested in the family is independent of risk aversion and family wealth.

Klibanoff et al. (2005) obtain the same results through numerical comparative statics in an example where the investor with smooth ambiguity aversion chooses between a risky asset and an ambiguous asset. They show that the impact of ambiguity aversion on the ratio of holdings of a riskier asset to a more ambiguous asset is unchanged regardless of the relationship between the expected return, but results regarding the risk aversion are sensitive to the relationship between the worst-case expected return in our model.

2.3. Compare with extant theory on the predictions of the family's exit decision

Extant theories build on agency costs, market frictions, private or non-pecuniary benefits to explain the concentrated ownership of family firms. Leland and Pyle (1977) argue that family owners hold large stakes to signal firm quality. DeMarzo and Urusevic (2006) highlight the trade-off between agency considerations and diversification in the family exit decision. Shleifer and Vishny (1986) rely on private payoffs to justify foregoing the benefits of diversification. Morck and Yeung (2003) and Roussanov (2010) indicate that family owners could receive non-pecuniary benefits, thus explaining their decision to hold a large, concentrated stake.

One of the common themes of these extant theories is that benefits from other channels compensate the cost of concentrated ownership for the risk-averse investor. In a classical investment model without ambiguity, costs of concentrated ownership increase with risk aversion and family wealth, so the family is more likely to exit if the family is more risk averse and wealthier. These predictions are in sharp contrast to Hypothesis 2.3-2.4 of our model.

Another popular explanation of the under-diversification puzzle of corporate insiders is overconfidence. Odean (1988) shows that overconfident investors who believe that the precision of their knowledge about the value of a security is greater than it is, hold riskier portfolios than rational investors with the same degree of risk aversion. However, the overconfident investors hold unrealistic beliefs about how high their returns will be and how precisely these can be estimated, unlike the ambiguity-averse investor in our model who rationally improves the precision of their estimates using relevant information. The models build on overconfidence to explain the under diversification, thus predict the family owners are more like to exit if the family is more risk averse or wealthier. These predictions are again in sharp contrast to the hypotheses of our model.

Furthermore, the conventional wisdom on family ownership suggests that the family will exit the firm as soon as possible after going public to obtain diversification benefits (Admati et al., 1994). Zingales (1995) models the family exit decision, suggesting the family continuously reduces their stake in the firm till they only hold a controlling stake, which they then sell at a premium. Similarly, Mello and Parsons (1998) describe family exits as sequential processes, sell small stakes to atomistic shareholders, and sell the controlling stake to an active investor at a higher price. Each of these settings implies that family experience (e.g., tenure or time) in the firm is positively related to the decision to sell their control stake and exit the firm. Hypothesis 2.2 of our model implies just the opposite.

Due to the very nature of ambiguity, it is neither directly observed nor easily differentiated from the conventional measure of risk in the data. However, the testable hypotheses obtained through comparative statics analysis allow us to test the model predictions by partitioning firms based on the easiness for family owners to exploit their private information to reduce ambiguity about the family firm. Furthermore, some of the hypotheses from our model are in sharp contrast to the predictions of models without ambiguity, so tests of the null hypothesis against the alternatives can effectively evaluate models. The next section discusses empirical analysis methods based on the matching samples to test the two sets of the hypothesis of our model.

3. Data and Primary Variable Measurement

In this section, we explore the relevance or applicability of our model by considering its empirical implications. Our model characterizes the effect of ambiguity, family experience, wealth, and risk aversion on the family's decision to hold a stake in a single firm versus holding a diversified portfolio.

To test our model, we use the 2,000 largest industrial firms in the U.S. with a fiscal year-end in 2001. To collect the sample, we pull all firms from CompuStat for data-year 2001 with information available for total assets. We exclude public utilities (SIC codes 4812, 4813, and 4911 through 4991), financial firms (SIC codes 6020 through 6799), firms listed as master limited partnerships (21-firms), foreign firms, and firms with a share price less than $0.25. These firms are excluded to make our work comparable to previous literature[23] on family ownership and because government regulation potentially affects firm ownership structure, corporate transparency, and performance.

We manually collect family-ownership data from corporate proxy statements and 10-k's from 2001 through 2010, including ownership level, dual-class share structures, voting power, and CEO type. Corporate histories (i.e., family lineage) are garnered from ReferenceforBusiness.com, FundingUniverse.com, and individual company websites.[24] To control for survivorship bias, we allow firms to exit and re-enter the sample. Our initial sample consists of the largest 2,000 non-

[23] For example, Anderson, Duru, and Reeb (2009), Anderson, Reeb and Zhao (2012) examine the largest 2000 non-financial and non-utility industrial firms, which are in Russell 3000 index. The Russell 3000 index tracks the performance of the 3,000 largest U.S.-traded stocks, which represent about 97% of all U.S.-incorporated equity securities.

[24] In designating family firms, we do not include shares held by charitable foundations as part of the family holdings. Foundations hold substantial equity stakes in several firms (e.g., Hershey Co, Eli Lilly & Co, Kellogg Co, Hormel Foods Corp, and etcetera – less than 20 firms) with the express intent of promoting public welfare rather than financially or economically benefiting family members.

utility, non-financial U.S. firms from 2001 through 2010, yielding 16,200 firm year observations.

Consistent with previous research, family firms are those where the founding family continues to maintain five percent or larger ownership stake in the firm (Shleifer and Vishny, 1986; Villalonga and Amit, 2006). To measure family presence, similar to McConnell and Servaes (1990), we use the fractional level of family ownership. Specifically, we determine the total number of shares held by the family (and their relatives), and divide by total outstanding shares – including both traded and untraded share classes (dual-class firms). Firms without family owners are referred to as diffuse shareholder firms, i.e., nonfamily firms with professional managers.

3.1. Level of family ownership

In the first set of empirical tests, we examine the relationship between family ownership and industry ambiguity. Studies using non-U.S. data typically use minimum thresholds for family ownership (e.g., 10% or 20%) to delineate between these controlling shareholders and diffuse shareholder firms (Claesssens et al., 2000; Dyck and Zingales, 2002). We choose an arbitrary benchmark of a 20% ownership stake to maintain firm control. Shleifer and Vishny (1997) indicate that the size of the equity stake needed for firm control depends on the degree of legal protection afforded to shareholders, suggesting that in the U.S., a 20% ownership would provide ample influence to control firm policies and direction. The median ownership stake of our family-firm subsample is 19.7%. We define high (low) levels of family ownership as an equity stake greater (less) than 20% of the firm's outstanding equity.[25]

3.2. Family owners exiting the firm

Across our subsample of family firms from 2001 through 2010, families sell their ownership stakes and exit the firm in 147 cases. Notably, in all of these cases, the firm continues to operate

[25] Different cut-offs levels for high and low ownership (10%, 15%, and 25%) provide similar results.

after the family's sell-off of their stakes, thus is reclassified as a non-family firm. In the family-exit sample, we observe that families hold a mean (median) ownership 5-years prior to their final exit of 15.7% (11.1%) of the firm's outstanding equity. The family's mean (median) ownership stake before their exit is 7.20% (5.34%). The year following the exit, the mean and median ownership stake drops to zero percent. In the sale of the family's shares to external investors, we note that in all cases that the outside investors are either institutional shareholders (i.e., mutual funds) or shareholders with less than a 5% ownership stake. Thus, the shares were *not* sold to another concentrated or undiversified investor, but rather the shares become diffusely held across a wide shareholder base. Since families typically make a gradual exit from the firm rather than abruptly selling their stake, we cannot define or delineate an exact event date for a pre- and post-examination on the relation between our ambiguity characteristics and family ownership. Rather, we use the 5-years preceding the family's exit as our testing period in our analysis.

We compare the family-exit firms to two matched samples. The first matched sample is firms where the family continuously maintains their ownership position over the entire sample period. The second matched sample is firms without family owners (diffuse shareholder firms).[26] To develop our matched samples, we use propensity score matching methodology. Specifically, we construct a propensity-score matched sample of family-exit firms to non-exit family firms and another one to diffuse shareholder firms. The matching criteria are calendar year, firm age, firm size (natural log of total assets), leverage (long-term debt divided by total assets), firm performance (ROA based on EBITDA), the level of equity issuance, and the standard deviation of the growth of income before extraordinary items for the prior 20-quarters for each firm. The propensity score model uses one-to-one firm matching, a caliper of 0.1, and a common support range of [0.1 to 0.9]

[26] We examine our matched sample of family firms and diffuse shareholder firms outside of the data collection period to ensure that that their designation as a family or diffuse shareholder firm did not change prior to- or after- the analysis period.

(Villalonga and Amit, 2010; Caliendo and Kopeining, 2008). The matching process yields a sample of 711 family-exit firm years, 711 family firm years, and 711 diffuse shareholder firm years.

3.3. Proxies for Industry-level exogenous ambiguity

Neither the family owners' ambiguity aversion nor the relative firm's ambiguity levels can be directly observed or measured. Based on the model implication, we develop proxies for relative importance of exogenous ambiguity based on industry-, firm-, and family- characteristics that potentially affect family shareholders' ability to exploit private information to assess the firm's future prospects and, thus, their willingness to concentrate their stakes in the family firm.

Based on the key insight of our model that family owners would concentrate more of their wealth in their firm if they can exploit information to further reduce learnable component of the ambiguity about their firm. For example, firms in highly innovative industries face greater potential disruption and thus greater exogenous ambiguity than firms in industries with little innovation (Tushman and Anderson, 1986), suggesting that family shareholders may not possess information advantage and are less willing to remain in such industries. We measure innovation ambiguity as the sum of R&D for all firms in each Fama-French industry group divided by the sum of total assets for all firms in each industry group. Similarly, industries with many new entrants are likely more difficult for family owners to forecast because new entrants maintain strong incentives to undertake radical actions that undermine existing industry patterns (Henderson, 1993). We measure ambiguity arising from industry entry as a dummy variable that equals one when the fraction of new firms entering each industry group is greater than 1% of the total firms in the industry and zero otherwise. The fraction of new firms entering each industry is calculated as the increase in the number of firms from time $t-1$ to t divided by the number of firms at time $t-1$.

In contrast to industry innovation and industry new entrants, family owners may better understand the nature of their firm's risk in industries with substantive corporate branding or barriers to entry. Dranove and Jin (2010) argue that branding is difficult to establish and costly to maintain, suggesting that the likely investment outcomes in the industry are well defined. We measure industry branding as the sum of advertising expenses for all firms in each Fama-French industry group divided by the sum of total assets for all firms in each industry group. Research in accounting indicates that in industries with greater barriers to entry, management appears better able to forecast future performance (Bamber and Cheon, 1998). We measure barriers to entry as the Herfindahl Index of intangible assets, which is defined as $\text{Herfindahl}_j = \sum s^2_{ij}$, where s_{ij} is the intangible asset share of each firm *i* in industry *j* (Hou and Robinson, 2006).

Anderson, Ghysels, and Juergens (2009) indicate that the dispersion of analysts' forecasts provides a measure of ambiguity. Our final measure of industry ambiguity focuses on analysts' ability to predict future earnings. It is more difficult for family owners to exploit information to reduce the ambiguity of their firms in industries where the exogenous ambiguity is larger or analysts forecast errors are more dispersed. We propose to use the *kurtosis* in addition to the standard deviation of analysts' forecast error to measure the dispersion of the analyst forecast errors[27]. Li and Zhu (2021) simulate stock returns following ambiguous normal distributions with a group of volatilities (ambiguous volatility), and find that the kurtosis of simulated data is positively related to the ambiguity of the probability distributions, while the standard deviation of simulated data is positively related to the average volatility of the probability distributions. Hence, the kurtosis of analyst forecast errors captures the ambiguity in addition to the conventional measure of volatility.

We compute the average kurtosis of analyst forecast errors for all firms in each Fama-French industry group divided by the value-weighted average book–to-market ratio of firms in the industry. We find similar results using the standard deviation of analyst forecast errors. All industry-level measures are based on the Fama-French 48-industry groupings except for regulated utilities and financials.

[27] Analyst forecast error is measured as the difference between actual annual earnings and earnings forecast scaled by the prior year-end stock price, where the forecast is made nine months to one year prior to firm's fiscal year-end.

3.4. Family-level wealth, risk aversion, experience, and ambiguity

Our model predicts that family owners (considering exit) are less likely to exit the firm as their total wealth increases. However, the family absolute wealth levels are not directly observable. Anderson and Reeb (2003) note that family shareholders, on average, hold about 69% of their total wealth in their firm's stock, suggesting that the majority of family wealth resides in the firm's stock. We measure family wealth as the natural log of the family's fractional equity stake multiplied by year-end total market value. To measure market value for firms with an untraded class of stock (dual-class shares), we use the share price of the traded class as a proxy for the price of the untraded class.

Our model further indicates that family shareholders are less likely to exit their ownership positions if they are more risk averse. However, family risk aversion cannot be directly observed. Following Borghans et al. (2009), who find that women are more risk averse than men, we instrument for risk aversion of family owners using the presence of females in family firms. More specifically, the family women dummy equals one when a female member of the family serves on the firm's board of directors and/or is listed in the proxy statement as holding a 5% or larger equity stake in the firm and zero otherwise.

Our model also predicts that family owners are less likely to exit the firm as family experience in the firm is longer. We proxy for family experience as the natural log of the number of years since the firm's inception, or more generally, the number of years since the family founded the company.

Lastly, our model predicts that family shareholders are more likely to exit as the exogenous ambiguity surrounding the profitability of the family firm increases. We use three proxies to capture the firm-level (learnable) ambiguity about the firm's future profitability. First, we use unexpected earning shocks measured as a dummy variable that equals one when the firm experiences four or more unexpected earnings shocks during the previous 12 quarters. An unexpected earnings shock is defined as falling into the top (positive shock) or bottom (negative shock) quintile of all earnings shocks. An earning shock is measured as the residual term ($\varepsilon_{i,q}$) from the following regression:

$$EPS_{i,q} = \alpha + \beta_1 EPS_{i,q-1} + \beta_2 EPS_{i,q-4} + \beta_3 EPS_{i,q-8} + \varepsilon_{i,q}$$

where $EPS_{i,q}$ is actual earnings per share of firm *i* in the announcement quarter *q*. The other two measures are the standard deviation and kurtosis of analyst earnings forecast errors.

3.5. Control variables

For both the family-exit tests and the level of family-ownership tests, we include several controls variables in our analysis. Firm size is measured as the natural log of total assets. Leverage is long-term total debt divided by total assets. Performance is earnings before interest, tax, depreciation, and amortization (EBITDA) divided by total assets, measured at *t-1*. Risk is the standard deviation of the growth of income before extraordinary times for the prior 20-quarters for each firm. Firm age is the natural log of the number of years since the firm's founding. Growth opportunities are measured as the market value of equity divided by total assets. Equity issuance is measured as the sum of common and preferred stock annual issuances divided by total assets. All regressions include industry-level dummies based on the Fama-French 48 industry grouping (excluding financial and regulated public utilities)[28].

Table 1 describes all the variables used in our analysis.

3.6. Descriptive statistics

In our initial sample of the 2,000 largest non-utility, non-financial industrial firms from 2001 through 2010 (16,200 firm-year observations), family firms constitute 5,371 firm-year observations or 33.2% of the sample. During the first year of the sample (2001), we have 788 family firms and 1,212 diffuse shareholder firms. We segregate family firms into low- and high-control stakes based on the level of family ownership. Families with a controlling stake of less than 20% of the firm's

[28] We obtain data and data definitions for the Fama-French industry groups from Kenneth French's website at: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/

equity are classified as low ownership (2,685 firm-year observations), and families with a 20% or large stake are classified as high ownership (2,686 firm-year observations).

Table 3 reports the summary statistics of the full sample, family firms, and non-family or diffuse-shareholder firms. The sample size of non-family firms is double that of family firms. A simple comparison (ignoring the sample-size difference) shows that family firms are smaller, younger, have lower debt ratios, low earnings volatility, and higher equity issuance. Table 3 also shows that for family firms, average industry branding, entry barriers are higher while industry innovation is lower. Although the average industry entrants of family firms are more, we should note that this is an unmatched-sample comparison.

Next, we exploit cross-sectional differences in the level of family ownership across our full sample of family firms. We proxy for exogenous ambiguity using industry innovation, industry branding, industry barriers to entry, industry entrants, and industry kurtosis of analyst earnings forecast errors. Table 4, Panel A presents summary statistics and univariate test results for family firms with high- and low- control stakes. Note that families with high control stakes (relative to families with low control stakes) appear to be associated with industries with less innovation and fewer entrants, suggesting less industry disruption and thus less ambiguity for family owners. Similarly, families with high control stakes are also associated with industries with greater branding and greater barriers to entry, again indicating less industry disruption and family ambiguity about the firm's prospects. Table 4, Panel B provides a correlation matrix of the key variables in our analysis and confirms the results from the univariate tests.

Next, Table 5 Panel A and Panel B provide summary statistics and a correlation matrix for the family-exit firms, the matched samples of non-exit family firms, and diffuse shareholder firms. The statistics are calculated by averaging each variable for the 5-years before the family's exit from the firm. The statistics are also calculated over the same 5-year period as the family-exit firms for the family firms and diffuse shareholder firms. The family-exit firms are matched to the non-exit family firms or diffuse shareholder firms based on propensity score matching techniques based on the calendar year, firm age, firm size, leverage, firm performance, volatility, equity issuance, and growth opportunities. Across the three subsamples, our average firm holds $2.35 billion in total assets, employs a debt ratio of 21.8%, returns 10.3% (EBITDA) as a fraction of total assets, and has been in existence for 34 years. Panel A indicates a strong match between family-exit firms and non-exit family firms, and between family-exit firms and diffuse shareholder firms. In particular, the results of the *t*-tests in the last two columns of Panel A show no significant differences for the matching variables across the different sub-samples (family-exit versus family firms, and family-exit firms versus diffuse shareholder firms).

4. Empirical Analysis Results

4.1. Cross-sectional analysis of family ownership and ambiguity

We first test Hypothesis 1.a-1.e regarding the relationship between industry-ambiguity and the level of family ownership across our full sample of family firms. The model indicates family investors tend to hold larger equity stakes in industries with less ambiguity about the firms' prospects. We examine this proposition using the following regression:

$$(\textit{High-Low Family Ownership})_{i,t} = \alpha + \beta(\textit{Industry Ambiguity})_{i,t} + \boldsymbol{X}_{i,t}\boldsymbol{\gamma} + \varepsilon_{i,t}$$

where the dependent variable is the level of family ownership less than 20%, as defined in Table I, and industry ambiguity are the five industry proxies developed in section 3.3. $\boldsymbol{X}$ representing a vector of control variables. We control for serial correlation and heteroscedasticity using the Huber-White sandwich estimator (clustered on firm-level identifier) for the standard errors on the coefficient estimates. The construction of our (industry-level) independent variables mitigates reverse causality concerns as the level of family ownership (dependent variable) is unlikely to determine the industrial structure.

Table 6 indicates a robust relation between the level of family ownership and industry-level ambiguity. Families with high control stakes (relative to families with low control stakes) appear to be associated with industries with less innovation (column 1), fewer entrants (column 2), greater branding (column 3), greater barriers to entry (column 4) and a lower kurtosis of analyst earnings forecast errors (column 5), that is, less (exogenous) ambiguity about prospects of firms in the industry. Overall, our cross-sectional analysis based on family-ownership levels supports the model's predictions and suggests that families tend to maintain larger control stakes, in the industries where the private information possessed by the family is more useful to reduce the ambiguity about the firm's prospects.

4.2. Family exit firms, family firms, and diffuse shareholder firms

The key insight of our model suggests that families will retain their ownership stakes in situations where the family's private information help reduce their ambiguity about the firm's prospects. Suppose the exogeneous ambiguity surrounding the family increases such that the family owners' private information is no longer useful to reduce the relative ambiguity, we expect the family to exit the firm and seek diversification benefits. To examine this proposition, we examine family-exit firms versus non-exiting family firms using the following logit specification:

$$(\mathit{Family\ Exit}_{i,t}) = \alpha + \beta_{1\text{-}3}(\mathit{Family\ Characteristics})_{i,t} + \beta_{4\text{-}8}(\mathit{Firm\ Characteristics})_{i,t} + \boldsymbol{X}_{i,t}\boldsymbol{\gamma} + \varepsilon_{i,t}$$

where the dependent variable equals one for family-exit firms and zero for non-exiting family firms. We control for serial correlation and heteroscedasticity using the Huber-White sandwich estimator (clustered on firm-level identifier) for the standard errors on the coefficient estimates.

Table 7 displays the logit regression results to examine what kind of families are more likely to exit. Our model predicts that wealthier families (families with a larger dollar wealth stake) are less likely to exit the firm. The results in column 1 clearly support our model's prediction, as families with more wealth invested in the firm are significantly less likely to exit their ownership stake.

Our model also predicts that more risk-averse family owners should be less likely to exit their ownership stake, all else equal. Column 2 of Table 7 suggests that families with more women on board or CEOs are less likely to exit their ownership position.[29]

[29] An alternative explanation for our finding on family women centers on the notion that because women are more risk averse than men, family owners may be more willing to place female family members on board in

Family members serving as the firm's CEO arguably provide the family shareholders with superior information regarding the firm's prospects versus an external professional manager serving as CEO. If so, we expect families a family member serves as CEO less likely to exit their ownership positions. Column 3 of Table 7 indicates that family owners are less likely to exit their ownership positions when a family member serves as CEO of the firm. Column 4 of Table 7 shows that our model predictions remain robust when family absolute wealth, female family representation, and family CEOs are simultaneously included in a single regression.

Our model further predicts that families with more experience in the firm should be less likely to exit. Our earlier tests examining family characteristics are based on a propensity-score-matched sample with firm age as one of the matching criteria. To alleviate confounding effects from matching on firm age, we generate a new propensity-score-matched sample for this test using the calendar year, firm size, debt ratio, previous-year ROA, volatility, and growth opportunities as the matching criteria. Column 5 of Table 7 clearly indicates that families are significantly less likely to exit the firm as their experience in the firm increases.

In sum, the family-exit tests using family characteristics yield strong evidence supporting predictions of our model (Hypothesis 2.2.-2.4) against other models without ambiguity.

Our final tests on family exits focus on firm characteristics, in particular, the ambiguity of the industry that the family firm belongs to and the ambiguity of the firm's future earnings. Our model suggests that families will be more likely to exit their ownership stakes as the ambiguity increases (Hypothesis 2.1).

Columns 1 of Table 8 indicates that families are more likely to exit as the level of industry innovation increases. The probability of a family's exit increases by 3.29% when the industry innovation increases by 1%. Column 2 of Table 7 indicates that families are significantly less likely

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sectors characterized by high (or low) risk. We examine this proposition by regressing firm volatility on female-family representation and our control variables (with and without industry controls). The analysis indicates that female-family representation is not significantly related to firm risk (volatility).

to exit in industries with greater branding recognition. A 1% increase in industry branding is associated with a 3.59% decrease in family exits[30]. Intuitively, family owners face fewer disruptions about the firm's prospects in industries with greater branding recognition and less innovation, thereby are more inclined to remain invested in the firm.

Next, we examine the impact of ambiguity about a firm's future earnings on the family exit decision. Intuitively, when a firm experiences large unexpected earnings shocks, the information possessed by family owners is less helpful to reduce the ambiguity regarding the firm's prospects, so families are more likely to exit. Column 3 of Table 8 indicates a significant positive relation between unexpected earnings shocks and family exit decisions.

Lastly, we study the relationship between the family-exit decision and the dispersion in analyst earnings forecast errors measured as the standard deviation and kurtosis of analyst forecast errors. More dispersed analyst forecast errors suggest less predictability and more ambiguity about the firm's prospects. Columns 4 and 5 of Table 8 show that families are more likely to leave the firm if analyst forecast errors are more dispersed.

In summary, the family-exit analysis based on firm characteristics also supports the predictions of our model that family owners are more likely to exit their ownership positions when firms are in the industry with greater ambiguity or the firm's prospects are more ambiguous.

5. Summary and conclusion

Classical finance theory and conventional wisdom suggest that investors should hold well-diversified portfolios versus holding large concentrated stakes in a single firm. Yet, in well-developed equity markets such as those in the U.S., we continue to observe that many founding-family owners invest the bulk of their wealth in just one stock. Moreover, with the ability to control senior management posts, board seats, and the prevalence of super-voting dual-class shares, the

[30] The calculation if these percent changes are based on marginal effects, and are available upon request.

notion that family shareholders need to maintain such large stakes to extract private benefits appears to be an incomplete explanation.

We propose an alternative explanation for this family ownership puzzle based on a robust portfolio-choice theory with ambiguity about both expected returns and risk (of conventional measure). The key insight of our model is that family owners may exploit private information about the firm to reduce the ambiguity about their firm relative to other firms, and optimally choose a portfolio where the majority of their investable funds are held in the family firm. In contrast, non-family owners without access to private information invest in a diversified portfolio.

Our model provides various testable implications, including several that appear contrary to the implications of models based on classical finance theory without ambiguity. Our model predicts that family owners are more likely to retain their ownership stakes where private information and experience allow them to better understand the nature of the firm's prospects. The model distinctly predicts that families will be less likely to sell off their ownership positions (exit the firm) as family wealth invested in the firm increases[31], as family risk aversion increases, and as family tenure in the firm increases.

We empirically investigate the model's predictions using two testing procedures. First, we examine family ownership using panel regression for the largest family firms in the U.S. Second, in a more robust test environment, we examine family decisions to sell off their ownership stakes and exit the firm. Our first set of tests suggests that families hold larger ownership stakes in industries with less innovation, fewer new entrants (competitors), greater branding, larger barriers to entry, and less dispersed analyst forecast errors. Our second set of tests examining family exits also confirms that families tend to remain in industries with less innovation, greater branding, less ambiguous earnings prospects. Notably, the exit analysis also indicates that families are less likely

[31] Bitler, Moskowitz and Vissing-Jorgensen (2005) argue that the standard agency model also implies a positive relationship between wealth and ownership, as absolute risk aversion is typically thought to decreasing in wealth. However, for investors with constant absolute risk aversion, this implied positive relationship between wealth and ownership does not hold.

to sell off their ownership stakes as their invested wealth increases, their tenure in the firm increases, and their risk aversion increases.

Our study contributes to the literature on the economic decisions under ambiguity, by deriving testable implications from a model with ambiguity about return volatility. We find that the ambiguity about volatility rather than expected return is critical to explaining the family ownership puzzle quantitatively. Furthermore, our empirical analysis yields evidence strongly supporting our model's prediction against other models without ambiguity, which indicates that the ambiguity and risk affect the investment decision in a radically different way, as Knight (1921) suggests. Finally, although it is challenging to measure and separate ambiguity from the conventional measure of risk due to the nature of ambiguity, our study shows that empirical tests based on comparative statics shed light on how to differentiate the impact of ambiguity and risks.

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Appendix 1: Model Implied Family Ownership

This appendix presents simulation results of the share of family wealth invested in the family firm in models with and without ambiguity. The implied family ownership is computed for different levels of ambiguity about the diversified portfolio relative to the family firm, and for three cases where the worst expected return of the family firm is the same, less, and more than that of the diversified portfolio. For comparison, we also present the implied ownership of a single stock in the model without ambiguity.

Table A.1.
Calibration of Stock Ownership

Model implied share of wealth invested in the single stock in models with and without ambiguity[32]. These estimates are computed in the benchmark model in Section 2. In the model without ambiguity, the true values of annual expected return ($\bar{r}_{2,0}$) and variance ($\sigma^2_{2,0}$) of the diversified portfolio are assumed to be 9% and 0.037, respectively. The true value of annual return variance ($\sigma^2_{1,0}$) of the single stock is assumed to be 24.2%, the correlation coefficient between diversified portfolio and single stock is assumed to be 0.123 (see Elton and Gruber, 1977). In the model with uncertainty, the worst-case values of annual expected return ($\bar{r}_{2,min}$) and variance ($\sigma^2_{2,max}$) of the diversified portfolio are assumed to be the same as the true values in the model without ambiguity.

Panel A: Model with Uncertainty			
Relative Ambiguity of Risk $(\sigma^2_{2,max}/\sigma^2_{1,max})$	Relative Ambiguity of Expected Returns $\bar{r}_{1,min} = \bar{r}_{2,min}$	$\bar{r}_{1,min} = 0.5\bar{r}_{2,min}$	$\bar{r}_{1,min} = 1.5\bar{r}_{2,min}$
Infinity	100%	81%	>100%
4.00	82%	65%	98%
2.00	68%	54%	82%
1.00	50%	39%	61%
0.50	30%	22%	38%
0.25	13%	8%	17%
Panel B: Model without Uncertainty about Risk			
Relative Risk $(\sigma^2_{2,0}/\sigma^2_{1,0})$	Relative Ambiguity of Expected Returns $\bar{r}_{1,min} = \bar{r}_{2,min}$	$\bar{r}_{1,min} = 0.5\bar{r}_{2,min}$	$\bar{r}_{1,min} = 1.5\bar{r}_{2,min}$
0.15	3%	0%	6%

[32] The risk aversion and wealth level are calibrated such that 30% of wealth is invested in the risky portfolio and 70% invested in the safe asset, in Markowitz (1952) framework that contains a safe asset with return of 2% and a risky asset with expected return of 9%.

Appendix 2: Family investment decision with Treasury Bills

In this appendix, we consider the optimal portfolio choice problem faced by the family owners who choose to invest in three assets, the family's firm with return r_1, a diversified portfolio with return r_2 and Treasury Bills with return r_3. We use Treasury Bills as a proxy for the risk-free asset with nonzero variance[33].

Table A.2.
Summary Statistics of Return on Treasury Bonds and Market Index

The returns are deflated using the inflation rate computed from CPI. The returns on treasuries, CPI, and the CRSP value-weighted market index are from the CRSP dataset available on WRDS.

	Market Index	*Treasury Bonds/Bills*				
		30 year	*10 year*	*5 year*	*2 year*	*30 day*
Annual Return (%) 1942-2013	9.15	2.11	1.86	1.80	1.31	0.15
Annual Volatility (%) 1942-2013	18.50	13.51	9.71	7.10	5.20	3.55
Annual Return (%) 1995-2013	9.56	6.10	4.38	3.64	2.16	0.41
Annual Volatility (%) 1995-2013	20.19	17.61	8.95	5.93	3.59	2.11

Table A.2 shows that the average return and standard deviation of 30-day Treasury Bills are significantly lower than that of the market portfolio. The standard deviation of the return on Treasury Bills is significantly different from zero. The standard deviations of longer-term Treasury Bonds are much higher.

We assume that returns on all three assets, r_1, r_2 and r_3 are independent and normally distributed. To focus on the investment choice between the family firm and the market portfolio, we assume no ambiguity regarding the return on Treasury Bills. That is, we assume the true value of the mean and variance of returns on the family firm and the market portfolio, r_1 and r_2, are unknown to investors, while the true value of the mean and variance of returns on Treasury Bills are unique and known to all investors. The true value of the mean and variance of the return on asset *i*, r_i, is denoted as $(\bar{r}_{i,0}, \sigma_{i,0})$ for *i* =1, 2, 3. Investors perceive that the mean and variance of r_1

[33] 30-day Treasury Bills is commonly used as proxy of the risk free asset in asset pricing and optimal portfolio choice literature.

and r_2 belong to a set of possible values $\Theta_1 = \{(r_{1,n}, \sigma_{1,m}), n = 1,2, \dots, N, m = 1,2, \dots, M\}$, and $\Theta_2 = \{(r_{2,n}, \sigma_{2,m}), n = 1,2, \dots, N, m = 1,2, \dots, M\}$, which contain the true values of mean and standard deviation $(\bar{r}_{i,0}, \sigma_{i,0})$, for i = 1, 2. That is, the true values $\bar{r}_{i,0}$ and $\sigma_{i,0}$ lie between $[\bar{r}_{i,min}, \bar{r}_{i,max}]$ and $[\sigma_{i,min}, \sigma_{i,max}]$, respectively, for i = 1, 2. We further assume that the mean return on Treasury Bills is smaller than the minimum mean returns on family firm and market portfolio, and the variance of return on Treasury Bills is smaller than the minimum variances of returns on family firm and market portfolio, that is,

$$\begin{gathered} \bar{r}_{1,min} > \bar{r}_3,\ \bar{r}_{2,min} > \bar{r}_3 \\ \sigma_{1,min} > \sigma_3,\ \sigma_{2,min} > \sigma_3 \end{gathered} \tag{A.1}$$

As in our benchmark model, we assume that family owners have private information regarding the fundamentals of the family firm, which allows them to reduce the ambiguity about the fundamentals of their own firms over time and through experience. However, the family owner cannot obtain material, non-public information about firms in the broad portfolio as they do not manage these other firms. In our setup, the reduction in ambiguity is captured by the shrinkage in the range of all possible means and variances on the return on the family firm.

A.1.1 The family's investment decision

For each time period t, the decision problem of the family owner with CARA utility can be formulated as

$$\max_{\alpha_1,\alpha_2} \min_{\theta_1 \in \Theta_1, \theta_2 \in \Theta_2} E_t[-exp(\gamma W_{t+1})]$$

$$s.t. W_{t+1} = W_t[\alpha_1 r_{1,t+1} + \alpha_2 r_{2,t+1} + (1 - \alpha_1 - \alpha_2) r_{3,t+1}]$$

(A.2)

where W_t is the wealth of the family at time t, $r_{1,t+1}$, $r_{2,t+1}$ and $r_{3,t+1}$ are the returns on the family firm, market portfolio, and Treasury Bills at time t+1, respectively. α_1 and α_2 are the shares of wealth invested in the family firm and market portfolio, respectively. We allow for short sale in all the assets, and do not restrict α_1 and α_2 to be positive and less than one.

It is straightforward to show that the decision problem of the family owner can be transformed as,

$$\max_{\alpha_1,\alpha_2} \min_{\bar{r}_{1,n_1}, \bar{r}_{2,n_2}, \sigma_{1,m_1}, \sigma_{2,m_2}} \Big\{ [\alpha_1(\bar{r}_{1,n_1} - \bar{r}_3) + \alpha_2(\bar{r}_{2,n_2} - \bar{r}_3) + \bar{r}_3] \\ - \frac{\gamma W_t}{2} [\alpha_1^2 \sigma_{1,m_1}^2 + \alpha_2^2 \sigma_{2,m_2}^2 + (1 - \alpha_1 - \alpha_2)^2 \sigma_3^2] \Big\} \tag{A.3}$$

Let's first examine the minimization problem of (A.3). Since the objective function in (A.3) is monotonically decreasing in σ_1^2 and σ_2^2, the maximum possible value of variance is always chosen, that is

$$\sigma_1^* = \sigma_{1,max} \equiv \max_{m=1,2,..,M}\{\sigma_{1,m}\}$$

$$\sigma_2^* = \sigma_{2,max} \equiv \max_{m=1,2,..,M}\{\sigma_{2,m}\}$$

As long as the family owner is long in the assets, then the minimum possible mean returns are chosen for this asset. If the family shorts the asset, then the maximum possible mean return of this asset is chosen, that is,

$$(\bar{r}_1^*, \bar{r}_2^*) = \begin{cases} (\bar{r}_{1,min}, \bar{r}_{2,max}), & if\ \alpha_1 > 0\ and\ \alpha_2 < 0 \\ (\bar{r}_{1,min}, \bar{r}_{2,min}), & if\ \alpha_1 > 0\ and\ \alpha_2 > 0 \\ (\bar{r}_{1,max}, \bar{r}_{2,min}), & if\ \alpha_1 < 0\ and\ \alpha_2 > 0 \end{cases}$$

Assumption (A.1) implies that it is never optimal for the family owners to short both family firm and market portfolio at the same time. Given the solution to the minimization problem (A.3), the optimal share invested in the family business and market portfolio can be then characterized as follows:

Case 1: If the following condition is satisfied,

$$(\bar{r}_{1,min} - \bar{r}_{2,max})\sigma_3^2 > (\bar{r}_{2,max} - \bar{r}_3)\sigma_{1,max}^2 + \gamma W_t \sigma_3^2 \sigma_{1,max}^2$$

then the family owners go long on the family stock and short the diversified portfolio

$$\alpha_1^* = \frac{(\bar{r}_{1,min} - \bar{r}_3)\sigma_{2,max}^2 + (\bar{r}_{1,min} - \bar{r}_{2,max})\sigma_3^2 + \gamma W_t \sigma_{2,max}^2 \sigma_3^2}{\gamma W_t(\sigma_{1,max}^2 \sigma_{2,max}^2 + (\sigma_{1,max}^2 + \sigma_{2,max}^2)\sigma_3^2)} > 0$$

$$\alpha_2^* = \frac{(\bar{r}_{2,max} - \bar{r}_3)\sigma_{1,max}^2 + (\bar{r}_{2,max} - \bar{r}_{1,min})\sigma_3^2 + \gamma W_t \sigma_{1,max}^2 \sigma_3^2}{\gamma W_t(\sigma_{1,max}^2 \sigma_{2,max}^2 + (\sigma_{1,max}^2 + \sigma_{2,max}^2)\sigma_3^2)} < 0$$

$$1 - \alpha_1^* - \alpha_2^* = \frac{-(\bar{r}_{1,min} - \bar{r}_3)\sigma_{2,max}^2 - (\bar{r}_{2,max} - \bar{r}_3)\sigma_{1,max}^2 + \gamma W_t \sigma_{1,max}^2 \sigma_{2,max}^2}{\gamma W_t(\sigma_{1,max}^2 \sigma_{2,max}^2 + (\sigma_{1,max}^2 + \sigma_{2,max}^2)\sigma_3^2)} \gtrless 0$$

In this case, family owner's private information allows them to significantly reduce the ambiguity of their family firm relative to the other firms, and the minimum expected return of family firm is much larger than the maximum possible expected return of the diversified portfolio (in excess of the return on Treasury Bills), and the family chooses to invest as much as possible in their firm. In this case, the share invested in the family firm decreases with the risk aversion and family wealth. However, we argue this is an extreme case and does not apply to the marginal family owner who may exit, as the family would never exit in this case. Furthermore, we argue this is a case rare in reality, as it is extreme for the family to think the minimum risk-adjusted expected return of the family firm is much larger than the maximum possible expected return of the diversified portfolio.

Case 2: If the following condition is satisfied,

$$-(\bar{r}_{1,min}-\bar{r}_3)\sigma_{2,max}^2-\gamma W_t\sigma_3^2\sigma_{2,max}^2<(\bar{r}_{1,min}-\bar{r}_{2,min})\sigma_3^2<(\bar{r}_{2,min}-\bar{r}_3)\sigma_{1,max}^2+\gamma W_t\sigma_3^2\sigma_{1,max}^2,$$

then the family owners invest positive shares in both the family stock and the diversified portfolio

$$\alpha_1^*=\frac{(\bar{r}_{1,min}-\bar{r}_3)\sigma_{2,max}^2+(\bar{r}_{1,min}-\bar{r}_{2,min})\sigma_3^2+\gamma W_t\sigma_{2,max}^2\sigma_3^2}{\gamma W_t(\sigma_{1,max}^2\sigma_{2,max}^2+(\sigma_{1,max}^2+\sigma_{2,max}^2)\sigma_3^2)}>0$$

$$\alpha_2^*=\frac{(\bar{r}_{2,min}-\bar{r}_3)\sigma_{1,max}^2+(\bar{r}_{2,min}-\bar{r}_{1,min})\sigma_3^2+\gamma W_t\sigma_{1,max}^2\sigma_3^2}{\gamma W_t(\sigma_{1,max}^2\sigma_{2,max}^2+(\sigma_{1,max}^2+\sigma_{2,max}^2)\sigma_3^2)}>0$$

$$1-\alpha_1^*-\alpha_2^*=\frac{-(\bar{r}_{1,min}-\bar{r}_3)\sigma_{2,max}^2-(\bar{r}_{2,min}-\bar{r}_3)\sigma_{1,max}^2+\gamma W_t\sigma_{1,max}^2\sigma_{2,max}^2}{\gamma W_t(\sigma_{1,max}^2\sigma_{2,max}^2+(\sigma_{1,max}^2+\sigma_{2,max}^2)\sigma_3^2)}\gtrless 0$$

In this case, the family owner still has less ambiguity about the expected return on their firm relative to other firms. However, the private information advantage is not as large as in Case 1, so the family firm invests positive wealth shares in both the family firm and the diversified portfolio.

Case 3: If the following condition is satisfied,

$$(\bar{r}_{1,max}-\bar{r}_{2,min})\sigma_3^2<-(\bar{r}_{1,max}-\bar{r}_3)\sigma_{2,max}^2-\gamma W_t\sigma_3^2\sigma_{2,max}^2$$

then the family owners short the family stock and long the diversified portfolio

$$\alpha_1^*=\frac{(\bar{r}_{1,max}-\bar{r}_3)\sigma_{2,max}^2+(\bar{r}_{1,max}-\bar{r}_{2,min})\sigma_3^2+\gamma W_t\sigma_{2,max}^2\sigma_3^2}{\gamma W_t(\sigma_{1,max}^2\sigma_{2,max}^2+(\sigma_{1,max}^2+\sigma_{2,max}^2)\sigma_3^2)}<0$$

$$\alpha_2^*=\frac{(\bar{r}_{2,min}-\bar{r}_3)\sigma_{1,max}^2+(\bar{r}_{2,min}-\bar{r}_{1,max})\sigma_3^2+\gamma W_t\sigma_{1,max}^2\sigma_3^2}{\gamma W_t(\sigma_{1,max}^2\sigma_{2,max}^2+(\sigma_{1,max}^2+\sigma_{2,max}^2)\sigma_3^2)}>0$$

$$1-\alpha_1^*-\alpha_2^*=\frac{-(\bar{r}_{1,max}-\bar{r}_3)\sigma_{2,max}^2-(\bar{r}_{2,min}-\bar{r}_3)\sigma_{1,max}^2+\gamma W_t\sigma_{1,max}^2\sigma_{2,max}^2}{\gamma W_t(\sigma_{1,max}^2\sigma_{2,max}^2+(\sigma_{1,max}^2+\sigma_{2,max}^2)\sigma_3^2)}\gtrless 0$$

In this scenario, the private information does not help to reduce the ambiguity of the family firm enough, so the minimum possible expected return of the family business is less than the minimum possible expected return of the diversified portfolio.

Let us again focus on Case 2, where family owners invest positive shares in the family firm and the diversified portfolio. In this case, we can rewrite the optimal investment share in the family firm as

$$\alpha_1^*=\frac{(\bar{r}_{1,min}-\bar{r}_3)/\sigma_3^2+(\bar{r}_{1,min}-\bar{r}_{2,min})/\sigma_{2,max}^2+\gamma W_t}{\gamma W_t(\sigma_{1,max}^2/\sigma_{2,max}^2+\sigma_{1,max}^2/\sigma_3^2+1)} \quad \text{(A.4)}$$

The optimal share depends on the Sharpe ratio of the family firm relative to the treasury bills and the market portfolio. When the family has less ambiguity about the variance and the expected return of the family firm, that is, the smaller are $\sigma_{1,max}^2/\sigma_{2,max}^2$ and $\sigma_{1,max}^2/\sigma_3^2$ or the larger are $(\bar{r}_{1,min} - \bar{r}_3)/\sigma_3^2$ and $(\bar{r}_{1,min} - \bar{r}_{2,min})/\sigma_{2,max}^2$, the family is more likely to continue to invest in the family firm. Thus, the concern about risk-adjusted return centers on the ambiguity of the variance rather than the level of the variance. Moreover, if the family has little information about the diversified portfolio (relative to their firm), then they would put a very high upper bound on the perceived variance of the return of this asset (extremely high $\sigma_{2,max}^2$), which implies that they would invest little in the diversified portfolio. Suppose the family is extremely averse to ambiguity, or the relative ambiguity about the risk of the diversified portfolio gets extremely large, then the family would invest nothing in the diversified portfolio, regardless of their risk aversion or wealth level.

To study the relationship between the share invested in the family firm and the characteristics of the family, such as risk aversion and wealth of the family, we found it is necessary to focus on Case 2. Taking the derivative of the optimal share in family share (α_1^*) with respect to risk aversion (γ) and wealth (W_t), we have

$$\frac{\partial \alpha_1^*}{\partial(\gamma W_t)} = -\frac{(\bar{r}_{1,min}-\bar{r}_3)/\sigma_3^2+(\bar{r}_{1,min}-\bar{r}_{2,min})/\sigma_{2,max}^2}{(\gamma W_t)^2(\sigma_{1,max}^2/\sigma_{2,max}^2+\sigma_{1,max}^2/\sigma_3^2+1)} \quad \text{(A.5)}$$

If the family owners have less ambiguity regarding the family firm and think the return on their family firm is so good that the minimum return is sufficiently larger than that of the market portfolio such that the Sharpe ratio relative to the market portfolio is larger than the Sharpe ratio relative to the Treasury Bills, that is,

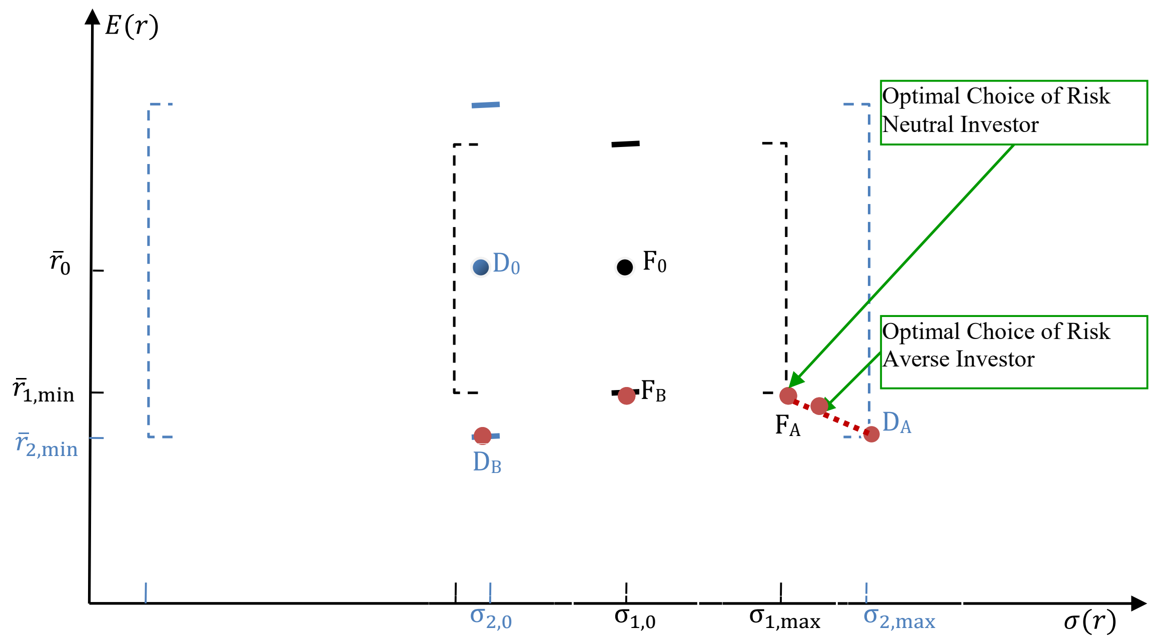
$$\frac{(\bar{r}_{1,min} - \bar{r}_{2,min})}{\sigma_{2,max}^2} > -\frac{(\bar{r}_{1,min} - \bar{r}_3)}{\sigma_3^2}$$

then the family will always invest a positive share in the family business and not consider exiting. However, we are more interested in the case where

$$\frac{(\bar{r}_{1,min} - \bar{r}_{2,min})}{\sigma_{2,max}^2} < -\frac{(\bar{r}_{1,min} - \bar{r}_3)}{\sigma_3^2} < 0$$

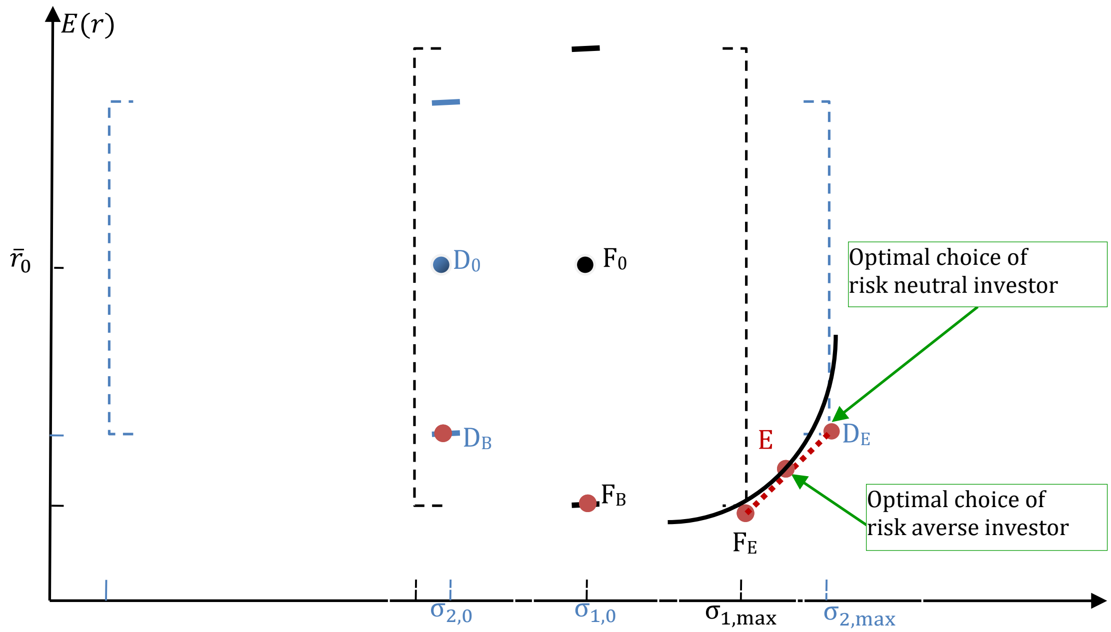
The family thinks the minimum expected return of the family business is smaller than that of the market portfolio. This is the case that captures the marginal family investors who might consider the exit decision. In this case, (A.5) is positive, so the more risk averse is the family owner, the more wealth the family has, the larger share of wealth invested in the family firm, the less likely the family to exit *ceteris paribu.* Hence, the predictions of our benchmark model still hold when the choice set of the family owners includes not only two risky assets with different levels of ambiguity but also Treasury Bills that are proxies for the relatively low-risk asset without ambiguity.

Figure 1. Optimal Portfolio Choice of Family Owners



Panel A: Non-Exiting Case ($\bar{r}_{1,min} - \bar{r}_{2,min} > 0$)

F_A corresponds to the worst-case scenario return-risk of family firm stock ($\bar{r}_{1,min}, \sigma_{1,max}$), D_A corresponds to the worst-case-scenario return-risk of diversified portfolio ($\bar{r}_{2,min}, \sigma_{2,max}$). F_B and D_B correspond to the worst-case scenario return-risk of family firm stock and diversified portfolio, respectively, in the case where there is no ambiguity about risk of the returns.



Panel B: Exiting Case ($\bar{r}_{1,min} - \bar{r}_{2,min} < 0$)

F_E corresponds to the worst-case scenario return-risk of family firm stock ($\bar{r}_{1,min}, \sigma_{1,max}$), D_E corresponds to the worst-case-scenario return-risk of diversified portfolio ($\bar{r}_{2,min}, \sigma_{2,max}$). F_B and D_B correspond to the worst-case scenario return-risk of family firm stock and diversified portfolio, respectively, in the case where there is no ambiguity about risk of the returns.

Table 1. Variable Definitions

Debt Ratio – year-end long-term debt divided by total assets for each firm.
Dual Class – the percent of firms that maintain dual-class share structures.
Earnings Volatility – standard deviation of the growth of income before extraordinary items for the prior 20-quarters for each firm.
Equity issuance – the sum of common and preferred stock annual issuances divided by total assets.
Family CEO –equals one when either the founder or a founder's descendant serves as the firm's CEO; equals zero when an outside professional manager serves as the firm's CEO.
Family Experience – natural log of the number of years since the firm's inception.
Family Firm –equals one when the family holds a five percent or larger ownership stake and zero otherwise.
Family Ownership – the percent of common equity held by the family.
Family Wealth – family fractional equity ownership stake multiplied by year-end firm total market value.
Family Women – equals one when a female member of the family serves on the firm's board of directors and/or holds a 5% or larger equity stake.
Firm Age – the number of years since the firm's founding.
Firm Kurtosis of Forecast Error –kurtosis of the analysts' earnings forecast error, the difference between actual annual earnings and earnings forecast scaled by the prior year-end stock price, where the forecast is made nine months to one year prior to the firm's fiscal year-end.
Kurtosis of Forecast Error (KurtFE) for firm *j* is defined as

$$KurtFE_j = \frac{(n+1)n(n-1)}{(n-2)(n-3)} \frac{\sum_{i=1}^{n}\left(FE_{i,j} - \overline{FE_j}\right)^4}{\left[\sum_{i=1}^{n}\left(FE_{i,j} - \overline{FE_j}\right)^2\right]^2} - 3\frac{(n-1)^2}{(n-2)(n-3)}$$

Firm Standard Deviation of Forecast Error- standard deviation of the analysts' earnings forecast error, the difference between actual annual earnings and earnings forecast scaled by the prior year-end stock price, where the forecast is made nine months to one year prior to the firm's fiscal year-end. Forecast Error (StdFE) for firm *j* is defined as

$$StdFE_j = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(FE_{i,j} - \overline{FE_j}\right)^2}$$

Where $FE_{i,j}$ = forecast error of analyst *i* for firm *j*
$\overline{FE_j}$ = mean of forecast error for firm *j*
n = number of analysts covering the firm *j*

Tobin's Q – market value of equity divided by total assets.
High-Low Family Ownership – the level of family ownership minus 20%.
Industry Barriers to Entry – is the Herfindahl Index of intangible assets, which is defined as $\text{Herfindahl}_j = \sum s^2_{ij}$, where s_{ij} is the intangible asset share of each firm i in industry j. We perform the above calculation for each Fama-French industry group for each year.
Industry Branding – average of advertising expenses divided by total assets for all firms in each of the Fama-French industry groups.
Industry Dummy - equals one for each Fama-French industry group and zero otherwise.
Industry Entrants– equals one when the fraction of new firms entering each Fama-French industry is greater than 10% of the total firms in the industry and zero otherwise. The fraction of new firms entering each of the Fama-French industry groups is calculated as the increase in the number of firms from time *t-1* to *t* divided by the number of firms in the industry at time *t-1*.
Industry Innovation – average of R&D expense divided by total assets for all firms in each of the Fama-French industry groups.
Industry Kurtosis of Forecast Errors – industry average kurtosis of the analysts' forecast error of all firms in each of the Fama-French industry groups, divided by the industry value-weighted average book-to-market ratio.

Return on Assets – earnings before interest, tax, depreciation, and amortization (EBITDA) divided by total assets for the prior year-end for each firm.

Total Assets – year-end total assets for each firm.

Unexpected Earnings Shocks – equals one when the firm experiences 4-or more EPS shocks during the previous 12-quarters. An EPS shock is defined as falling into the top (positive shocks) or bottom (negative shocks) quintiles of all earnings shocks. Earnings shocks are measured as the residual term from the following regression:

$$EPS_{i,q} = a + \beta_1 EPS_{i,q-1} + \beta_2 EPS_{i,q-4} + \beta_3 EPS_{i,q-8} + \varepsilon_{i,t}$$

where EPS is actual earnings per share of the announcement quarter (*q*), the prior 1, 4, and 8 quarter (*q-1*, *q-4*, *q-8*).

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Table 2. Industry Distribution of Family Firms

This table reports the percentage of family firms and the average family ownership of family firms in each of the Fama-French 48 industry groups.

Industry Code	Industry Classifications	Family Firms (%)	Family Ownership (%)	Industry Code	Industry Classifications	Family Firms (%)	Family Ownership (%)
16	Textiles	78.9	13.9	47	Trading	31.3	10.1
4	Beer & Liquor	77.3	20.4	17	Construction Materials	30.9	8.5
46	Real Estate	65.7	27.4	22	Electrical Equipment	30.2	5.7
3	Candy & Soda	64.2	30.6	35	Computers	27.9	6.3
8	Printing and Publishing	63.4	21.7	38	Business Supplies	27.4	8.4
32	Communication	62.8	20.1	24	Aircraft	26.9	3.8
10	Apparel	61.7	21.8	36	Electronic Equipment	25.2	5.0
7	Entertainment	54.7	22.2	11	Healthcare	25.1	5.8
6	Recreation	45.5	5.9	12	Medical Equipment	24.8	5.6
25	Shipbuilding, Railroad Equipment	44.6	9.1	1	Agriculture	24.3	8.3
45	Insurance	44.6	10.9	23	Automobiles and Trucks	23.8	6.0
44	Banking	44.4	7.6	37	Measuring and Control Equipment	23.4	5.6
33	Personal Services	44.3	10.0	26	Defense	22.7	6.5
42	Retail	42.0	12.6	13	Pharmaceutical Products	21.9	3.8
18	Construction	41.3	10.7	30	Petroleum and Natural Gas	21.8	5.0
2	Food Products	39.4	12.1	14	Chemicals	21.4	5.7
40	Transportation	38.4	10.2	19	Steel Works	20.9	3.5
20	Fabricated Products	37.9	8.8	29	Coal	18.7	5.9
34	Business Services	37.2	8.6	21	Machinery	17.8	3.8
41	Wholesale	36.9	9.8	48	Other	12.7	4.4
39	Shipping Containers	35.4	10.8	28	Non-Metallic and Industrial Metal Mining	12.6	1.8
15	Rubber and Plastic Products	34.4	9.3	31	Utilities	5.7	1.1
9	Consumer Goods	33.6	7.9	5	Tobacco Products	0	0
43	Restaurants, Hotels, Motels	33.1	6.3	27	Precious Metals	0	0

Table 3. Summary Statistics

This table provides summary statistics for the full sample, family firm, and non-family firm samples.

	Full Sample	*Family Firms*	*Non-Family Firms*
Firm-Year Obs.	16200	5371	10829
Unique Firm Number in 2001	2000	788	1212
Family Ownership (%)	8.57	25.97	-
Industry Innovation (%)	3.09	2.18	3.28
Industry Branding (%)	1.03	1.42	0.97
Industry Barriers to Entry	9.14	10.68	9.06
Industry Entrants	0.33	0.38	0.30
Family Wealth ($million)	206	632	-
Ln(total assets)	7.43	6.82	7.60
Debt Ratio(%)	23.61	20.66	25.09
Return on Assets(t-1)	0.11	0.11	0.11
Earnings Volatility(%)	23.43	21.65	24.24
Firm Age	47.12	42.42	50.67
Tobin's Q	1.17	1.15	1.17
Equity Issuance/Assets(%)	1.48	1.59	1.48

Table 4. Summary Statistics for Low- and High- Family Ownership

Panel A: This panel provides annual summary statistics and difference of mean tests for Low- and High- Family Ownership. Low Ownership is defined as family ownership stakes less than 20%. High Ownership is defined as family ownership stakes greater than 20%. *t*-statistics are corrected for serial correlation and heteroscedasticity by clustering on the firm-level identifier. ***, **, and * indicate significance at the 1%, 5% and 10% levels, respectively.

Summary Statistics for High-Low Family Ownership				
	All Family Firms	*Low Ownership*	*High Ownership*	*t-statistics*
Observations	5,371	2,685	2,686	-
Family Ownership	25.97	11.20	40.89	33.83***
Industry Innovation	2.18	2.54	1.82	3.89***
Industry Branding	1.42	1.30	1.55	2.31**
Industry Barriers to Entry	10.68	10.05	11.34	1.98**
Industry Kurtosis of Forecast Errors	2.655	2.705	2.593	1.77*
Industry Entrants	0.38	0.40	0.33	1.51
Ln(total assets)	6.82	6.93	6.71	2.40***
Debt Ratio (%)	20.66	19.70	21.62	1.39
Return on Assets$_{t-1}$ (%)	11.29	10.89	11.69	1.40
Earnings Volatility (%)	21.65	16.45	26.88	0.63
Firm Age	42.42	41.44	43.41	0.87
Tobin's Q	1.15	1.20	1.11	1.43
Equity Issuance/Assets (%)	1.59	1.93	1.21	4.87***

Panel B: This panel provides Spearman correlation coefficients for the annual measures used in the Low- and High- Family Ownership Analysis. ***, **, and * indicate significance at the 1%, 5% and 10% levels, respectively.

	(1)	*(2)*	*(3)*	*(4)*	*(5)*
(1) Family Ownership	1.00				
(2) Ind. Innovation	-0.17*	1.00			
(3) Ind. Entry	-0.02***	-0.08***	1.00		
(4) Ind. Branding	0.15***	-0.11***	-0.06***	1.00	
(5) Ind. Barriers to Entry	0.05***	-0.12***	0.12***	0.08***	1.00
(6) Ind. Kurtosis of Forecast Errors	-0.04***	0.39***	-0.13***	0.11***	-0.12***

Table 5. Summary Statistics for exit family firms

Panel A: This panel provides annual summary statistics and difference of mean tests for exit family firms, non-exit family firms, and diffuse shareholders. Exit family firms are matched to non-exit family firms based on propensity score matching techniques using the calendar year, firm age, firm size (natural log of total assets), leverage (long-term debt divided by total assets), firm performance (ROA_{t-1} based on EBITDA), the standard deviation of income before extraordinary items for the prior 20-quarters, equity issuance, and the market value of equity divided by total assets. The propensity score model uses one-to-one firm matching, nearest neighbor, a caliper of 0.02, and a common support range of [0.1 to 0.9]. *t*-statistics are corrected for serial correlation and heteroscedasticity by clustering on the firm-level identifier. ***, **, and * indicate significance at the 1%, 5% and 10% levels, respectively.

Summary Statistics for Family-Exit Firms, Family Firms, and Diffuse Shareholder Firms

	(1)	*(2)*	*(3)*	*(4)*	*(5)*
	Family-Exit Firms	*Non-Exit Family Firms*	*Diffuse Shareholder Firms*	*t-test Family-Exit vs. Non-Exit Family Firms*	*t-test Family Exit vs. Diffuse Shareholder*
Observations	711	711	711	-	-
Industry Innovation (%)	3.28	2.18	3.22	4.39***	0.07
Industry Branding (%)	1.11	1.38	0.92	1.44	1.17
Family Wealth ($ millions)	265	1,047	0.00	5.20***	-
Family Women	7.58	25.21	0.00	5.75***	-
Family CEO	51.11	72.15	0.00	5.25***	-
Unexpected Earning Shocks	0.658	0.592	0.615	2.59***	0.96
Std. Dev. of Forecast Errors	0.011	0.007	0.012	1.80*	0.19
Kurtosis of Forecast Errors	0.829	0.460	0.501	2.06**	1.76*
Number of Analysts	6.33	7.55	7.02	2.27**	1.55
Ln(total assets)	6.50	6.28	6.50	0.67	0.02
Debt Ratio (%)	21.52	21.84	22.14	0.17	0.35
Return on Assets$_{t-1}$ (%)	10.02	10.17	10.80	0.12	0.64
Earnings Volatility(%)	9.49	10.67	9.01	0.43	0.18
Firm Age	34.15	32.32	24.00	0.75	0.19
Tobin's Q	1.56	1.39	1.64	1.01	0.43
Equity Issuance/Assets (%)	0.034	0.033	0.041	0.172	1.122

Table 5. Summary Statistics for exit family firms (Continued)

Panel B: This panel provides Spearman correlation coefficients for the annual measures of family-exit firms and non-exit family firms. Family-exit firms are matched to family firms (and diffuse shareholder firms) based on propensity score matching techniques using the calendar year, firm age, firm size (natural log of total assets), leverage (long-term debt divided by total assets), firm performance (ROA_{t-1} based on EBITDA), the standard deviation of income before extraordinary items for the prior 20-quarters, equity issuance, and the market value of equity divided by total assets. The propensity score model uses one-to-one firm matching, nearest neighbor, a caliper of 0.02, and a common support range of [0.1 to 0.9]. ***, **, and * indicate significance at the 1%, 5% and 10% levels, respectively.

	(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*
(1) Exit Family Firms	1.00						
(2) Industry Innovation	0.18***	1.00					
(3) Industry Branding	-0.11***	-0.10***	1.00				
(4) Ln (Family Wealth)	-0.25***	-0.06***	0.16***	1.00			
(5) Family Women	-0.24***	-0.10***	0.06**	0.27***	1.00		
(6) Unexpected Earning Shocks	0.07***	-0.09***	-0.07***	-0.04**	-0.08***	1.00	
(7) Std. Dev. of Forecast Errors	0.02***	-0.04	-0.02	-0.03	-0.04	0.04	1.00
(8) Kurtosis of Forecast Errors	0.06	-0.01	-0.01	0.02	-0.04	0.02	-0.05

Table 6. High-Low Ownership Analysis

This table presents regression results of High-Low Family Ownership on industry-level characteristics. High-Low family ownership is the level of family ownership less 20%. These regressions include only family firms. *t*-values are reported in parentheses and are corrected for serial correlation and heteroskedasticity by clustering on firm-level identifiers. ***, **, and * indicate significance at the 1%, 5% and 10% levels, respectively.

	Dependent Variable = High-Low Family Control				
	(1)	*(2)*	*(3)*	*(4)*	*(5)*
Industry Innovation	-1.078***	-	-	-	-
	(4.24)				
Industry Entry	-	-0.460***	-	-	-
		(2.41)			
Industry Branding	-	-	1.117***	-	-
			(2.34)		
Industry Barriers to Entry	-	-	-	0.235***	-
				(2.22)	
Ind. Kurtosis of Forecast Errors	-	-	-	-	-0.449***
					(3.50)
Ln (Total Assets)	-0.027***	-0.015***	-0.025***	-0.025***	-0.018***
	(5.20)	(4.06)	(4.82)	(4.64)	(4.24)
Debt Ratio(%)	0.060	0.036	0.088***	0.072**	0.061***
	(1.50)	(1.55)	(2.60)	(2.14)	(3.46)
Return on Assets$_{t-1}$(%)	0.023	0.056	0.086*	0.094*	0.103***
	(0.47)	(1.56)	(1.71)	(1.87)	(2.62)
Tobin's Q	0.003	-0.002	-0.005	-0.005	0.007
	(0.47)	(0.57)	(0.87)	(0.90)	(0.99)
Ln (Firm Age)	0.017**	0.018***	0.018**	0.018**	0.009**
	(1.94)	(2.85)	(2.00)	(1.96)	(2.49)
Equity Issuance/Assets (%)	-0.297***	-0.194***	-0.321***	-0.318***	-0.381***
	(4.52)	(4.08)	(4.88)	(4.81)	(4.17)
Constant	0.193***	0.041	0.130***	0.128***	0.113***
	(3.93)	(1.25)	(2.76)	(2.69)	(4.00)
Year Dummies	Yes	Yes	Yes	Yes	Yes
R^2	0.065	0.039	0.052	0.051	0.034
Observations	5,371	5,371	5,371	5,371	5,371

Table 7. Family Exit Analysis: Family Characteristics

This table reports logit regression results of family-exit firms and family firms on family characteristics. Family-exit firms are matched to non-exit family firms based on propensity score matching techniques using the calendar year, firm age, firm size (natural log of total assets), leverage (long-term debt divided by total assets), firm performance (ROA_{t-1} based on EBITDA), the standard deviation of income before extraordinary items for the prior 20-quarters, equity issuance, and the market value of equity divided by total assets. The propensity score model uses one-to-one firm matching, nearest neighbor, a caliper of 0.02, and a common support range of [0.1 to 0.9]. *z*-values are reported in parentheses and are corrected for serial correlation and heteroskedasticity by clustering on firm-level identifiers. ***, **, and * indicate significance at the 1%, 5% and 10% levels, respectively.

	(1)	*(2)*	*(3)*	*(4)*	*(5)*
Ln (Family Wealth)	-0.878***	-	-	-0.731***	-
	(6.81)			(5.42)	
Family Women	-	-1.531***	-	-1.163***	-
		(4.75)		(3.48)	
Family CEO	-	-	-1.122***	-1.092***	
			(5.32)	(5.02)	
Ln (Family Experience)	-	-	-	-	-0.333**
					(2.17)
Ln (Total Assets)	0.934***	0.109	0.012	0.768***	0.117
	(5.60)	(1.15)	(0.11)	(4.51)	(1.19)
Debt Ratio(%)	-0.646	0.540	0.711	-0.508	0.446
	(1.11)	(1.01)	(1.33)	(0.88)	(1.08)
Return on $Assets_{t-1}$ (%)	1.485	-0.334	0.146	1.383	1.154
	(1.61)	(0.42)	(0.18)	(1.49)	(1.45)
Tobin's Q	0.269***	0.052*	0.037	0.206***	-0.125
	(2.90)	(1.71)	(1.10)	(2.69)	(1.30)
Ln (Firm Age)	0.064	0.212	-0.047	0.052	-
	(0.41)	(1.33)	(0.31)	(0.32)	
Equity Issuance/Assets (%)	-0.542	-0.663	-0.863	-0.402	2.087
	(0.70)	(0.91)	(1.05)	(0.49)	(1.55)
Earnings Volatility(%)	0.105	0.126	0.081**	0.093	-0.098
	(1.39)	(1.36)	(1.98)	(1.40)	(0.58)
Constant	-18.321***	-17.666***	-16.323***	-17.277***	-13.864***
	(17.23)	(22.92)	(17.32)	(16.04)	(11.25)
Industry Dummies	Yes	Yes	Yes	Yes	Yes
R^2	0.201	0.145	0.151	0.241	0.103
Observations	1,422	1,422	1,422	1,422	1,422

Table 8. Family Exit Analysis: Firm Characteristics

This table reports logit regression results of family-exit firms and family firms on firm characteristics. Family-exit firms are matched to non-exit family firms based on propensity score matching techniques using the calendar year, firm age, firm size (natural log of total assets), leverage (long-term debt divided by total assets), firm performance (ROA_{t-1} based on EBITDA), the standard deviation of income before extraordinary items for the prior 20-quarters, equity issuance, and the market value of equity divided by total assets. The propensity score model uses one-to-one firm matching, nearest neighbor, a caliper of 0.02, and a common support range of [0.1 to 0.9]. *z*-values are reported in parentheses and are corrected for serial correlation and heteroskedasticity by clustering on firm-level identifiers. ***, **, and * indicate significance at the 1%, 5% and 10% levels, respectively.

	Family Exit vs. Family Firms				
	(1)	*(2)*	*(3)*	*(4)*	*(5)*
Industry Innovation	12.986***	-	-	-	-
	(3.61)				
Industry Branding	-	-15.402***	-	-	-
		(3.69)			
Unexpected Earning Shocks	-	-	0.584***	-	-
			(3.03)		
Std. Dev. of Forecast Errors	-	-	-	1.153***	-
				(2.51)	
Kurtosis of Forecast Errors	-	-	-	-	0.084**
					(2.21)
Ln (Total Assets)	-0.022	-0.048	0.066	-0.041	-0.127
	(0.26)	(0.99)	(0.69)	(0.35)	(1.54)
Debt Ratio (%)	0.229	-0.071	0.412	0.521	0.168
	(0.51)	(0.25)	(0.77)	(0.70)	(0.28)
Return on $Assets_{t-1}$(%)	0.252	-0.096	0.305	0.703	-0.337
	(0.33)	(0.20)	(0.40)	(0.61)	(0.33)
Tobin's Q	0.021	0.045	0.050*	-0.161	-0.147
	(0.74)	(1.64)	(1.71)	(1.51)	(1.45)
Ln (Firm Age)	0.153	0.130*	0.038	0.055	0.177
	(1.09)	(1.69)	(0.25)	(0.27)	(1.13)
Equity Issuance/Assets (%)	-0.530	-0.387	-0.758	0.598	1.614
	(0.74)	(0.58)	(1.01)	(0.39)	(1.00)
Earnings Volatility(%)	0.076*	0.080***	0.083*	0.598	-0.334
	(1.79)	(3.27)	(1.87)	(0.65)	(0.49)
Ln (Number Analyst)	-	-	-	0.150	0.026
				(0.89)	(0.11)
Constant	-0.785	0.060	-18.201***	-0.599***	0.606
	(1.15)	(0.17)	(16.82)	(0.47)	(0.69)
Industry Dummies	No	No	Yes	Yes	Yes
R^2	0.028	0.013	0.121	0.244	0.188
Observations	1,422	1,422	1,422	881	544