Skewness and time-varying second moments in a nonlinear production network: theory and evidence

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Abstract

This paper studies asymmetry in economic activity over the business cycle. It develops a tractable multisector model in which complementarity across inputs causes aggregate activity to be left skewed. It then examines implications for time-series skewness, cyclicality of cross-sectional dispersion and skewness, and the conditional covariances of sector growth rates, finding support for each in the data. Empirically, skewness grows with the level of aggregation, consistent with the model's implication that it is due to nonlinearity in the production structure. Other prominent models of asymmetry are not able to match the range of empirical facts that the network model can.

1 Introduction

A defining feature of the business cycle is the existence of recessions as distinct episodes. Rather than simply experiencing symmetric random fluctuations around a trend, output, employment, and other aggregate measures of the state of the economy display sharp declines and relatively smooth expansions. In other words, levels and growth in real activity are skewed left.[1] At the same time, many measures of volatility are countercyclical. As

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[1] See Sichel (1993), McKay and Reis (2008), Morley and Piger (2012), Berger, Dew-Becker, and Giglio (2020), and Dupraz, Nakamura, and Steinsson (2021), among others.

a mathematical matter, there is a mechanical link between skewness and countercyclical volatility—high volatility in bad times leads to a long left tail of outcomes—but only a handful of models capture that feature of the economy.

In general, negative skewness endogenously arises in models when they feature a form of concavity, mapping symmetrical shocks into asymmetrical outcomes. That can happen, for example, when there are occasionally binding constraints or capacity limits.[2] In past work the concavity typically arises either because the mapping from shocks to output is concave for individual firms (e.g. Ilut, Kehrig, and Schneider (2018)), or because aggregate shocks have a nonlinear effect on output (e.g. Kozeniauskas, Orlik, and Veldkamp (2018)).

This paper studies a multisector production network in the tradition of Long and Plosser (1983) and Acemoglu et al. (2012). When intermediate inputs are gross complements, we show that aggregate output is a concave function of the micro shocks, even though shocks linearly affect micro units. That is, concavity arises *endogenously* due the aggregation of shocks through the production network. Baqaee and Farhi (2019) show that a model like the one we study generates skewness in aggregate output. The core contribution of this paper is to develop and test a broad range of predictions not just for GDP but also for sector-level output and employment. Those predictions also help theoretically and empirically distinguish the network model from other mechanisms generating concavity and asymmetry.

Mathematically, the equilibrium of the economy has a simple representation. For a given realization of sector-level shocks $(\varepsilon_1, \ldots, \varepsilon_n)$, aggregate and sectoral outputs are given by

$$\log \mathrm{GDP} = f(\varepsilon_1, \ldots, \varepsilon_n) \quad (1)$$

$$\log y_i = a_0 \log \mathrm{GDP} + a_1 \varepsilon_i, \quad (2)$$

respectively, where and a_0 and a_1 are constants that depend on model primitives but not the realization of the shocks. The function f is concave in each ε_i (though also linearly homogeneous; for example, a CES aggregator with elasticity <1). As an extreme example, when production is Leontief in inputs,[3] $f(\{\varepsilon_i\}) = \min_i \varepsilon_i$. Throughout, we assume that sectoral shocks $(\varepsilon_1, \ldots, \varepsilon_n)$ are drawn from a symmetric distribution.

The paper argues that the concave aggregation arising in a network model fits a wide range of features of the economy. In particular, it develops three sets of theoretical predictions.

[2]See, among many others, Kiyotaki and Moore (1997), Gilchrist and Williams (2000), Kocherlakota (2000), Hansen and Prescott (2005), and Bianchi (2011). See also Acemoglu and Scott (1997) and Dupraz, Nakamura, and Steinsson (2021) for alternative mechanisms.

[3]Along with other restrictions on the model.

1. *Unconditional skewness:* Aggregate and sector activity are both skewed left, but the effect is stronger at the aggregate level. The sector-specific component of activity is unskewed.

2. *Cross-sectional moments:* The cross-sectional variance of output is countercyclical and cross-sectional skewness is procyclical.

3. *Conditional covariances:* When a sector receives a negative shock, it subsequently covaries more strongly with other sectors and with aggregate activity.

The key mechanism driving all three predictions is that the (endogenous) function f is concave, immediately generating negative skewness and countercyclical volatility.[4] The fact that sector output has an independent symmetrically distributed component explains why it is less negatively skewed than aggregate output. Empirically, we confirm past results on the negative skewness of aggregate time series, but further show that for industrial production, employment, and stock returns, skewness is significantly more negative at high than low levels of aggregation, by factors of two to five.[5]

On the other hand, when we examine sector-specific shocks in the data—the ε_i—skewness is near zero with tight confidence intervals: all the skewness observed at the sector level is explained by exposure to an aggregate factor. These observations imply that skewness is an aggregate phenomenon, rather than being due to, for example, skewed micro shocks.

In the model, countercyclical cross-sectional variance and procyclical skewness are generated simply because aggregate output, as a concave function of the sector-level shocks, tends to be lower in periods when the (sample) cross-sectional standard deviation happens to be higher or the skewness is lower. The model is thus able to generate countercyclical cross-sectional dispersion—which is observed empirically and often taken as evidence for countercyclical uncertainty—even if the shocks are homoskedastic. That result calls into question empirical analyses of uncertainty based on cross-sectional standard deviations of realized output or employment.[6]

The third set of results on conditional covariances directly addresses the key mechanism in the model. In the presence of complementarities, when the supply of one input shrinks, downstream output becomes relatively more sensitive to it and less sensitive to other inputs. Empirically, following a positive statistical innovation in output in a given sector and month

[4] On countercyclical aggregate volatility, see, recently, Jurado, Ludvigson, and Ng (2015), among many others.

[5] Albuquerque (2012) discusses that fact for stock returns.

[6] E.g. see the discussion in Bloom (2014). Note that the results are also consistent with the findings of Dew-Becker and Giglio (2020) on the acyclicality of conditional cross-sectional volatility.

(which, in the model, identifies ε_i), that sector's industrial production, employment, stock returns, and TFP covary less strongly with other sectors and with aggregate activity for the next three to 12 months.

A single simple idea, then, that production features complementarities, generates a wide range of predictions that can be used to compare the network production model both to the data and other models. While the main theoretical predictions come from a highly stylized (yet tractable) model, using numerical simulations we show that a more realistic model is quantitatively consistent with the empirical results.

Since the paper's starting point is business cycle asymmetry, our last question is whether other mechanisms that generate such asymmetry can also match the additional predictions that this paper generates and tests.

To generate aggregate skewness, one might naturally assume that there are skewed aggregate shocks, such as rare disasters,[7] or perhaps skewness in a universal input (e.g. Brunnermeier and Sannikov (2014)). But such a model does not necessarily generate cyclical cross-sectional moments. Conversely, models of micro uncertainty shocks, such as Bloom (2009) and Christiano, Motto, and Rostagno (2014), imply that cross-sectional moments are cyclical, but they do not have any implications for aggregate skewness.

Finally, one might also consider a model in which skewness arises due to concave decision rules at the *micro* level, but without any nonlinearity in interactions across economic units, as in Ilut, Kehrig, and Schneider (2018). That model can match many of our empirical results, but it counterfactually predicts that the magnitude of skewness is greater at the micro than the macro level. Specifically, linear aggregation in that model causes micro skewness to wash out.

In other words, all three alternative models we consider fail to match the data on at least one dimension, whereas the network production model can match all three sets of facts parsimoniously. The failures of the models are instructive: the results on time-series skewness point to the existence of a skewed common component in output—that is, skewness arises at the aggregate rather than micro level—while the cross-sectional results imply that the common component is *endogenous* to sector shocks. The explicit aggregation emphasized by Baqaee and Farhi (2019), represented in the reduced form (1–2), creates that endogeneity.

Related work As already mentioned, the paper is related to the literature on time-variation in time-series and cross-sectional moments of output.[8] It also belongs to the

[7] E.g. Barro (2006), Gourio (2012, 2013), and Wachter (2013)

[8] For work on unconditional skewness, see Sichel (1993), McKay and Reis (2008), Morley and Piger (2012), and Berger, Dew-Becker, and Giglio (2020). For recent work on time-varying volatility in the real

growing literature studying the role of production networks in propagating and amplifying shocks (Long and Plosser, 1983; Acemoglu et al., 2012).[9] Particularly relevant is the body of work that emphasizes complementarities in production, such as Horvath (2000), Jones (2011), Atalay (2017), and Baqaee and Farhi (2019).

Empirically, Atalay (2017) and Atalay et al. (2018) estimate the elasticity of substitution between intermediate inputs using sectoral data and find evidence for strong complementarity, as do Barrot and Sauvagnat (2016), Boehm, Flaaen, and Pandalai-Nayar (2019), Carvalho et al. (2021), and Peter and Ruane (2020) at the firm level. Those papers focus on testing the micro implications of network models, while this paper's contribution is to test the model's predictions for the joint distribution of output, employment, and stock returns at the sector and aggregate levels, and also to distinguish it from other theories. It thus builds on the literature studying the joint behavior of aggregate and sectoral output.[10]

More generally, this paper is related to the broader literature that studies the macroeconomic impacts of microeconomic shocks, such as Gabaix's (2011) work on granularity (among many others). More recently, Gourieroux et al. (2020) examine how shocks to systemically important firms may affect aggregate consumption and asset prices (see also Seo and Wachter (2018)).

2 Skewness in aggregate activity

The fact that both levels and growth rates of real activity and stock returns are skewed left has been established in previous work.[11] This section provides a brief overview of some evidence on asymmetry in aggregate activity.

The first column in Table 2 reports skewness coefficients (the scaled third moment) for growth rates (Panel A) and levels (Panel B) of six measures of aggregate activity: industrial production, employment, stock market returns, GDP, consumption, and investment. In

economy, see Justiniano and Primiceri (2008), Clark and Ravazzolo (2015), and Schorfheide, Song, and Yaron (2018). Work on time-varying cross-sectional moments includes Guvenen, Ozkan, and Song (2014), Salgado, Guvenen, and Bloom (2020), and Dew-Becker and Giglio (2021).

[9] For other recent related work, see Grassi and Sauvagnat (2019), Frohm and Gunnella (2021), and Liu and Tsyvinski (2020).

[10] E.g. Horvath (2000), Foerster, Sarte, and Watson (2011), Carvalho and Gabaix (2013), Atalay, Drautzburg, and Wang (2018), and Caliendo et al. (2018)

[11] Berger, Dew-Becker, and Giglio (2020) show that growth rates of employment, capacity utilization, industrial production, GDP, durable and non-durable consumption, and residential and nonresidential investment are all skewed left. Furthermore, returns on the S&P 500 are skewed left, as is their option-implied distribution. Morley and Piger (2012) provide a much more thorough analysis of asymmetry in the output gap—that is, on skewness in levels, rather than growth rates—and finding similar results— while Sichel (1993) provides an earlier analysis distinguishing asymmetry in levels from growth rates. See also references therein for the literature on business cycle asymmetry.

all cases, here and below, levels are detrended with an exponentially weighted moving average. p-values for a two-sided test against the null of zero skewness from a block bootstrap are reported in brackets. Across the six series, in both levels and growth rates, the skewness coefficients are negative in all cases. In terms of magnitudes, industrial production features the most skewed distribution (-1.22) followed by investment (-0.86) for growth rates. Investment (-1.20) and stock returns (-1.13) are most negatively skewed in levels.

The remaining columns in Table 2 report results for a nonparametric measure of asymmetry. Denote the mean and standard deviation of some variable x by μ_x and σ_x, respectively, and the empirical cumulative distribution function as $\hat{F}_x(z)$. The table reports the following tail probability ratios:

$$\text{tail probability ratio} = \frac{\hat{F}_x(\mu_x - k\sigma_x)}{1 - \hat{F}_x(\mu_x + k\sigma_x)} \tag{3}$$

for various values of k. This ratio measures the relative probability that x is k standard deviations below its mean compared to the probability it is k standard deviations above its mean. If the left tail is asymptotically heavier than the right, in the sense that the probability ratio diverges to ∞, then choosing a large value of k will produce a larger ratio. At the same time, though, when k is larger, the probability ratio is calculated based on fewer observations. So the ability to reject the null that the ratio is equal to 1 will, heuristically, peak for some finite k.[12]

The cutoff k ranges in the table between 1 and 3, with power appearing to peak around values of 1.5 and 2. In all cases, as k rises, the relative probability of negative deviations rises. In other words, large negative deviations in both levels and growth rates of the six variables studied in Table 2 have been far more common than large positive deviations. At $k = 1.5$ and $k = 2$, a number of the ratios are statistically significantly larger than 1 at conventional levels, but that statistical significance is not uniform across all estimates. Compared to the results for the coefficient of skewness above, the tail probability ratios are relatively more weakly measured statistically. So while the tail measures are attractive for being simple and nonparametric, they face the usual tradeoff of also having somewhat lower power. The tail measures have the advantage, though, that they map directly into certain theoretical results developed below.

To summarize, Table 2 shows that in the empirical sample we study, major measures of aggregate activity are consistently skewed left according to a range of different measures.

[12]For example, for $k = 10$ the sample CDFs will both be equal to zero and the ratio undefined—we have no 10-standard-deviation events in our sample.

While the results are individually only marginally statistically significant, they overall tell a consistent story of a long left tail, and we take that left skewness as the basic starting point for the remainder of the analysis.

Time-series skewness is closely related to countercyclical volatility. If volatility rises when a variable falls, it will tend to have a long left tail (since $\mathbb{E}[x^3] = \mathbb{E}[x \cdot x^2]$, negative skewness means that the level of a series covaries negatively with its square). There is a large literature studying countercyclicality of aggregate volatility. Jurado, Ludvigson and Ng (2015), for example, show that the volatility of forecast errors for aggregate outcomes is significantly countercyclical.

3 Nonlinear network model

This section presents our benchmark model. Baqaee and Farhi (2019) and Dew-Becker (2021) show that production networks with complementarity generate negative skewness in general. The goal of this paper's theoretical analysis is to develop a range of additional predictions that describe the broader behavior of the economy. To do so, we set the model up to be as simple as possible, which will allow us to obtain transparent analytic solutions and formally derive testable predictions, using a form very similar to that of Jones (2011). Section 6 examines results from a numerical solution of a richer and more realistic specification.

Consider an economy consisting of n sectors, each producing a distinct product according to the technology

$$y_{i,t} = z_{i,t}\zeta_i \ell_{i,t}^{1-\alpha} \left(\sum_{j=1}^{n} a_j^{1/\xi} x_{ij,t}^{(\xi-1)/\xi} \right)^{\alpha\xi/(\xi-1)}, \tag{4}$$

where $z_{i,t}$ is sector i's productivity on date t, ℓ_i is sector i's use of labor, $x_{i,j}$ is sector i's use of material input j, and ξ is the elasticity of substitution across material inputs. $\zeta_i = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}$ is a normalization constant.

The nonnegative constants $\{a_j\}$ determine the importance of inputs in the intermediate input bundle and are normalized such that $\sum_{j=1}^{n} a_j = 1$. Note that these weights do not depend on the identity of the using sector i, implying that all sectors use the same mix of inputs for production. While restrictive, that symmetry assumption allows closed-form characterization of equilibrium.

Final consumption, which constitutes all of GDP, is

$$\text{GDP}_t = \left(\sum_{j=1}^{n} a_j^{1/\xi} c_{j,t}^{(\xi-1)/\xi} \right)^{\xi/(\xi-1)}, \tag{5}$$

where $c_{j,t}$ is consumption of good j at time t. The elasticity of substitution across goods and the weight of each good is the same in final consumption as in sector production. In addition to consuming, households inelastically supply a single unit of labor to the firms.[13]

The competitive equilibrium of the economy is a collection of prices and quantities such that (i) firms in all sectors maximize their profits taking all prices as given, (ii) the representative household maximizes utility, taking prices as given, and (iii) all markets clear, with the market-clearing condition for good j given by

$$c_{j,t} + \sum_{i=1}^{n} x_{ij,t} = y_{j,t}. \tag{6}$$

Since the model is fully static, we drop the time subscripts when they are unnecessary. The appendix shows that, in equilibrium, sector and aggregate output are given by

$$\log y_i = (1 - \xi + \alpha\xi) \log(\text{GDP}) + \xi \log z_i + \delta_i \tag{7}$$

$$\log(\text{GDP}) = \frac{1}{(\xi - 1)(1 - \alpha)} \log \sum_{i=1}^{n} a_i z_i^{\xi-1}, \tag{8}$$

respectively, where δ_i is a constant that is independent of the shocks. Employment in each sector satisfies

$$\log \ell_i = (1 - \xi)(1 - \alpha) \log(\text{GDP}) + (\xi - 1) \log z_i + \log a_i. \tag{9}$$

Two observations are immediate. First, equation (8) illustrates that aggregate output is a CES aggregate over the sector productivities with elasticity ξ. When the inputs are gross complements—i.e., when $\xi < 1$—aggregate output is a concave function of the sector productivities.

Second, equations (7) and (9) show that sector output and employment have a factor structure: they are both simply equal to a multiple of aggregate output plus a shock. That factor structure will be the source of some of the key results, but the core feature is not just the existence of a factor structure but rather that the common factor—aggregate output—is a *concave* function of the sector shocks.[14]

Since log output and employment in each sector have the same basic factor structure (though with different coefficients), we discuss results in what follows for sector output, but

[13] In Appendix A.2, we show that our results continue to hold if we instead assume that labor is sector-specific (in the sense that it cannot be relocated among sectors) and is supplied elastically.

[14] The factor structure for labor may initially be surprising, given that aggregate employment is fixed at 1. Note, though, that the factor representation is in terms of logs.

the results all go through equivalently for sector employment and the empirical analysis will study both.

For the rest of the paper we use $\varepsilon_{i,t} = \log z_{i,t}$ to denote the log productivity shock to sector i. We assume that the $\varepsilon_{i,t}$ are distributed symmetrically around zero with full support on the real line, both conditionally and unconditionally. This assumption guarantees that any skewness in log output will then arise endogenously through the production and aggregation process.

4 Testable predictions of the model

This section uses the closed-form solution to the model to develop a series of empirical predictions. The predictions are divided into three basic categories, all of which take advantage of sector-level information: the unconditional distribution of sector activity; time-variation in the cross-sectional distribution of activity; and conditional variances and covariances.

4.1 The unconditional distribution of real activity

Let $\mu = \mathbb{E}[\log \text{GDP}]$ and $\sigma = \text{stdev}(\log \text{GDP})$

Proposition 1. *Suppose log productivity shocks are drawn from a common distribution* $F(\cdot)$ *with density* $f(\cdot)$ *such that* $\lim_{\tau\to\infty} \frac{f(\tau+k)}{F(-\tau)f(\tau)} = \infty$ *for all* k. *If* $\xi < 1$, *then* $\log \text{GDP}$ *is negatively skewed in the sense that*

$$\lim_{\tau\to-\infty} \frac{\mathbb{P}(\log \text{GDP} < \mu - \tau\sigma)}{\mathbb{P}(\log \text{GDP} > \mu + \tau\sigma)} = \infty. \tag{10}$$

If $\xi > 1$, *then aggregate output is positively skewed, in the sense that above limit is equal to zero.*

Note that proposition 1 uses a concept of skewness different from the more common scaled third moment (though there are many measures of skewness studied in the literature). The limit in (10) describes the tails of the distribution of aggregate output. The advantage of this approach is that it is possible to derive formal results that hold irrespective of the specific details of intermediate input mix and elasticities, and for a wide range of shock distributions.[15] Furthermore, due to its asymptotic nature, the result in (10) is also

[15]For example, the distributional restriction in Proposition 1 holds for the entire family of Weibull-tailed distributions (such as normal, exponential, and subexponential Weibull distributions), as well as Pareto-tailed distributions with finite mean and standard deviation.

insensitive to changes in any finite moment of the distribution: for example, any change in the mean or variance of any of the productivity shocks has no effect on the limit.

Proposition 1 refers to levels, due to the static setup of the model. Appendix A.2.5 provides results for growth rates.

Under the assumption that intermediate inputs are gross complements, Proposition 1 leads to the following prediction:

Prediction 1a. Log aggregate and sector output are both skewed left, and the magnitude is greater for aggregate output.

The intuition for that result is simple: log sector output is equal to log aggregate output plus a symmetrically distributed shock. As a result, the skewness of $\log y_i$ has to be less than that of $\log \mathrm{GDP}$. Appendix A.2.2 extends the model to more explicitly have multiple layers of aggregation and shows that the increasing magnitude of skewness holds across those layers.

To push this result further, note that the decomposition in equation (7) indicates that sector output inherits its skewness from the that of the aggregate output. This is because it is fundamentally the aggregation process (through the nonlinearities in the economy's production structure) that generates skewness in the first place, as sectoral shocks themselves are not skewed. So if we extract the sector-specific component of activity, we should not observe any skewness:

Prediction 1b. The residual from a regression of log sector output on log aggregate output is not skewed.

Equation (7) says a regression of $\log y_i$ on $\log \mathrm{GDP}$ identifies, as its residual, $\varepsilon_i = \log z_i$ (up to an affine transformation). Examining the relative skewness of the ε and $\log \mathrm{GDP}$ then tests a core feature of the model: that macro skewness is caused by the aggregation process (i.e. it is systemic), rather than by skewness in sector- or firm-level shocks.

4.2 The cross-sectional distribution of activity

There is a long-running literature that studies the cyclicality of cross-sectional dispersion in outcomes. The network model has strong implications for dispersion. Since aggregate output is a concave function of the sector-level shocks, an increase in their dispersion mechanically reduces aggregate output, simply through a Jensen's inequality effect. To see that, a cumulant-type expansion for output yields,

$$\log \mathrm{GDP} \approx \frac{1}{(1-\alpha)} \sum_{m=1}^{3} \frac{1}{m!} (\xi - 1)^{m-1} \mu_m (\{\varepsilon_j\}), \tag{11}$$

where $\mu_m(\{\varepsilon_j\})$ is the sample mth moment of the cross-sectional distribution of productivity. The approximation shows that high dispersion and negative skewness in the cross-sectional distribution of productivity reduce GDP when $\xi < 1$. That can be used to derive the following formal result (see appendix A.1):

Proposition 2. *Let μ_2 and μ_3 denote the cross-sectional variance and skewness of the realized log productivity shocks. If $\xi < 1$, then, to a first-order approximation in $\xi - 1$,*

$$\mathrm{cov}(\mu_2, \log \mathrm{GDP}) < 0 \tag{12}$$

and if, in addition, the sectoral shocks have positive excess kurtosis,

$$\mathrm{cov}(\mu_3, \log \mathrm{GDP}) > 0 \tag{13}$$

In other words, when inputs are complements, the cross-sectional variance and skewness of realized shocks are, respectively, countercyclical and procyclical. Furthermore, when coupled with equation (7), Proposition 2 also implies that the same cyclical behavior also holds for the cross-sectional distribution of sectoral output. This is a simple consequence of the fact that, according to (7), the cross-sectional distribution of sectoral output is a scaled version of the cross-sectional distribution of shocks.[16]

Proposition 2 then leads to the following pair of predictions:

Prediction 2a. The cross-sectional variance of log sector output (and the sector-specific residuals) is countercyclical.

Prediction 2b. The cross-sectional skewness of log sector output (and the sector-specific residuals) is procyclical.

As above, the cyclicality of the cross-sectional distribution is not caused by changes in conditional distributions or "time-varying uncertainty." The cross-sectional distribution is a random variable that is correlated with output, even though there are no shocks to the volatility of sector-level shocks. Proposition 2 shows that there is a mechanical relationship between the cross-sectional sample moments and aggregate output.

[16] The fact that the unweighted moments (e.g. without any reference to sector size) appear here is notable, and in this case justifies the usual empirical practice of just looking at raw cross-sectional moments.

Finally, note that the predictions here rely on the fact that aggregate output is a function of the sector shocks. If aggregate output were independent of the sector-specific shocks—e.g. if the sector shocks "washed out" through linear aggregation, as in the Cobb–Douglas benchmark—then the cross-sectional distribution of shocks would be uncorrelated with aggregate output.

4.3 Conditional covariances

The results so far are about the contemporaneous relationship between sectoral and aggregate variables. The last result is about the time-series implications of the model. To this end, and just in this section, we impose a further assumption that the log productivities, $\varepsilon_{i,t}$, follow AR(1) processes (up to now, nothing has depended on the dynamic properties of productivity), which allows a characterization of the dynamic properties of conditional moments:

Proposition 3. *Suppose $\xi < 1$ and that log productivity shocks follow AR(1) processes with positive persistence. Then, for all distinct pairs of sectors $i \neq k$,*

$$\frac{\mathrm{d}\Delta_t^{ik}}{\mathrm{d}\varepsilon_{i,t}} < 0 \qquad \text{and} \qquad \frac{\mathrm{d}\Xi_t^{ik}}{\mathrm{d}\varepsilon_{i,t}} < 0, \tag{14}$$

where

$$\Delta_t^{ik} = \sum_{j \neq i} \mathrm{cov}_t(\log y_{i,t+1}, \log y_{j,t+1}) - \sum_{j \neq k} \mathrm{cov}_t(\log y_{k,t+1}, \log y_{j,t+1}) \tag{15}$$

$$\Xi_t^{ik} = \mathrm{cov}_t(\log y_{i,t+1}, \log \mathrm{GDP}_{t+1}) - \mathrm{cov}_t(\log y_{k,t+1}, \log \mathrm{GDP}_{t+1}), \tag{16}$$

and cov_t denotes the covariance conditional on information available on date t.

That result leads to the following predictions:

Prediction 3a. Following a negative innovation in $\varepsilon_{i,t}$, sector i's conditional covariances with other sectors rise relative to other sectors' covariances with one another.

Prediction 3b. Following a negative innovation in $\varepsilon_{i,t}$, sector i's conditional covariance with aggregate activity rises.

Intuitively, Proposition 3 and predictions 3a and 3b follow from the fact that when a sector receives a negative shock, it becomes relatively more important in determining variation in aggregate output when $\xi < 1$.[17]

Proposition 3 formalizes the idea that when inputs are gross complements, when a sector receives a negative shock it becomes more "central" than other sectors in the sense that sectoral and aggregate outputs covary more strongly with sector i's output. This is yet another core prediction of the model, in that it directly tests the idea that aggregate output is a concave function of the sector shocks. When aggregation is concave, it is exactly the sectors that receive negative shocks that should rise most in importance. We show formally below that most other models of aggregate skewness do not generate such a prediction for time-varying centrality.

4.4 Extensions

Appendix A.2 reports results for two variations of the baseline model that relax some of the strict assumptions above. Appendix A.2.3 shows that the characterization in equations (7) and (8) remain valid with minor modifications in the presence of (i) elastic labor supply or (ii) sector-specific factors of production (say, capital or inflexible labor). Those results show that Predictions 1–3 are robust to the specific assumptions made on factor elasticities and mobility.

Appendix A.2.4 shows that sectoral and aggregate payments to any fixed factor share the same characteristics as sectoral and aggregate output. To the extent that stock returns move with payments to capital, then, the predictions in Section 4 also apply to stock returns, which we study in the empirical analysis. That said, the analysis here is certainly far from a fully realistic model of equity markets.

Finally, appendices A.2.1 and A.2.2 extend the model to more explicitly allow multiple layers of aggregation, as in the data studied below. Section A.2.1 models firms within each sector and derives results for skewness at the firm, sector, and aggregate levels when the outputs of firms within each sector are substitutes while those across sectors remain complements. Section A.2.2 allows for multiple layers of aggregation across sectors and shows that with complementarity skewness becomes progressively more negative across levels of aggregation.

[17] Note that levels and growth rates generate the same predictions since they have the same conditional distributions up to a level shift.

5 Empirical analysis

5.1 Data

Macroeconomic data: The analysis focuses on measures of activity that have data at the monthly frequency or higher and are measured at a high level of sectoral detail. Higher-frequency data yields more power for estimating skewness, but we also report results for some lower-frequency series.

The two monthly series are industrial production (IP), available from the Federal Reserve website, which is measured at up to the five-digit NAICS level of detail in manufacturing industries, and employment, available from the Current Employment Survey of the U.S. Bureau of Labor Statistics (BLS), which is measured up to the six-digit NAICS level and covers the entire economy. For industrial production, we follow Foerster, Sarte, and Watson (2011) and study data since 1972. For employment, the sample with detailed NAICS coverage begins in 1990, while data on two-digit BLS-defined supersectors is available since 1972.

Stock returns: Stock returns have the drawback that they do not directly measure activity, being driven not just by current conditions but also by expectations for the future (and discount rates). However, returns are measured at much higher frequencies—we use up to daily data—which is useful for estimating time-variation in conditional moments. For sector-level measures of stock returns, we construct value-weighted portfolios according to SIC sectors, requiring at least five firms in a given sector/month pair to include it in the analysis.

The fact that the sample extends to 1972 means that it includes six recessions, giving a sample long enough that we can have some confidence that the results are not just driven by a single episode. In addition, the sample ends in 2019, so that the large movements in the economy associated with Covid are excluded.

Measures of sector activity have both high- and low-frequency features, corresponding to transitory shocks and also permanent changes in the structure of the economy. By focusing on monthly data, the analysis is primarily aimed at understanding high-frequency features, which is consistent with the fact that the model has a fixed network structure. At lower frequencies, production technologies change, affecting the structure of the economy, which is outside the scope of this paper's analysis.

5.2 Prediction 1: Time-series skewness

Figure 1 reports skewness across different levels of aggregation for industrial production, employment, and stock returns. At a given level of aggregation, we calculate the skewness coefficient (the scaled third moment) in each sector's time-series, and then report the average of those skewness coefficients at each level of aggregation. 90-percent confidence bands are plotted for each estimate.[18]

5.2.1 Prediction 1a: Skewness declines with the level of aggregation

The first and second columns of panels in Figure 1 plot estimates of time-series skewness at the aggregate and sector levels for IP, employment, and stock returns, in both growth rates and levels. Squares represent point estimates and circles represent the difference between each sector's average skewness and overall aggregate skewness.

The top panels report results for industrial production. The skewness of total industrial production growth is -1.22. At the two-digit level—just three sectors: durable and nondurable manufacturing and mining—average skewness is -0.96. At the three- and four-digit levels, where there are 43 and 81 total sectors, respectively, skewness declines in magnitude to -0.55 then -0.45. Finally, at the five-digit level skewness is only -0.41.

At the point estimates, the skewness of IP growth is *three times* greater at the aggregate level than for the most disaggregated sectors. The red series show confidence bands for those differences, and three of the four are statistically significant.

The second and third rows report results for employment growth and stock returns. For aggregate employment, skewness is -1.49, compared to -0.30 at the five-digit level.[19] The pattern is similar for stock returns, where aggregate skewness is -0.65, while average skewness at the five-digit level is only -0.30. The results for employment are statistically slightly weaker than for industrial production, while those for stock returns are the strongest. Since returns are uncorrelated over time, while employment and IP are substantially serially correlated, the number of effective observations for returns is much larger, increasing statistical power.

If there is measurement error in sector-level data, and it is greater at lower levels of aggregation, then that could potentially explain the skewness patterns in figure 1. However,

[18] The standard errors are calculated with a blockwise jackknife that clusters by date. Specifically, each jackknife replication removes 50 consecutive months of data from the sample—the same 50 months for all sectors—and we iterate over all possible starting months for the excluded dates; see Lahiri (2003).

[19] Recall that in the model aggregate employment is fixed, so the model cannot generate the negative skewness observed in aggregate empoyment. However, appendix A.2.2 shows that skewness still becomes more negative across levels of aggregation, just not including the full economy.

to do so would require so much noise that the monthly data would in fact have more noise than signal (see appendix A.3 for the calculation). Since we are using the fully revised data (rather than preliminary estimates) for industrial production and employment, errors of that magnitude are unlikely. And for stock returns, the most likely source of "measurement" error would be bid/ask spreads, which are small in comparison to the monthly returns used for Figure 1. In addition, Figure A.1 shows that the same results hold in annual data, where measurement error is minimal, especially for employment since it is based on complete population data from the Census.

To summarize, we find strong evidence for the first prediction of the model: skewness is greater—by a factor of two to five—at higher levels of aggregation.

5.2.2 Prediction 1b: Sector residuals are unskewed

In the model, sector shocks can be identified from a regression of sectoral on aggregate activity. To that end, we estimate the regressions

$$\begin{aligned} y_{i,t} &= a_i + b_i y_t + \nu_{i,t}, && (17) \\ \Delta y_{i,t} &= a_{\Delta,i} + b_{\Delta,i} \Delta y_t + \nu_{\Delta,i,t}, && (18) \end{aligned}$$

where $y_{i,t}$ denotes some measure of activity in sector i and Δ is the first-difference operator. The third and fourth columns of panels in Figure 1 plot skewness for the residuals $\nu_{i,t}$ and $\nu_{\Delta,i,t}$ as well as the difference between skewness for the residuals and skewness for the original data, $y_{i,t}$, at the same level of aggregation. Recall that the prediction of the theoretical model is that $\nu_{i,t}$ and $\nu_{\Delta,i,t}$ are unskewed, since skewness in sector activity in the model is due to exposure to the aggregate component.

Figure 1 shows that the skewness of the residuals is, across all variables, economically close to zero. In each case it is less negative than the skewness for the original data, and it is typically substantially smaller than -0.5. For the returns data, skewness for residuals is indistinguishable from zero, and for industrial production it is less than half the magnitude that is observed in the original data. The blue data series display the difference in skewness between the residuals and the raw data, showing that the difference is consistently statistically meaningful.

So while there is strong evidence for negative skewness in raw growth rates in line with prediction 1a, the distribution of residuals displays little to no skewness, consistent with prediction 1b. This finding further strengthens the notion that skewness is an endogenous outcome: sector-specific shocks themselves add little to asymmetry across sectors. Rather, it is how those shocks combine into aggregate outcomes that matters.

The analysis here is fairly stark in that the only common factor used to isolate the sector-specific growth rates in equations (17) and (18) is the aggregate series. Figure A.1 in the appendix reports results for a version of the analysis where the first five principal components of the cross-section of sector-level data are included as additional factors and shows that the findings are nearly identical. Atalay et al. (2018) also estimate sectoral productivity shocks, again based on a richer model that has more than just a single factor, and find, for a low elasticity of substitution as studied here, that average skewness is close to zero, consistent with the findings in Figure 1. Finally, in terms of the model, the estimated residuals in (17) and (18) represent TFP shocks. In the NBER-CES manufacturing dataset, TFP shocks, measured through growth accounting, also have skewness close to zero. A variety of different estimation schemes thus all find that sector-level shocks appear to be close to symmetrically distributed.

5.3 Prediction 2: The cross-sectional distribution

The second set of predictions of the model is about the cyclicality of the cross-sectional distribution of sector activity. In each month t, we calculate the cross-sectional variance and skewness of monthly growth rates of industrial production and employment at the four-digit NAICS level. We also calculate the mean across days within each month of the cross-sectional variance of sector-level stock returns.[20] In all three cases, we calculate cross-sectional moments not just for total sector activity, but also from residuals from a regression of sector activity on aggregate activity ($\nu_{\Delta,i,t}$ in (18)). That eliminates the effects of exposure to any common component, and in the model corresponds to sector-level technology shocks (again, the results are highly similar when more factors are included).

It is important to emphasize that these are *realized* sample moments—they do not measure a conditional distribution, so they do not tell us whether or not the conditional probability density from which the sector growth rates are drawn changes over time. We are simply measuring sample moments—which are random variables—and examining their contemporaneous relationship with the state of the business cycle. To measure the cyclicality

[20]That is, using daily returns in sector i, $r_{i,d}$, the monthly cross-sectional variance is $\frac{1}{\#(days \in t)} \sum_{d \in t} \text{var}_d\left(r_{i,d}\right)$ where var_d is the cross-sectional standard deviation on day d. We use 140 three-digit SIC sectors for stock returns for data availability reasons—there are not enough firms in the CRSP dataset to calculate sector returns with too much detail.

of cross-sectional variance and skewness, we estimate the regressions

$$\text{variance}_t = \alpha_v + \beta_v \times \text{economic activity}_t + \eta_{v,t}, \tag{19}$$

$$\text{skewness}_t = \alpha_s + \beta_s \times \text{economic activity}_t + \eta_{s,t}, \tag{20}$$

where economic activity$_t$ is proxied either by an NBER recession indicator or aggregate employment growth and the η's are residuals.

Panel A of Table 3 reports results from univariate regressions of the cross-sectional variances of both levels and growth rates of industrial production and employment on an NBER recession indicator and aggregate employment growth as two different measures of the state of the business cycle. The cross-sectional variance, skewness, and aggregate employment growth are normalized to have unit variance to help in interpreting the coefficients.

Consistent with the predictions of the model and with previous work (Davis and Haltiwanger (1992), Ilut, Kehrig, and Schneider (2018), and Salgado, Guvenen, and Bloom (2020)), we find that there are statistically and economically significant increases in cross-sectional dispersion when the economy is weak. Across the various estimates, in both levels and growth rates, and for raw measures and sector-specific residuals, variance is on average higher by 0.79 standard deviations during recessions and the correlation with aggregate employment growth is -0.31. Moreover, across all coefficients, the point estimate has the predicted sign in every case (and is statistically significant in all but four).

Beyond what has been documented in past work, Table 3 also shows that the cyclicality of the cross-sectional variance holds not just for total activity at the sector level, but also the sector-specific residuals. That is, the cyclicality in cross-sectional variance is not due just to exposures to a common factor. After extracting that factor, and looking just at the sector-specific part—which corresponds to the sector shock in the model—the cross-sectional variance remains countercyclical. That fact is not consistent with all structural models, as we discuss further in section 7.

Table 3 also reports analogous results for cross-sectional skewness. In this case, we find that skewness is procyclical. On average, the estimates imply that skewness is more negative by 0.28 standard deviations in recessions and has a correlation of 0.10 with aggregate employment growth. The results are statistically weaker than for variances, which is to be expected as skewness is more poorly estimated than variance. Again, while cross-sectional skewness for total activity has been found to be procyclical elsewhere, these results are novel for showing that the same result holds for the sector-specific component of activity.

5.4 Prediction 3: Conditional moments

This final section tests a core prediction of the network model: since GDP is a concave function of the sector shocks, when a sector receives a negative shock it should become more central.

We examine variation in centrality based on conditional covariances for three datasets: (1) employment and industrial production, directly measuring activity at the monthly level; (2) stock returns, indirectly measuring activity, but at the daily level; and (3) total factor productivity, which is most tightly linked to the model primitives, but available only at the annual level.

5.4.1 Industrial production and employment

Define Σ_t to be the (unobservable) conditional covariance matrix of sector-level growth rates on date t. Define $\Sigma_{t,i}$ to be the average of the i'th column of Σ_t, excluding the (i,i) element. $\Sigma_{t,i}$ is the average of the covariances of sector i with all other sectors. When we say that sector i covaries more strongly with other sectors, we mean $\Sigma_{t,i}$ rises. We also define $\beta_{i,t}$ to be the conditional covariance of activity in sector i with aggregate activity.

The goal is to estimate relationships of the form

$$\tilde{x}_{i,t} = \tilde{a}_i + \sum_{j=0}^{J-1} \tilde{b}_j \varepsilon_{i,t-j} + \tilde{c}_t + \tilde{\eta}_{i,t}, \tag{21}$$

for $\tilde{x}_{i,t}$ equal to $\Sigma_{t,i}$ or $\beta_{i,t}$ and where $\varepsilon_{i,t}$ measures the innovation to the level of activity in sector i on date t (included up to lag $J-1$). $\tilde{a}_i$, $\tilde{b}_j$ and $\tilde{c}_t$ are coefficients and $\tilde{\eta}_{i,t}$ is a residual. The problem is that $\Sigma_{t,i}$ and $\beta_{i,t}$ are not observable. We therefore proxy for them with date-t products, similar to the literature on heteroskedasticity and feasible generalized least squares.

More specifically, define $\varepsilon_{i,t}$ to be the statistical innovation in activity in sector i (i.e. from a forecasting regression).[21] Note that $\varepsilon_{i,t}$ here is not meant to capture the sector-specific TFP shock. Rather, it is just the innovation in sector output conditional on date-$(t-1)$ information, which will contain both sector and aggregate components.

We then proxy for $\Sigma_{t,i}$ with $\sum_{j\neq i} \varepsilon_{i,t+1}\varepsilon_{j,t+1}$ and $\beta_{i,t}$ with $\varepsilon_{i,t+1}\varepsilon_{agg,t+1}$ (where $\varepsilon_{agg,t}$ is the statistical innovation in aggregate activity). Those products are single-observation sample

[21] Specifically, we forecast activity in each sector using four lags of sector activity and the lagged value of the first three principal components of activity across all sectors.

moments when the conditional expectation of $\varepsilon_{i,t}$ is zero, with the property that

$$\mathbb{E}_t\left[\sum_{j\neq i}\varepsilon_{i,t+1}\varepsilon_{j,t+1}\right] = \Sigma_{t,i}, \tag{22}$$

$$\mathbb{E}_t\left[\varepsilon_{i,t+1}\varepsilon_{agg,t+1}\right] = \beta_{i,t}. \tag{23}$$

This leads to the following regression

$$x_{i,t} = a_i + \sum_{j=0}^{J-1} b_j \varepsilon_{i,t-j} + c_t + \eta_{i,t}, \tag{24}$$

$$\text{where } x_{i,t} = \sum_{j\neq i}\varepsilon_{i,t+1}\varepsilon_{j,t+1} \text{ or } \varepsilon_{i,t+1}\varepsilon_{agg,t+1}, \tag{25}$$

and $\eta_{i,t}$ captures both the true residual, $\tilde{\eta}_{i,t}$, and also the measurement error in the dependent variable, $x_{i,t} - \tilde{x}_{i,t}$ (e.g., $\varepsilon_{i,t+1}\varepsilon_{agg,t+1} - \beta_{i,t}$). The errors are therefore in general non-Gaussian. Because there may be common components across sectors in the innovations, $\varepsilon_{i,t}$, we include time fixed effects in the estimation (c_t) and cluster the standard errors by date. Similarly, some sectors will covary with others more strongly on average, so we also include sector fixed effects, a_i. The inclusion of time fixed effects means that changes in the conditional moments are all interpreted as changes relative to those in other sectors. For example, since the date-t mean of $\Sigma_{t,i}$ is equal to the mean of all pairwise covariances, positive values for the b_j coefficients mean that a positive shock to sector i raises its covariances *relative to those between other sectors*. Because the $x_{i,t}$ variables are functions of date-$t+1$ observations, the regressions all represent forecasts and hence conditional moments. That is, the fitted value of the right-hand side is a date-t conditional expectation.

The top three rows of Table 4 report results of the forecasting regressions for IP and employment. In each case, we use the level of aggregation that yields the largest number of sectors. For IP it is the 4-digit level. For employment, we use 2-digit data that extends to 1972 and 5-digit when using data since 1990 in separate regressions. In all cases, we use three monthly lags of activity on the right-hand side ($J = 3$) and report the sum of the coefficients in the table. Standard errors clustered by date are reported in brackets.

For the regressions forecasting $\Sigma_{t,i}$ and $\beta_{i,t}$, the estimated coefficients are negative for both IP and employment. For the first two rows, the coefficients are of similar magnitude, about -0.05, while they are close to zero for the short employment sample. Since the variables are all standardized, a value of -0.05 implies that when a sector's activity rises by one standard deviation, the product on the left-hand side variable falls by 0.05 standard deviations. The regressions thus give consistent support to the model's prediction that following a negative shock, a sector becomes more central and more correlated with aggregate activity.

5.4.2 Stock returns

Since IP and employment are only available at the monthly frequency, forcing us to use a single observation to proxy for a covariance, one might naturally worry that the proxies for the moments in (22)–(23) would have a substantial amount of measurement error.[22] Stock returns have the advantage that they are available at the daily frequency and thus allow us to measure the covariance matrix for each month more accurately. We denote the sample covariance in month t by $\hat{\Sigma}_t$, and then $\hat{\Sigma}_{t,i}$ is again the sum of the i'th row, excluding the diagonal element. Similarly, the covariance of each sector's returns with returns on the overall market can be estimated using daily data within each month, $\hat{\beta}_{i,t}$, and the sector's variance can be estimated from the monthly sample variance. The fourth row of Table 4 reports results for regressions of the form

$$x_{i,t} = a_i + \sum_{j=0}^{J-1} b_j r_{i,t-j} + c_t + \eta_{i,t}, \tag{26}$$

for $x_{i,t} = \hat{\Sigma}_{t,i}$ or $\hat{\beta}_{i,t}$, where $r_{i,t}$ is the return in sector i in month t. That is, we use the same specification as for IP and employment, just replacing $\sum_{j \neq i} \varepsilon_{i,t}\varepsilon_{j,t}$ with $\hat{\Sigma}_{t,i}$, etc.

The results are highly similar to those for IP and employment, with coefficients again close to -0.05. While there is less measurement error in this regression, we also have fewer observations since we use a higher level of aggregation (due to the number of firms available), so the magnitude of the standard errors is similar to that for our proxies of economic activity.

5.4.3 TFP shocks

To get a more direct measure of productivity shocks in each sector, this section uses the NBER-CES manufacturing database, which measures productivity at the 6-digit level among manufacturing industries, but is only available at the annual frequency (Acemoglu, Akcigit, and Kerr, 2015).

We estimate regressions using the same specification as for IP and employment above (equations (24) and (25)), but with two modifications. First, we use data on real gross output and hours of production workers instead of IP and employment on the left-hand

[22] Notice, however, that such measurement error appears in the residual in the regression, and therefore is accounted for in the standard errors.

side. Second, the independent variable, instead of being the lagged statistical innovation in activity, is the lagged statistical innovation in total factor productivity, as reported in the NBER-CES database.

The bottom two rows of Table 4 report the results from this exercise. The results are very similar to what we report for IP and employment. In particular, for gross output, as with IP (which also measures gross output), the coefficients are close to -0.05 and statistically significant, though in this case the coefficient for sector volatility is close to zero. For hours worked, as with employment, the point estimates are negative, and -0.05 is close to boundary of the confidence bands, but the point estimates themselves are close to zero.

In summary, there is strong evidence for gross output following the predictions of the model, in terms of negative shocks increasing covariances, but for employment the results are again weak at best.

5.4.4 Robustness

The coefficient estimates are similar sectors are weighted by their relative size, when the dependent variable is winsorized, and when alternative levels of aggregation are used.

6 Quantitative illustration

This section develops a more general and quantitatively realistic version of the benchmark model that relaxes the various symmetry assumptions and explores the extent to which it matches the actual numbers reported in Section 5.

6.1 Model and calibration

The specification closely follows that of Baqaee and Farhi (2019), with three key generalizations from our baseline model: heterogeneity in the intermediate input mix used by each industry, differential elasticities of substitution for production inputs versus final consumption, and differential elasticities for labor versus material inputs.

The following pair of equations replace equations (4) and (5) in Section 3:

$$y_{i,t} = z_{i,t} \left((1-\alpha_i)^{1/\beta} \ell_i^{(\beta-1)/\beta} + \alpha_i^{1/\beta} \left(\sum_{j=1}^{n} a_{ij}^{1/\xi} x_{ij,t}^{(\xi-1)/\xi} \right)^{\frac{(\beta-1)\xi}{\beta(1-\xi)}} \right)^{\beta/(1-\beta)} \tag{27}$$

$$\text{GDP}_t = \left(\sum_{j=1}^{n} a_{cj}^{1/\hat{\xi}} c_{j,t}^{(\hat{\xi}-1)/\hat{\xi}} \right)^{\hat{\xi}/(\hat{\xi}-1)} . \tag{28}$$

The market-clearing condition for good j continues to be given by (6). In another departure from the baseline model, we assume that labor is sector-specific and cannot be reallocated across sectors, with the total supply of labor available to firms in industry i given by $\bar{L}_i$.[23] That assumption helps make the magnitude of the various effects quantitatively realistic by reducing the degree to which the economy can absorb technology shocks, since it inhibits reallocation of resources to low-productivity sectors.

The calibration follows Baqaee and Farhi (2019). The production weights are chosen to match the 1982 input-output table. TFP shocks are calibrated to match the relative variance of TFP by industry along with the observed correlations. Their overall scale is chosen to match the volatility of industrial production growth (and the moments we will examine will be matched to the IP data). The log productivity shocks are drawn from a t-distribution with four degrees of freedom, which generates fat tails consistent with observed sector-specific IP growth. The autocorrelation of sector productivity is set to 0.85 (at the monthly frequency) to match the dynamics of sector-level IP growth.

The elasticities of substitution are set to $\xi = 0.1$, $\hat{\xi} = 0.9$, and $\beta = 0.5$, implying that goods are less substitutable in firms' production technology than in the consumption good bundle, which is close to a Cobb-Douglas specification. The strong complementarity in sectoral production technologies—in line with the estimates of Atalay (2017)—means that the mix of material inputs is not amenable to adjustment.

6.2 Results

Table 5 reports moments of the model corresponding to the results from Figure 1 and Tables 3–4 along with the associated empirical results for industrial production. The top section shows that, as in the data, skewness is higher at the aggregate than the sector level, though in the model sector skewness is somewhat higher than in the data. Residual and aggregate

[23]Appendix A.2, shows that the theoretical results in Propositions 1–3 extend to economies with fixed factors.

skewness in the model is well within the empirical confidence bands.

The middle section examines the cyclicality of cross-sectional variance and skewness. NBER recessions in the model are defined as periods when aggregate output growth is in the bottom 15 percent of the unconditional distribution, to replicate the empirical frequency of recessions. The signs and magnitudes of the coefficients are highly similar between the model and the data, both for skewness and variance. The model replicates the empirical result that cross-sectional variance is countercyclical and cross-sectional skewness is procyclical, and the magnitudes are empirically realistic.

Finally, the bottom section reports results for the conditional covariance regressions. Similar to the data, the coefficients in the two versions of the regressions are similar. In the model, they are equal to -0.025, compared to approximately -0.05 in the data. The magnitude of that difference is about the same size as the empirical standard errors, so the data and model again yield quantitatively similar results.

Overall, the quantitative model performs well in matching the time-series, cross-sectional, and conditional moments, given that we made few choices in the calibration. Table 5 therefore shows that a richer version of the model, designed to be closer to quantitative realism than the highly restricted setup analyzed theoretically above, is able to broadly match the empirical behavior of the economy documented in Figure 1 and Tables 3–4.

7 Alternative models of skewness

This section examines alternative models of skewness. We examine relatively stylized forms of models meant to capture different potential economic mechanisms that could be driving aggregate or cross-sectional skewness. The analysis shows that none of the alternatives is able to generate all of the same predictions—or match all of the empirical results—developed above. The table at the end of the section summarizes the results for the various models.

7.1 Skewed aggregate shocks

Consider the following simple reduced-form specification for aggregate and sector output, y_t and $y_{i,t}$:

$$y_t = \eta_t, \tag{29}$$

$$y_{i,t} = b_i y_t + \varepsilon_{i,t}, \tag{30}$$

where η_t is a shock to aggregate output that is skewed left and $\varepsilon_{i,t}$ is a symmetrically distributed idiosyncratic shock. We assume that there are many individual sectors that aggregate linearly so that $y_t = \mathrm{xE}\left[y_{i,t}\right]$ (with the average of $b\{i\}$ equal to 1), where xE denotes the cross-sectional mean. This type of reduced-form could be generated by a number of different structural models.[24]

Time-series skewness: The skewness of η_t and symmetry of $\varepsilon_{i,t}$ immediately implies that $skew\left(y_t\right) < skew\left(y_{i,t}\right) < 0$, as in the data.

Cyclicality of cross-sectional moments: On any date, the cross-sectional variance of sector output is

$$\mathrm{xvar}\left(y_{i,t}\right) = \eta_t^2\,\mathrm{xvar}\left(b_i\right) + \mathrm{xvar}\left(\varepsilon_{i,t}\right) + 2\,\mathrm{xcov}\left(b_i, \varepsilon_{i,t}\right), \tag{31}$$

where $\mathrm{xvar}\left(y_{i,t}\right)$ denotes the cross-sectional variance of $y_{i,t}$ (on date t). It then immediately follows that

$$\mathrm{cov}\left(y_t, \mathrm{xvar}\left(y_{i,t}\right)\right) = \mathrm{xvar}\left(b_i\right)\mathbb{E}\left[\eta_t^3\right]. \tag{32}$$

In other words, cross-sectional variance is countercyclical as long as aggregate output is skewed left. Recall, though, that the empirical results in Table 3 apply not just to sector growth rates, but also to residuals from regressions of sector growth rates on aggregate growth. That is, in the data the cyclicality of cross-sectional variance is not due to exposure to a common shock, but rather to cyclicality in the variance of the sector-specific component. In the model with skewed aggregate shocks described in (29)–(30), the assumption that η_t and $\varepsilon_{i,t}$ are independent means that the cross-sectional moments of the $\varepsilon_{i,t}$ are acyclical:

$$\mathrm{cov}\left(y_t, \mathrm{xvar}\left(\varepsilon_{i,t}\right)\right) = 0 \tag{33}$$

which is inconsistent with the empirical results.

The failure of the model described in (29)–(30) to match the cyclicality of the cross-sectional distribution of the residuals is instructive. Its basic structure is superficially similar to the equilibrium of our model in (7)–(8), but it has a critical difference: the common component, η_t, is exogenous and independent of the sector-specific shocks, $\varepsilon_{i,t}$, whereas in

[24] Technology or policy shocks, represented by η_t, could be skewed left, due to rare disasters (Rietz (1988), Barro (2006)), or with less extreme but still left-tilter asymmetry as in Berger, Dew-Becker, and Giglio (2020). Alternatively, some input to production used by all sectors, e.g., the output of the financial sector (financial intermediation) could be skewed to the left. For example, the financial sector might face occasionally binding constraints (see, e.g., Kocherlakota (2000) or Brunnermeier and Sannikov (2014)).

the network model the common component is endogenous to the sector-specific shocks.

The model also fails to match the variation in the conditional covariances across sectors, again simply due to the independence of the sector and aggregate shocks.

7.2 Sector output is a concave function of symmetric shocks

Ilut, Kehrig, and Schneider (2018) (henceforth, IKS) study a model in which firms have concave responses to economic shocks, such that firm output or employment takes the form

$$y_{i,t} = g\left(\eta_t + \varepsilon_{i,t}\right) \tag{34}$$

for a concave and increasing function g, where η_t is an aggregate shock and $\varepsilon_{i,t}$ is an idiosyncratic shock. The shocks are mean-zero and independent with symmetrical distributions. IKS then assume that aggregate output is simply the sum over many firms of $y_{i,t}$—i.e. it is an expectation across values of $\varepsilon_{i,t}$, conditional on the value of η_t:

$$y_t = \mathrm{xE}\left[y_{i,t}\right], \tag{35}$$

where again $\mathrm{xE}\left[y_{i,t}\right]$ denotes a mean across values of i on date t.

Unlike our network production model, the model of IKS generates skewness through micro decisions, so it provides a useful benchmark for understanding the difference between skewness arising from concave *aggregation* of symmetric micro shocks and skewness arising from concave micro *responses* to shocks.

Time-series skewness: Sector (or firm) output, $y_{i,t}$, in this model is skewed left due to the concavity of g. However, in this case it is straightforward to show that skewness *decreases* with the level of aggregation.[25] The follows simply from the fact aggregate output is modeled as a simple average over sector- (or firm-) level outputs, so that the micro skewness washes out (this can be shown formally by following the methods in IKS).

These results would also obtain in a granular model, as in Gabaix (2011), in which sector shocks are skewed left. That is, suppose there is effectively a small number of sectors, so that the sector shocks have nontrivial effects on aggregate output. Then even if they are skewed, after (linear) aggregation, aggregate output will be less skewed than sector output, due to simple averaging. In other words, the fact that skewness increases with aggregation is inconsistent with a simple form of micro granularity.

[25] Indeed, this is what IKS find in their simulation (see their Table 9).

Cyclicality of cross-sectional moments: IKS give a formal derivation of the result that cross-sectional variance is countercyclical. A simple, though informal, way to see it is to examine a linear approximation to sector output (i.e. use the delta method),

$$\text{xvar}_t\left(y_{i,t}\right) \approx g'\left(\eta_t\right)^2 \text{xvar}\left(\varepsilon_{i,t}\right). \tag{36}$$

By assumption, $g'\left(\eta_t\right)$ strictly increases as η_t declines, making cross-sectional variance countercyclical. The model also generates countercyclicality for the variance of the sector-specific component of output, as in the data.[26]

Variation in conditional covariances: Consider a linear approximation of $y_{i,t}$ around the point that the variance of the aggregate shock, η_t, is negligible compared to that of the idiosyncratic shocks: $y_{i,t} \approx g(\varepsilon_{i,t}) + g'(\varepsilon_{i,t})\eta_t$. Under such an approximation,

$$\text{cov}_t(y_{i,t+1}, y_{j,t+1}) \approx \mathbb{E}_t[g'(\varepsilon_{i,t+1})]\mathbb{E}_t[g'(\varepsilon_{j,t+1})]\text{var}_t(\eta_{t+1}). \tag{37}$$

As long as shocks exhibit positive persistence, a negative shock to sector i increases $g'(\varepsilon_{i,t+1})$. Therefore, a negative shock to sector i raises the covariance of sector i's output with all other sectors, without having any effect on covariances between other sectors. This result is not noted by IKS, but it represents an additional pair of empirical facts that the model matches.

Summary: The model of concave decision rules generates negative skewness, and can match the empirical cyclicality of cross-sectional moments and the behavior of conditional covariances. However, it fails to generate the result that skewness increases with the level of aggregation. The key difference between this model and the model of complementarity in production is the underlying origin of skewness. With concave decision rules, the skewness arises at the firm or sector level. Under linear aggregation, that skewness washes out at the aggregate level. In the network model with complementarity, it is fundamentally *created by* aggregation, leading to greater skewness at the aggregate than the firm level.

7.3 Time-varying uncertainty

Consider a simple model of time-varying cross-sectional volatility,

$$y_t = \sigma_{t-1}\eta_t - k\sigma_t^2, \tag{38}$$

$$y_{i,t} = y_t + \sigma_{t-1}\varepsilon_{i,t}, \tag{39}$$

[26] IKS also show that cross-sectional skewness is procyclical under a restriction on g.

where $y_{i,t}$ and y_t denote sectoral and aggregate variables (such as output or log output), η_t and $\varepsilon_{i,t}$ are the corresponding innovations with variances normalized to 1, and k is a constant that determines how output responds to variation in cross-sectional volatility relates to the level of output. Once again, we assume that η_t and $\varepsilon_{i,t}$ are independent with unskewed distributions and that aggregation is linear over many sectors so that $y_t = \mathbb{E}_t [y_{i,t}]$.

Time-series skewness: Time-series skewness can occur in this model either if shocks to σ_t^2 are skewed to the right—so that $-k\sigma_t^2$ is skewed left—or if η_t and σ_t are correlated, so that the distribution of $\sigma_{t-1}\eta_t$ is skewed left. These features are not universal characteristics of models of uncertainty shocks. For example, the model of Bloom et al. (2018) does not generate skewness in aggregate output growth, because both the term $\sigma_t\eta_t$ and changes in volatility are symmetrically distributed. That said, if the conditions necessary for y_t to be skewed left are satisfied, then $y_{i,t}$ is also skewed left, and again by less than y_t, because of the fact that $\varepsilon_{i,t}$ is symmetrically distributed. So the model *can*, though does not necessarily, match the data on time-series skewness of aggregate and sector output.

Time-varying cross-sectional moments: Cross-sectional variance in the model described by (39) is time-varying by assumption. Furthermore, cross-sectional variance is countercyclical for $k > 0$. This result holds both for total sector output and for the sector specific component, $\varepsilon_{i,t}$.

To generate time-varying cross-sectional skewness, as in the data, an additional skewness process would need to be added to the model. Salgado, Guvenen, and Bloom (2020) examine such a setup.

Time-series skewness in the model is determined by the parameter k. Procyclical cross-sectional skewness, on the other hand, would require an extra free parameter. So the model requires a new parameter (or assumption) for each of the empirical results it matches. In that sense, it is less parsimonious than the network model, which just requires an assumption about a single parameter (i.e. that inputs are complements).

Conditional moments: The covariance of output between sectors is

$$\operatorname{cov}(y_{i,t}, y_{j,t}) = \operatorname{var}(\varepsilon_t) = \sigma_{t-1}^2. \tag{40}$$

In other words, the covariances are all identical on any date. They change over time due to σ_t^2, and they are all countercyclical, but there is no variation across sectors. In both the network and IKS models, the source of the increased covariance following negative shocks is that a sector loads more heavily on the common component. This illustrates the importance of the common component being endogenous to the sector shocks, unlike here, where it is purely exogenous.

Summary: In a model where aggregate output responds negatively to uncertainty, it is possible to generate negative skewness. The model also, by assumption, generates countercyclical cross-sectional and time-series volatility. However, the model has no prediction for differences in covariances across sectors.[27]

7.4 Implications

Table 1 summarizes the empirical results and the ability of the models to qualitatively match them.

Table 1: **Summary Alternative Models**

	Model			
Fact	Complementary network	Skewed shocks	Concave responses	Uncertainty shocks
Increasing skewness with aggregation	✓	✓	×	?
No skewness for residuals	✓	✓	×	✓
Cyclicality of cross-sectional variance	✓	✓	✓	✓
Cyclicality of cross-sectional resid. var.	✓	×	✓	✓
Centrality rises after negative shocks	✓	×	✓	×

The equilibrium of the network model has two key features that allow it to match the empirical facts: there is skewness in the common but not sector-specific components, and the common component is endogenous to the sector shocks.

8 Conclusion

The goal of this paper is to understand the sources of asymmetries in aggregate output. One theoretical way to generate that asymmetry is for aggregate output to be a concave function of micro shocks. A network model in which inputs are complements is a particular

[27] The analysis in this section is based on a reduced-form representation, but in simulations of the general-equilibrium model of Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2018) the results are similar. In fact, in the simulations output and employment, in both levels and growth rates, are significantly *positively* skewed.

example of that mechanism. The paper shows that it naturally generates time-series skewness in aggregate and sector output. It then goes on to test a range of additional empirical predictions, all of which have support in the data. The idea of complementarity is powerful in understanding both the aggregate and cross-sectional behavior of the economy.

The model implies that second and third moments change over time and are cyclical. In the past, it has sometimes been argued that the observed cyclicality of those moments implies that there are exogenous shocks to *uncertainty*, and that uncertainty then has negative effects on the economy. Models in which aggregate output is a concave function of micro shocks, as in the network model studied here, can generate that result without volatility itself having any independent effect on the level of output – output is purely driven by the technology shocks themselves.

A second important implication, which is supported by the empirical results, is that the centrality of sectors changes over time. In some models, recessions have common causes, e.g. aggregate technology shocks. Here, however, every episode is different. When a sector receives a negative shock, it becomes relatively more important. So in a period where oil stocks are low, shocks to the oil sector become a major driving force (e.g., Hamilton (2003), Kilian (2008)), whereas in periods when the financial sector is highly constrained, financial shocks become most relevant (e.g., Brunnermeier and Sannikov (2014)). A key insight of this paper is that complementarity means that the aggregate effects of shocks change in important ways over time, those changes can be measured from the covariances of sector growth rates, and many models fail to match them.

References

Acemoglu, Daron, Ufuk Akcigit, and William Kerr (2015), "Networks and the macroeconomy: An empirical exploration." *NBER Macroeconomics Annual*, 30(1), 273–335.

Acemoglu, Daron, Vasco M Carvalho, Asuman Ozdaglar, and Alireza Tahbaz-Salehi (2012), "The network origins of aggregate fluctuations." *Econometrica*, 80(5), 1977–2016.

Acemoglu, Daron and Andrew Scott (1997), "Asymmetric business cycles: Theory and time-series evidence." *Journal of Monetary Economics*, 40(3), 501–533.

Albuquerque, Rui (2012), "Skewness in stock returns: reconciling the evidence on firm versus aggregate returns." *Review of Financial Studies*, 25(5), 1630–1673.

Atalay, Enghin (2017), "How important are sectoral shocks?" *American Economic Journal: Macroeconomics*, 9(4), 254–80.

Atalay, Enghin, Thorsten Drautzburg, and Zhenting Wang (2018), "Accounting for the sources of macroeconomic tail risks." *Economics Letters*, 165, 65–69.

Baqaee, David Rezza and Emmanuel Farhi (2019), "The macroeconomic impact of microeconomic shocks: Beyond Hulten's theorem." *Econometrica*, 87(4), 1155–1203.

Barro, Robert J. (2006), "Rare disasters and asset markets in the twentieth century." *Quarterly Journal of Economics*, 121(3), 823–866.

Barrot, Jean-Noël and Julien Sauvagnat (2016), "Input specificity and the propagation of idiosyncratic shocks in production networks." *Quarterly Journal of Economics*, 131(3), 1543–1592.

Berger, David, Ian Dew-Becker, and Stefano Giglio (2020), "Uncertainty shocks as second-moment news shocks." *Review of Economic Studies*, 87(1), 40–76.

Bianchi, Javier (2011), "Overborrowing and systemic externalities in the business cycle." *American Economic Review*, 101(7), 3400–3426.

Bloom, Nicholas (2014), "Fluctuations in uncertainty." *Journal of Economic Perspectives*, 28(2), 153–76.

Bloom, Nicholas, Max Floetotto, Nir Jaimovich, Itay Saporta-Eksten, and Stephen J Terry (2018), "Really uncertain business cycles." *Econometrica*, 86(3), 1031–1065.

Boehm, Christoph, Aaron Flaaen, and Nitya Pandalai-Nayar (2019), "Input linkages and the transmission of shocks: Firm-level evidence from the 2011 tōhoku earthquake." *Review of Economics and Statistics*, 101(2), 60–75.

Brunnermeier, Markus K and Yuliy Sannikov (2014), "A macroeconomic model with a financial sector." *American Economic Review*, 104(2), 379–421.

Caliendo, Lorenzo, Fernando Parro, Esteban Rossi-Hansberg, and Pierre-Daniel Sarte (2018), "The impact of regional and sectoral productivity changes on the us economy." *Review of Economic Studies*, 85(4), 2042–2096.

Carvalho, Vasco and Xavier Gabaix (2013), "The great diversification and its undoing." *American Economic Review*, 103(5), 1697–1727.

Carvalho, Vasco M, Makoto Nirei, Yukiko Saito, and Alireza Tahbaz-Salehi (2021), "Supply chain disruptions: Evidence from the great east japan earthquake." *Quarterly Journal of Economics*, 136(2), 1255–1321.

Christiano, Lawrence J., Roberto Motto, and Massimo Rostagno (2014), "Risk shocks." *American Economic Review*, 104(1), 27–65.

Clark, Todd E and Francesco Ravazzolo (2015), "Macroeconomic forecasting performance under alternative specifications of time-varying volatility." *Journal of Applied Econometrics*, 30(4), 551–575.

Cont, Rama and P. Tankov (2004), *Financial Modelling with Jump Processes*. Chapman and Hall/CRC.

Davis, Steven J and John Haltiwanger (1992), "Gross job creation, gross job destruction, and employment reallocation." *Quarterly Journal of Economics*, 107(3), 819–863.

Dew-Becker, Ian (2021), "The effects of large shocks in production networks." Working paper.

Dew-Becker, Ian and Stefano Giglio (2021), "Cross-sectional uncertainty and the business cycle: evidence from 40 years of options data." Working paper.

Dupraz, Stéphane, Emi Nakamura, and Jón Steinsson (2021), "A plucking model of business cycles." Working paper.

Foerster, Andrew T, Pierre-Daniel G Sarte, and Mark W Watson (2011), "Sectoral versus aggregate shocks: A structural factor analysis of industrial production." *Journal of Political Economy*, 119(1), 1–38.

Frohm, Erik and Vanessa Gunnella (2021), "Spillovers in global production networks." *Review of International Economics*.

Gabaix, Xavier (2011), "The granular origins of aggregate fluctuations." *Econometrica*, 79(3), 733–772.

Gilchrist, Simon and John C Williams (2000), "Putty-clay and investment: a business cycle analysis." *Journal of Political Economy*, 108(5), 928–960.

Gourieroux, Christian, Alain Monfort, Sarah Mouabbi, and Jean-Paul Renne (2020), "Disastrous defaults." Working paper.

Gourio, Francois (2012), "Disaster risk and business cycles." *American Economic Review*, 102(6), 2734–2766.

Gourio, Francois (2013), "Credit risk and disaster risk." *American Economic Journal: Macroeconomics*, 5(3), 1–34.

Grassi, Basile and Julien Sauvagnat (2019), "Production networks and economic policy." *Oxford Review of Economic Policy*, 35(4), 638–677.

Guvenen, Fatih, Serdar Ozkan, and Jae Song (2014), "The nature of countercyclical income risk." *Journal of Political Economy*, 122(3), 621–660.

Hamilton, James D (2003), "What is an oil shock?" *Journal of Econometrics*, 113(2), 363–398.

Hansen, Gary D and Edward C Prescott (2005), "Capacity constraints, asymmetries, and the business cycle." *Review of Economic Dynamics*, 8(4), 850–865.

Horvath, Michael (2000), "Sectoral shocks and aggregate fluctuations." *Journal of Monetary Economics*, 45(1), 69–106.

Ilut, Cosmin, Matthias Kehrig, and Martin Schneider (2018), "Slow to hire, quick to fire: Employment dynamics with asymmetric responses to news." *Journal of Political Economy*, 126(5), 2011–2071.

Jones, Charles I (2011), "Intermediate goods and weak links in the theory of economic development." *American Economic Journal: Macroeconomics*, 3(2), 1–28.

Jurado, Kyle, Sydney Ludvigson, and Serena Ng (2015), "Measuring uncertainty." *American Economic Review*, 105(3), 1177–1216.

Justiniano, Alejandro and Giorgio E Primiceri (2008), "The time-varying volatility of macroeconomic fluctuations." *American Economic Review*, 98(3), 604–41.

Kilian, Lutz (2008), "The economic effects of energy price shocks." *Journal of Economic Literature*, 46(4), 871–909.

Kiyotaki, Nobuhiro and John Moore (1997), "Credit cycles." *Journal of Political Economy*, 105(2), 211–248.

Kocherlakota, Narayana (2000), "Creating business cycles through credit constraints." *Federal Reserve Bank of Minneapolis Quarterly Review*, 24(3), 2–10.

Kozeniauskas, Nicholas, Anna Orlik, and Laura Veldkamp (2018), "What are uncertainty shocks?" *Journal of Monetary Economics*, 100, 1–15.

Lahiri, S.N. (2003), *Resampling Methods for Dependent Data*. Springer-Verlag.

Liu, Ernest and Aleh Tsyvinski (2020), "Dynamical structure and spectral properties of input-output networks." Working paper.

Long, John B and Charles I Plosser (1983), "Real business cycles." *Journal of Political Economy*, 91(1), 39–69.

McKay, Alisdair and Ricardo Reis (2008), "The brevity and violence of contractions and expansions." *Journal of Monetary Economics*, 55(4), 738–751.

Morley, James and Jeremy Piger (2012), "The asymmetric business cycle." *Review of Economics and Statistics*, 94(1), 208–221.

Newey, Whitney K. and Kenneth D. West (1987), "A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix." *Econometrica*, 55(3), 703–708.

Peter, Alessandra and Cian Ruane (2020), "The aggregate importance of intermediate input substitutability." Working paper.

Rietz, Thomas A. (1988), "The equity risk premium: A solution." *Journal of Monetary Economics*, 22(1), 117–131.

Salgado, Sergio, Fatih Guvenen, and Nicholas Bloom (2020), "Skewed business cycles." Working paper.

Schorfheide, Frank, Dongho Song, and Amir Yaron (2018), "Identifying long-run risks: A bayesian mixed-frequency approach." *Econometrica*, 86(2), 617–654.

Seo, Sang Byung and Jessica A. Wachter (2018), "Do rare events explain cdx tranche spreads?" *The Journal of Finance*, 73, 2343–2383.

Sichel, Daniel E. (1993), "Business cycle asymmetry: a deeper look." *Economic Inquiry*, 31(2), 224–236.

Wachter, Jessica A. (2013), "Can time-varying risk of rare disasters explain aggregate stock market volatility?" *Journal of Finance*, 68(3), 987–1035.

Table 2: **Measures of Aggregate Time-Series Skewness**

	Skewness	Tail Probability Ratios				
		$k = 1$	$k = 1.5$	$k = 2$	$k = 2.5$	$k = 3$
	Panel A: Growth Rates					
IP	-1.22	1.03	0.92	1.75	2.50	9.00
	[0.01]	[0.84]	[0.70]	[0.27]	[0.20]	[0.10]
Employment	-0.52	1.16	1.78	3.43	6.00	4.00
	[0.25]	[0.43]	[0.08]	[0.07]	[0.15]	[0.43]
Stock Returns	-0.57	1.08	1.56	1.50	2.00	4.00
	[0.06]	[0.34]	[0.01]	[0.32]	[0.25]	[0.22]
GDP	-0.36	0.92	1.30	2.00	3.00	2.00
	[0.45]	[0.72]	[0.42]	[0.32]	[0.41]	[0.70]
Consumption	-0.78	0.74	0.80	2.33	4.00	∞
	[0.14]	[0.14]	[0.45]	[0.14]	[0.24]	[0.52]
Investment	-0.86	0.79	1.3.0	1.17	∞	∞
	[0.16]	[0.18]	[0.32]	[0.78]	[0.22]	[0.52]
	Panel B: Levels					
IP	-0.95	0.79	1.78	∞	∞	∞
	[0.08]	[0.27]	[0.27]	[0.08]	[0.34]	[0.49]
Employment	-0.68	1.32	1.81	3.14	∞	∞
	[0.12]	[0.38]	[0.36]	[0.48]	[0.53]	[0.43]
Stock Returns	-1.14	1.03	3.29	∞	∞	∞
	[0.04]	[0.88]	[0.02]	[0.05]	[0.26]	[0.40]
GDP	-0.45	0.86	1.75	8.00	∞	.
	[0.18]	[0.53]	[0.30]	[0.21]	[0.57]	[.]
Consumption	-0.44	1.30	1.56	6.00	∞	.
	[0.14]	[0.28]	[0.48]	[0.45]	[0.43]	[.]
Investment	-1.20	0.96	5.00	∞	∞	.
	[0.06]	[0.88]	[0.02]	[0.12]	[0.44]	[.]

Notes: The first column reports skewness for growth rates (Panel A) and levels (Panel B) for six variables: Industrial production, employment, stock returns, GDP, consumption, and investment. p-values against a symmetric null from a block bootstrap are reported in brackets. Data is monthly (quarterly for GDP, consumption, and investment) and runs from January 1972 to December 2019.

Table 3: **Cross-Sectional Moments and Cyclicality**

	IP	IP resid	Employ	Employ resid	Returns resid	Returns
	Panel A: Growth Rates					
	Variance					
NBER	0.48***	0.41***	0.95***	0.56***	1.00**	1.00**
	[0.16]	[0.14]	[0.23]	[0.16]	[0.41]	[0.41]
Employment growth	−0.14	−0.11	−0.39***	−0.28***	−0.36**	−0.36**
	[0.09]	[0.08]	[0.06]	[0.07]	[0.15]	[0.15]
	Skewness					
NBER	−0.23**	−0.02	−0.44***	−0.40***	−0.17	−0.16
	[0.09]	[0.10]	[0.13]	[0.14]	[0.16]	[0.16]
Employment growth	0.07*	0.00	0.13***	0.11**	0.05	0.05
	[0.04]	[0.05]	[0.05]	[0.05]	[0.06]	[0.06]
# Obs	566	566	352	352	588	588
	Panel B: Levels					
	Variance					
NBER	0.75***	1.00***	0.66	1.18		
	[0.20]	[0.27]	[0.41]	[0.73]		
Employment growth	−0.23***	−0.30***	−0.32***	−0.57***		
	[0.07]	[0.10]	[0.11]	[0.20]		
# Obs	566	566	352	352		
	Skewness					
NBER	0.09	0.08	−0.38***	−1.21***		
	[0.23]	[0.22]	[0.08]	[0.27]		
Employment growth	0.00	0.00	0.13***	0.41***		
	[0.07]	[0.07]	[0.03]	[0.11]		
# Obs	566	566	352	352		

Notes: This table reports regression results from regressing cross-sectional variance or skewness from growth rates (Panel A) or levels (Panel B) on economic activity. Each entry in the table is a univariate regression coefficient. NBER is a dummy variable which takes a value of one in recessions and zero otherwise. Employment growth is standardized to have unit variance, as is the dependent variable in each regression (cross-sectional variance or skewness). Standard errors, reported in brackets, are calculated using the Newey–West (1987) method with 12 monthly lags. The columns labeled residuals use the cross-sectional variance of residuals from regressions of sector growth rates on aggregate growth. * indicates significance at the 10- ** 5-, and *** 1-percent level, respectively.

Table 4: **Conditional Covariance**

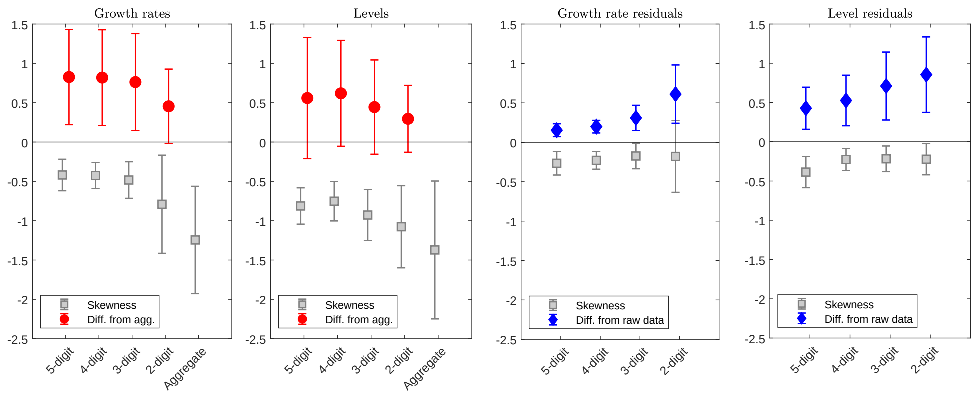
	$\Sigma_{t,i}$		$\beta_{t,i}$	
IP	-0.047^{**}	[0.020]	-0.053^{***}	[0.021]
Employment (1972-2019)	-0.109^{**}	[0.046]	-0.062^{*}	[0.039]
Employment (1990-2019)	-0.004	[0.017]	-0.014	[0.016]
Stock Returns	-0.059^{***}	[0.022]	-0.075^{***}	[0.027]
Shipments	-0.043^{**}	[0.018]	-0.040^{**}	[0.019]
Hours	-0.010	[0.014]	-0.012	[0.014]

Notes: Each row reports results of regressions measuring the response of conditional covariances to lagged innovations. For each variable, we use the level of aggregation that yields the largest number of sectors. $\Sigma_{t,i}$ reports results where the dependent variable is the covariance of each sector's growth rate with the sum of those for all other sectors. $\beta_{t,i}$ reports results for covariances with aggregate growth rates. For the first four rows, the independent variable is the lagged statistical innovation in the sector's growth rate. In the bottom section it is the lagged statistical innovation in TFP. All regressions include time and sector fixed effects and standard errors, reported in brackets, are clustered by date. The first four rows use monthly data and report the sum of the coefficients on three monthly lags. The final two rows use annual data and report the coefficient on a single annual lag. * indicates significance at the 10- ** 5-, and *** 1-percent level, respectively.

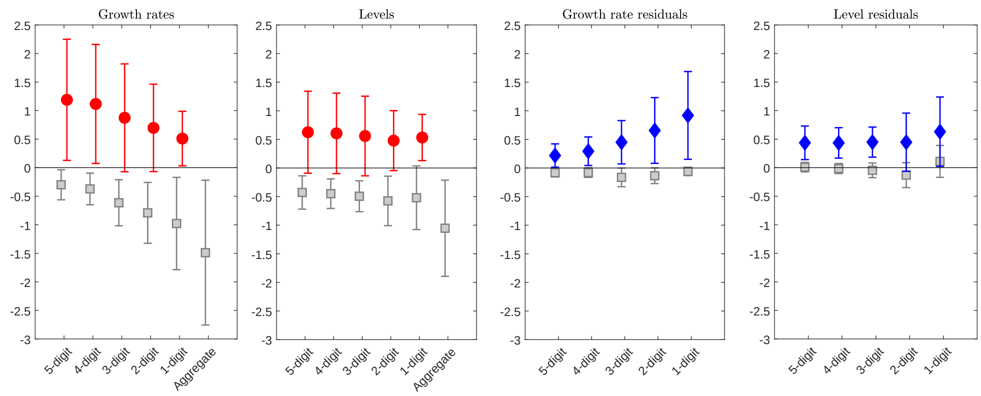
Table 5: **Model Simulation**

	Panel A: Time-Series Skewness					
	Model		Data		std err	
Aggregate	-1.72		-1.25		[0.68]	
Sector	-0.94		-0.42		[0.20]	
Residual	-0.17		-0.27		[0.15]	
	Panel B: Cross-Sectional Moments and Cyclicality					
	Growth Rates			Residuals		
	Model	Data	std err	Model	Data	std err
	Variance					
Recession	0.61	0.48	[0.16]	0.40	0.41	[0.14]
IP	-0.24	-0.14	[0.09]	-0.17	-0.11	[0.08]
	Skewness					
Recession	-0.20	-0.23	[0.09]	-0.07	-0.02	[0.10]
IP	0.10	0.07	[0.04]	0.03	0.00	[0.05]
	Panel C: Conditional Covariance					
	Model	Data	std err	Model	Data	std err
		$\Sigma_{t,i}$			$\beta_{i,t}$	
IP	-0.26	-0.05	[0.02]	-0.02	-0.05	[0.02]

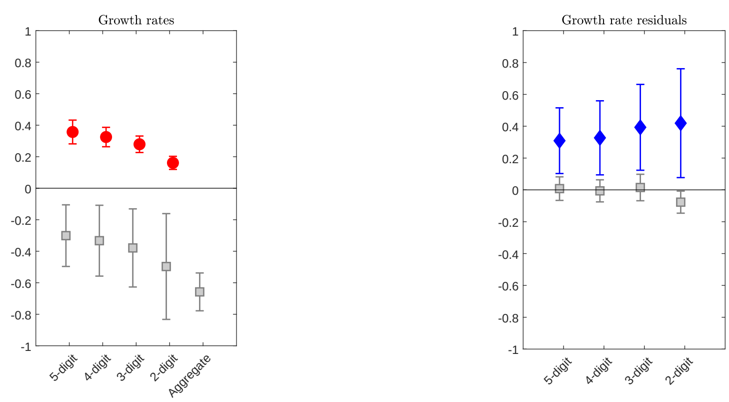
Notes: The "model" columns report moments from a simulation of 10,000 periods from the numerical solution to the model. The "data" columns report corresponding empirical estimates and standard errors. The sector-level estimates in the top and bottom sections are for industrial production for 4-digit industries.



(a) Industrial Production



(b) Employment



(c) Stock Returns

Figure 1: **Time-Series Skewness**

Notes: This figure plots skewness for different levels of aggregation for industrial production, employment, and stock returns for growth rates (left two panels) or in levels (right two panels) together with 90-percent confidence intervals. Data is monthly and starts in January 1972 and ends in December 2019 for industrial production and stock returns and starts in January 1990 and ends in December 2019 for employment.

A.1 Proofs

Equilibrium Characterization

In what follows we characterize the competitive equilibrium of the economy described in Section 3 and derive equations (7) and (8). Throughout, we let the consumption good bundle as the numeraire.

Let p_i denote the equilibrium price of good i and z_i be the productivity shock to sector i. Since all firms are competitive with constant returns technologies, it is immediate that the equilibrium price of good i satisfies

$$p_i = \frac{1}{z_i} w^{1-\alpha} \left(\sum_{j=1}^{n} a_j p_j^{1-\xi} \right)^{\alpha/(1-\xi)} = \frac{1}{z_i} w^{1-\alpha}, \tag{A.1}$$

where the second equality follows from our choice of the consumption good as the numeraire. Consequently,

$$\sum_{i=1}^{n} a_i p_i^{1-\xi} = w^{(1-\alpha)(1-\xi)} \sum_{i=1}^{n} a_i z_i^{\xi-1}. \tag{A.2}$$

Once again, the choice of the consumption good bundle as the numeraire implies that the left-hand side of the above equation is equal to 1. Furthermore, since we normalized the total supply of labor to 1, the household's budget constraint implies that $\mathrm{GDP} = w$. As a result, log aggregate output in this economy is given by

$$\log(\mathrm{GDP}) = \frac{1}{(\xi-1)(1-\alpha)} \log \left(\sum_{i=1}^{n} a_i z_i^{\xi-1} \right). \tag{A.3}$$

This establishes (8). To establish (7), first note that household's demand for good j is equal to $c_j = a_j p_j^{-\xi}\,\mathrm{GDP}$. Similarly, sector i's demand for good j is given by $x_{ij} = \alpha a_j p_j^{-\xi} p_i y_i$. Consequently, market-clearing for good j implies that

$$y_j = a_j p_j^{-\xi}\,\mathrm{GDP} + \alpha a_j p_j^{-\xi} \sum_{i=1}^{n} p_i y_i. \tag{A.4}$$

Multiplying both sides of the above equation by p_j, summing over all j, and using the fact that the consumption good bundle is the numeraire, implies that $\sum_{i=1}^{n} p_i y_i = \mathrm{GDP}/(1-\alpha)$. Plugging this into the previous equation we therefore obtain

$$y_j = \frac{1}{1-\alpha} a_j p_j^{-\xi}\,\mathrm{GDP}. \tag{A.5}$$

Replacing for the equilibrium price from (A.1) and using the fact that the wage is equal to aggregate output then establishes (7).

To derive the behavior of employment in each sector, note that optimal labor demand is

$$\ell_i = (1-\alpha)\, p_i y_i / w. \tag{A.6}$$

Combining the above with equations (A.5) and (A.1) then establishes (9). □

Proof of Proposition 1

Let $a_{\min} = \min\{a_1, \dots, a_n\} > 0$. Since $\xi < 1$, equation (8) implies that

$$\log \mathrm{GDP} \le \frac{1}{(\xi-1)(1-\alpha)} \left(\log a_{\min} + \log \sum_{i=1}^{n} z_i^{\xi-1} \right) \le k_0 + \frac{1}{1-\alpha} \varepsilon_{\min}, \tag{A.7}$$

where $\varepsilon_{\min} = \min\{\varepsilon_1, \dots, \varepsilon_n\}$, $\varepsilon_i = \log z_i$ is the log productivity shock to sector i, and $k_0 = \frac{\log a_{\min}}{(\xi-1)(1-\alpha)}$ is a positive constant. The above inequality implies that, for any $\tau \ge 0$,

$$\frac{\mathbb{P}(\log \mathrm{GDP} < \mu - \tau\sigma)}{\mathbb{P}(\log \mathrm{GDP} > \mu + \tau\sigma)} \ge \frac{\mathbb{P}(\varepsilon_{\min} < -(1-\alpha)(\tau\sigma - \mu + k_0))}{\mathbb{P}(\varepsilon_{\min} > (1-\alpha)(\tau\sigma + \mu - k_0))} \tag{A.8}$$

where $\mu = \mathbb{E}[\log \mathrm{GDP}]$ and $\sigma = \mathrm{stdev}(\log \mathrm{GDP})$ denote the expected value and standard deviation of log aggregate output, respectively. Since all log productivity shocks are symmetrically distributed around their mean of zero, the above inequality implies that

$$\lim_{\tau\to\infty} \frac{\mathbb{P}(\log \mathrm{GDP} < \mu - \tau\sigma)}{\mathbb{P}(\log \mathrm{GDP} > \mu + \tau\sigma)} \ge \lim_{\tau\to\infty} \frac{1 - F^n(\tau + k_1)}{F^n(-\tau + k_1)}, \tag{A.9}$$

where $F(\cdot)$ denotes the common cumulate distribution function of the log productivity shocks and $k_1 = (1-\alpha)(k_0 - \mu)$. As a result,

$$\lim_{\tau\to\infty} \frac{\mathbb{P}(\log \mathrm{GDP} < \mu - \tau\sigma)}{\mathbb{P}(\log \mathrm{GDP} > \mu + \tau\sigma)} \ge \lim_{\tau\to\infty} \frac{f(\tau + k_1)}{F^{n-1}(-(\tau - k_1)) f(\tau - k_1)} \ge \lim_{\tau\to\infty} \frac{f(\tau + k_1)}{F(-(\tau - k_1)) f(\tau - k_1)} \tag{A.10}$$

where $f(\cdot)$ denotes the density function corresponding to $F(\cdot)$ and the last inequality follows from the fact that $n \ge 2$. Now the assumption on the distribution function $F(\cdot)$ guarantees that the right-hand side of the above inequality diverges to infinity, thus establishing (10). □

Proof of Proposition 2

Fix the realization of log productivity shocks $\varepsilon = (\varepsilon_1, \dots, \varepsilon_n)$ and let ξ denote the elasticity of substitution between intermediate inputs. Define function $h : (0, \infty) \to \mathbb{R}$ as $h(\xi) = (1-\alpha)(\xi-1) \log \mathrm{GDP}$, where GDP, where recall log aggregate output satisfies (8). Therefore,

$$h(\xi) = \log \left(\sum_{i=1}^{n} a_i e^{(\xi-1)\varepsilon_i} \right). \tag{A.11}$$

It is straightforward to verify that a second-order Taylor expansion of $h(\xi)$ around $\xi = 1$ is given by

$$h(\xi) = (\xi - 1)\kappa_1 + \frac{1}{2}(\xi - 1)^2 \kappa_2 + o((\xi - 1)^2), \tag{A.12}$$

where κ_1 and κ_2 denote, respectively, the first and second central moment of log productivity shocks $(\varepsilon_1, \dots, \varepsilon_n)$ with weights $(a_1, \dots, a_n)$:

$$\kappa_1 = \sum_{i=1}^{n} a_i \varepsilon_i \qquad , \qquad \kappa_2 = \sum_{i=1}^{n} a_i \varepsilon_i^2 - \left(\sum_{i=1}^{n} a_i \varepsilon_i \right)^2. \tag{A.13}$$

Therefore, equation (A.12) implies that, to a second-order approximation, log aggregate output can be written as

$$\log(\mathrm{GDP}) = \frac{1}{1-\alpha}\kappa_1 + \frac{1}{2(1-\alpha)} (\xi - 1) \kappa_2 + o(\xi - 1). \tag{A.14}$$

Given the realization of the shocks $\varepsilon = (\varepsilon_1, \dots, \varepsilon_n)$, let $\mu_2 = \frac{1}{n} \sum_{i=1}^{n} \varepsilon_i^2 - \left(\frac{1}{n} \sum_{i=1}^{n} \varepsilon_i \right)^2$ denote the cross-sectional variance of the realized log productivity shocks. Equation (A.14) then implies that the covariance between log aggregate output and the cross-sectional variance of the realized shocks is given by

$$\mathrm{cov}(\mu_2, \log(\mathrm{GDP})) = \frac{\xi - 1}{2(1-\alpha)} \mathrm{cov}(\mu_2, \kappa_2) + o(\xi - 1), \tag{A.15}$$

where we are using the fact that since all shocks have symmetric distributions around their mean of zero, $\mathbb{E}[\varepsilon_i^k] = 0$ for all odd k. Note that

$$\mathbb{E}[\mu_2] = \frac{n-1}{n} \mathbb{E}[\varepsilon_i^2] \qquad , \qquad \mathbb{E}[\kappa_2] = \left(1 - \sum_{i=1}^{n} a_i^2 \right) \mathbb{E}[\varepsilon_i^2]. \tag{A.16}$$

Furthermore,

$$\mathbb{E}[\mu_2\kappa_2] = \frac{1}{n}\sum_{i,j=1}^{n} a_i\mathbb{E}[\varepsilon_i^2\varepsilon_j^2] - \frac{1}{n}\sum_{i,j,k=1}^{n} a_j a_k\mathbb{E}[\varepsilon_i^2\varepsilon_j\varepsilon_k] - \frac{1}{n^2}\sum_{i,j,k=1}^{n} a_i\mathbb{E}[\varepsilon_i^2\varepsilon_j\varepsilon_k] + \frac{1}{n^2}\sum_{i,j,k,r=1}^{n} a_i a_j\mathbb{E}[\varepsilon_i\varepsilon_j\varepsilon_k\varepsilon_r]. \tag{A.17}$$

Since productivity shocks are independent with symmetric distributions around their mean of zero,

$$\mathbb{E}[\mu_2\kappa_2] = \frac{n-1}{n^2}\left(1-\sum_{j=1}^{n} a_j^2\right)(\mathbb{E}[\varepsilon_i^4] + (n-1)\mathbb{E}^2[\varepsilon_i^2]) + \frac{1}{n^2}\sum_{\substack{i,k=1\\ j\neq i, r\neq k}}^{n} a_i a_j\mathbb{E}[\varepsilon_i\varepsilon_j\varepsilon_k\varepsilon_r] \tag{A.18}$$

$$= \frac{n-1}{n^2}\left(1-\sum_{j=1}^{n} a_j^2\right)(\mathbb{E}[\varepsilon_i^4] + (n-1)\mathbb{E}^2[\varepsilon_i^2]) + \frac{2}{n^2}\sum_{\substack{i=1\\ j\neq i}}^{n} a_i a_j\mathbb{E}[\varepsilon_i^2\varepsilon_j^2], \tag{A.19}$$

which in turn implies that

$$\mathbb{E}[\mu_2\kappa_2] = \frac{1}{n^2}\left(1-\sum_{j=1}^{n} a_j^2\right)((n-1)\mathbb{E}[\varepsilon_i^4] + ((n-1)^2+2)\mathbb{E}^2[\varepsilon_i^2]). \tag{A.20}$$

The above equation together with (A.15) and (A.16) implies that the covariance between the cross-sectional variance of the shocks and log aggregate output is given by

$$\text{cov}(\mu_2, \log\text{GDP}) = \frac{1}{2n^2}(\xi-1)\left(1-\sum_{i=1}^{n} a_i^2\right)\left((n-1)\mathbb{E}[\varepsilon_i^4] - (n-3)\mathbb{E}^2[\varepsilon_i^2]\right). \tag{A.21}$$

Since $\mathbb{E}[\varepsilon_i^4] \geq \mathbb{E}^2[\varepsilon_i^2]$, it is immediate that the right-hand side of the above equation is always negative whenever $\xi < 1$. Therefore, when intermediate inputs are gross complements, the covariance between the cross-sectional variance of the shocks and log aggregate output is negative whenever $\xi < 1$. This establishes (12).

To establish (13), let μ_3 denote the cross-sectional skewness of realized shocks and recall that log aggregate output satisfies (A.14). As a result,

$$\text{cov}(\mu_3, \log\text{GDP}) = \frac{1}{1-\alpha}\mathbb{E}[\mu_3\kappa_1] + o(\xi-1), \tag{A.22}$$

where once again we are using the fact that $\mathbb{E}[\varepsilon_i^k]=0$ for all odd k. Next, note that

$$\mathbb{E}[\mu_3\kappa_1]=\frac{1}{n}\sum_{i,j=1}^{n}a_i\mathbb{E}[\varepsilon_i\varepsilon_j^3]-\frac{3}{n^2}\sum_{i,j,k=1}^{n}a_i\mathbb{E}[\varepsilon_i\varepsilon_j\varepsilon_k^2]+\frac{2}{n^3}\sum_{i,j,k,r=1}^{n}a_i\mathbb{E}[\varepsilon_i\varepsilon_j\varepsilon_k\varepsilon_r] \tag{A.23}$$

$$=\frac{1}{n}\mathbb{E}[\varepsilon_i^4]-\frac{3}{n^2}\left(\mathbb{E}[\varepsilon_i^4]+(n-1)\mathbb{E}^2[\varepsilon_i^2]\right)+\frac{2}{n^3}(\mathbb{E}[\varepsilon_i^4]+3(n-1)\mathbb{E}^2[\varepsilon_i^2]). \tag{A.24}$$

Putting the above together with equation (A.22) therefore implies that

$$\text{cov}(\mu_3,\log\text{GDP})=\frac{(n-1)(n-2)}{n^3(1-\alpha)}(\mathbb{E}[\varepsilon_i^4]-3\mathbb{E}^2[\varepsilon_i^2]) \tag{A.25}$$

to a first-order approximation in $\xi-1$, which is always strictly positive when $n>2$ and log productivity shocks have a positive excess kurtosis. □

Proof of Proposition 3

By assumption, log productivity shock to sector i follows an AR(1) process. Let $\rho_i>0$ denote the persistence parameter of the shock to sector i, i.e., $\varepsilon_{i,t+1}=\rho_i\varepsilon_{i,t}+\eta_{i,t+1}$, where $\eta_{i,t+1}$ is productivity innovation at sector i. Recall that the log output of sector i has a factor structure given by equation (7). Therefore,

$$\begin{aligned}\text{cov}_t(\log y_{i,t+1},\log y_{j,t+1})&=\xi(1-\xi+\alpha\xi)\text{cov}_t(\varepsilon_{i,t+1}+\varepsilon_{j,t+1},\log\text{GDP}_{t+1})\\&\quad+(1-\xi+\alpha\xi)^2\text{var}_t(\log\text{GDP}_{t+1}),\end{aligned} \tag{A.26}$$

where we are using the fact that sectoral shocks are independent. Summing both sides of the above equation over all sectors $j\neq i$ implies that

$$\begin{aligned}\sum_{j\neq i}\text{cov}_t(\log y_{i,t+1},\log y_{j,t+1})&=(n-2)\xi(1-\xi+\alpha\xi)\text{cov}_t(\varepsilon_{i,t+1},\log\text{GDP}_{t+1})\\&\quad+(n-1)(1-\xi+\alpha\xi)^2\text{var}_t(\log\text{GDP}_{t+1})\\&\quad+\xi(1-\xi+\alpha\xi)\,\text{cov}_t\left(\sum_{j=1}^{n}\varepsilon_{j,t+1},\log\text{GDP}_{t+1}\right).\end{aligned} \tag{A.27}$$

Consequently, for any pair of sectors $k\neq i$,

$$\Delta_t^{ik}=(n-2)\xi(1-\xi+\alpha\xi)\text{cov}_t(\varepsilon_{i,t+1}-\varepsilon_{k,t+1},\log\text{GDP}_{t+1}). \tag{A.28}$$

Differentiating the above equation with respect to $\varepsilon_{i,t}$ then implies that

$$\frac{\mathrm{d}\Delta_t^{ik}}{\mathrm{d}\varepsilon_{i,t}} = (n-2)\xi(1-\xi+\alpha\xi)\mathrm{cov}_t\left(\varepsilon_{i,t+1}-\varepsilon_{k,t+1}, \frac{\mathrm{d}}{\mathrm{d}\varepsilon_{i,t}}\log \mathrm{GDP}_{t+1}\right). \tag{A.29}$$

On the other hand, recall that log aggregate output is given by (8) and log productivity shock to sector i follows an AR(1) process with persistence parameter ρ_i. Therefore,

$$\frac{\mathrm{d}}{\mathrm{d}\varepsilon_{i,t}}\log \mathrm{GDP}_{t+1} = \frac{\rho_i a_i}{1-\alpha}\frac{e^{(\xi-1)\varepsilon_{it+1}}}{\sum_{j=1}^n a_j e^{(\xi-1)\varepsilon_{j,t+1}}}. \tag{A.30}$$

Since $\xi < 1$, it is immediate that the right-hand side of the above expression is decreasing in $\varepsilon_{i,t+1}$ and increasing in $\varepsilon_{k,t+1}$ for all $k \neq i$. Therefore,

$$\mathrm{cov}_t\left(\varepsilon_{i,t+1}, \frac{\mathrm{d}}{\mathrm{d}\varepsilon_{i,t}}\log \mathrm{GDP}_{t+1}\right) < 0 \qquad \text{and} \qquad \mathrm{cov}_t\left(\varepsilon_{k,t+1}, \frac{\mathrm{d}}{\mathrm{d}\varepsilon_{i,t}}\log \mathrm{GDP}_{t+1}\right) > 0, \tag{A.31}$$

where we are using the fact that if $g(\cdot)$ is strictly increasing, then $\mathrm{cov}(x, g(x)) > 0$ for any non-degenerate random variable x.[1] The above inequalities thus imply that when $\xi < 1$, the right-hand side of (A.29) is strictly negative. This establishes the first inequality in (14).

To establish the second inequality in (14), note that equation (7) implies that

$$\mathrm{cov}_t(\log y_{i,t+1}, \log \mathrm{GDP}_{t+1}) = (1-\xi+\alpha\xi)\mathrm{var}_t(\log \mathrm{GDP}_{t+1}) + \xi\mathrm{cov}_t(\varepsilon_{i,t+1}, \log \mathrm{GDP}_{t+1}). \tag{A.32}$$

Therefore, $\Xi_t^{ik} = \xi\mathrm{cov}_t(\varepsilon_{i,t+1}, \log \mathrm{GDP}_{t+1}) - \xi\mathrm{cov}_t(\varepsilon_{k,t+1}, \log \mathrm{GDP}_{t+1})$, which in turn implies that

$$\frac{\mathrm{d}\Xi_t^{ik}}{\mathrm{d}\varepsilon_{i,t+1}} = \xi\mathrm{cov}_t(\varepsilon_{i,t+1}, \frac{\mathrm{d}}{\mathrm{d}\varepsilon_{i,t}}\log \mathrm{GDP}_{t+1}) - \xi\mathrm{cov}_t(\varepsilon_{k,t+1}, \frac{\mathrm{d}}{\mathrm{d}\varepsilon_{i,t}}\log \mathrm{GDP}_{t+1}). \tag{A.33}$$

The inequalities in (A.31) then imply that $\mathrm{d}\Xi_t^{ik}/\mathrm{d}\varepsilon_{i,t+1} < 0$. □

A.2 Extensions and Variations

This section examines a number of extensions to the baseline setup.

First, we examine a version of the model with differentiated firms within each sector. The sector-level results from the main text are unchanged (though the sector productivity

[1] To see this, note that $\mathrm{cov}(x, g(x)) = \mathbb{E}[xg(x)] - \mathbb{E}[x]\mathbb{E}[g(x)] = \mathbb{E}[(x-\mathbb{E}[x])g(x)] = \mathbb{E}[(x-\mathbb{E}[x])(g(x) - g(\mathbb{E}(x))]$. Since $g(\cdot)$ is strictly increasing, $(x-\mathbb{E}[x])(g(x)-g(\mathbb{E}(x)) > 0$ for all $x \neq \mathbb{E}[x]$. Therefore, $\mathrm{cov}(x, g(x)) > 0$.

shocks become endogenous), and we discuss the implications for firm data.

Second, we examine a version of the model that explicitly allows for multiple layers of aggregation among sectors, building on the previous extension. This gives a more concrete mapping to the empirical analysis across levels of aggregation.

Finally, in our baseline model in Section 3, we assumed that labor can be flexibly reallocated across sectors and is supplied by the representative household inelastically. In this appendix, we show that the characterization in equations (7) and (8) remain valid with minor modifications if we allow for elastic labor supply and sector-specific factors (say, capital). We then show that sectoral and aggregate payments to this fixed factor also share the same characteristics as sectoral and aggregate output. Therefore, to the extent that stock returns move with payments to capital, the predictions in Section 4 also apply to stock returns.

A.2.1 Differentiated firms within each sector

This section examines a version of the model with multiple firms within each sector. As in the model from the main text, there is a set of goods indexed by j, which we refer to as the sector outputs. Those goods are combined into GDP according to the same function,

$$\mathrm{GDP}_t = \left(\sum_{j=1}^{n} a_j^{1/\xi} c_{j,t}^{(\xi-1)/\xi} \right)^{\xi/(\xi-1)}. \tag{A.34}$$

Now, though, instead of production occurring at the sector level, we assume that there is a set of firms within each sector that each produce a differentiated output, denoted $y_{j,i}$, according to the production function

$$y_{j,i} = z_{j,i} l_{j,i}^{\alpha} \left(\sum_k a_k^{1/\xi} x_{j,i,k}^{(\xi-1)/\xi} \right)^{\alpha\xi/(\xi-1)}, \tag{A.35}$$

where $z_{j,i}$ is the productivity of firm i in sector j, $l_{j,i}$ is its use of labor, and $x_{j,i,k}$ is its use of good k. The total output of sector j is a CES aggregate of the outputs of the firms in that sector,

$$y_j = \left(\sum_i y_{j,i}^{(\gamma-1)/\gamma} \right)^{\gamma/(\gamma-1)}, \tag{A.36}$$

where γ is the elasticity of substitution across firms.

Lemma 4. *In equilibrium, each sector j has an as-if production function of the form*

$$y_j = z_j \ell_j^{1-\alpha} \left(\sum_{k=1}^{n} a_k^{1/\xi} x_{j,k}^{(\xi-1)/\xi} \right)^{\alpha\xi/(\xi-1)}, \tag{A.37}$$

$$\text{where } z_j = \left(\sum_i z_{j,i}^{(\gamma-1)} \right)^{1/(\gamma-1)}, \ l_j = \sum_i l_{j,i}, \text{ and } x_{j,k} = \sum_i x_{j,i,k} \tag{A.38}$$

That is, the model has a reduced-form representation in which sector output has an as-if production function of the exact same form as in the main analysis, with the only difference being that sector productivity is now an endogenous function of the firm-level shocks. But sector and aggregate output in this model are observationally equivalent to sector and aggregate output in the model in the main text.

Proof. Given prices, it is straightforward to show that each firm will use the same mix of inputs. If the share of inputs used by firm i is denoted by $\omega_{j,i}$, then each firm's output is

$$y_{j,i} = z_{j,i} X_j \omega_{j,i} \tag{A.39}$$

where X_j is the total use of inputs by sector j. In particular,

$$X_j = l_j^{\alpha} \left(\sum_k a_k^{1/\xi} x_{j,k}^{(\xi-1)/\xi} \right)^{\alpha\xi/(\xi-1)} \tag{A.40}$$

where l_j is sector j's total use of labor and $x_{j,k}$ is sector j's total use of good k.

A social planner's objective is then to maximize sector output conditional on sector inputs, subject to the constraint that $\sum_i \omega_j = 1$, which has the Lagrangian,

$$\max \sum_i (z_i \omega_i)^{(\gamma-1)/\gamma} - \lambda \sum_i \omega_i \tag{A.41}$$

Optimizing yields the result that

$$y_j = \left(\sum_i z_{j,i}^{(\gamma-1)} \right)^{1/(\gamma-1)} l_j^{\alpha} \left(\sum_k a_k^{1/\xi} x_{j,k}^{(\xi-1)/\xi} \right)^{\alpha\xi/(\xi-1)}. \tag{A.42}$$

□

Proposition 5. *In equilibrium, sector and firm output satisfy*

$$\log y_j = (1-\xi+\alpha\xi)\log \mathrm{GDP} + \xi \log\left(\sum_i z_{j,i}^{(\gamma-1)}\right)^{1/(\gamma-1)} + \delta_i \tag{A.43}$$

$$\log y_{j,i} = (1-\xi+\alpha\xi)\log \mathrm{GDP} + (\xi-\gamma)\log\left(\sum_i z_{j,i}^{(\gamma-1)}\right)^{1/(\gamma-1)} + \delta_i + \gamma \log z_i \tag{A.44}$$

Proof:

Continuing the analysis in the proof of lemma 4, one can also show that firm output satisfies

$$y_{j,i}/y_j = z_{j,i}^{\gamma}\left(\sum_i z_{j,i}^{\gamma-1}\right)^{-\gamma/(\gamma-1)} \tag{A.45}$$

Now due to the result in lemma 4, sector output is the same function of GDP as in the main text, yielding the first part of the proposition. Combining that with the relationship between firm and sector output yields the result for firm output. □

In the case where firm outputs are substitutes and sector outputs are complements, $\gamma > 1$ and $\xi - \gamma < 0$, so that the coefficient on sector productivity is negative, while the coefficient on the firm's on productivity is positive. In the case where $\log\left(\sum_i z_{j,i}^{(\gamma-1)}\right)^{1/(\gamma-1)}$ is skewed right, firm output will be negatively skewed, and moreso than sector output (how firm skewness compares to skewness in $\log \mathrm{GDP}$ is ambiguous). If the firm shocks are aggregated according to a CES aggregator with elasticity greater than 1, that would also imply that the sector-specific residuals are positively skewed, which we do not observe.

All of that said, the empirical evidence at the firm level from Barrot and Sauvagnat (2016) and Carvalho et al. (2021) implies that in the short-run, substitution across firms is in fact limited, implying that γ is finite, and potentially even smaller than 1.

A.2.2 Multiple layers of aggregation

The analysis in the previous section has multiple layers of aggregation in the sense that there are both firms and sectors. One would just as easily, however, refer to the firms in the previous section as "subsectors". That is, in the terminology of sector data, one could think of the $y_{j,i}$ units as two-digit level data and the y_j units as one-digit data. Output at those different levels of aggregation is then, just copying directly from proposition 5

$$\text{One-digit output: : } \log y_j = (1-\xi+\alpha\xi)\log \text{GDP} + \xi \log \left(\sum_i z_{j,i}^{(\gamma-1)}\right)^{1/(\gamma-1)} + \delta_i \tag{A.46}$$

$$\text{Two-digit output: : } \log y_{j,i} = (1-\xi+\alpha\xi)\log \text{GDP} + (\xi-\gamma) \log \left(\sum_i z_{j,i}^{(\gamma-1)}\right)^{1/(\gamma-1)} + \delta_i + \gamma \log z_i \tag{A.47}$$

Now we have that one-digit output is equal to aggregate output plus a shock, while two-digit output is equal to aggregate output plus two different shocks. The additional noise present at the two-digit level, due to the additional shock, intuitively explains why the model can generate less negative skewness at lower levels of aggregation.

Note, though, that this is naturally parameter-dependent. One simple way to get the desired result is to have $\xi = \gamma$. Then as long as $\text{var}(\log z_i) > \text{var}\left(\log\left(\sum_i z_{j,i}^{(\gamma-1)}\right)^{1/(\gamma-1)}\right)$ – i.e. if the low-level productivity shocks are less volatile than their aggregate – then skewness will become more negative with aggregation. On the other hand, as discussed in the previous section, there are obviously parameterizations that would also fail to match the data, where skewness at the two-digit level could be more negative than skewness at the one-digit level (e.g. if $\gamma > 1$ and $\xi < 1$).

As a simple example, consider a case where $\log z_{j,i} \sim N(0, 0.1)$, $\xi = 0.75$, $\gamma = 0.5$, and $\alpha = 0.5$. Set $a_i = 1 \ \forall \ i$ for simplicity, and assume $\log z_i$ has a Laplace distribution with standard deviation of 0.28 and that there are 20 one-digit sectors and 20 two-digit sectors within each one-digit sector . Then the skewness of two-digit output is -0.08, while for one-digit output it is -0.05. Obviously none of this is quantitatively realistic (see section 6 for something much more realistic), but it illustrates how the model can work when it formally includes multiple levels of aggregation.

A.2.3 Model with elastic and fixed factors

We consider an economy such that the production function of firms in sector i is given by

$$y_i = z_i \zeta k_i^{1-\alpha-\beta} \ell_i^{\beta} \left(\sum_{j=1}^{n} a_j^{1/\xi} x_{ij,t}^{(\xi-1)/\xi}\right)^{\alpha\xi/(\xi-1)}, \tag{A.48}$$

where ℓ_i is the labor input (which, as in the baseline model, can be flexibly allocated across sectors), k_i is a fixed factor of production that is specific to firms in sector i (which we refer to as capital), and ζ is a normalization constant. Throughout, we normalize the installed value of sector-specific capital to $k_i = 1$ for all i. In the above specification, α and β parametrize share of labor and intermediate inputs and are such that $\alpha + \beta \leq 1$. Finally, we assume that the consumption bundle is given by (5) and the representative household supplies labor elastically at some exogenously-specified real wage w.[2] To simplify the expressions, we choose the normalization constant $\zeta = \alpha^{\beta-1}\beta^{-\beta}w^{\beta}$.

To characterize the equilibrium of this economy, note that since the supply of all sector-specific capital is normalized to one, the economy is equivalent an alternative economy in which firms in sector i have a decreasing returns production function given by

$$y_i = z_i \zeta \ell_i^{\beta} s_i^{\alpha}, \tag{A.49}$$

where s_i denotes the intermediate input bundle of sector i. In this economy, the profit of firms in sector i is given by $\pi_i = p_i y_i - w l_i - s_i$, where p_i denotes the price of good i. The first-order conditions corresponding to sector i's problem imply that sector i's demand for the intermediate input bundle and labor are given by

$$\ell_i = \beta p_i y_i / w \qquad \text{and} \qquad s_i = \alpha p_i y_i, \tag{A.50}$$

respectively. From the above it is immediate that the firm i's expenditure on labor and profits are given by $w l_i = \beta s_i/\alpha$ and $\pi_i = (1 - \alpha - \beta)s_i/\alpha$, respectively. Consequently, aggregate output in this economy is equal to

$$\text{GDP} = \sum_{i=1}^{n} \pi_i + \sum_{i=1}^{n} w\ell_i = (1/\alpha - 1)\sum_{i=1}^{n} s_i. \tag{A.51}$$

Next, note that the market-clearing condition for good j in equation (6) can be written as

$$\alpha^{-1} z_j s_j^{\alpha+\beta} = a_j p_j^{-\xi}\,\text{GDP} + a_j p_j^{-\xi} \sum_{i=1}^{n} s_i, \tag{A.52}$$

where we are using the fact that the household's and sector i's demand for good j are given

[2] A simple microfoundation for this assumption is to assume that household preferences are given by $u(C, L) = C - \chi \frac{1}{1+1/\eta} L^{1+1/\eta}$ for some parameter $\chi > 0$ and consider the limit that the Frisch elasticity of labor supply $\eta \to \infty$. In such an economy, the real wage is given by $w = \chi$.

by $c_j = a_j p_j^{-\xi}\,\text{GDP}$ and $x_{ij} = a_j p_j^{-\xi} s_i$, respectively. Thus, by (A.51),

$$z_j s_j^{\alpha+\beta} = a_j p_j^{-\xi} \sum_{i=1}^{n} s_i. \tag{A.53}$$

Next note that the first-order conditions in (A.50) imply that sector j's demand for the intermediate input bundle is given by

$$s_j = (p_j z_j)^{1/(1-\alpha-\beta)}.$$

Therefore, solving for p_j from the above equation and plugging in into (A.53) implies that

$$s_j = \left(a_j z_j^{\xi-1} \sum_{i=1}^{n} s_i \right)^{\frac{1}{1+(\xi-1)(1-\alpha-\beta)}}. \tag{A.54}$$

Summing both sides of the above equation over all j, solving for $\sum_{i=1}^{n} s_i$ and plugging the result into equation (A.51) implies that

$$\log \text{GDP} = \delta + \frac{1+(\xi-1)(1-\alpha-\beta)}{(\xi-1)(1-\alpha-\beta)} \log \sum_{j=1}^{n} \hat{a}_j z_j^{\frac{\xi-1}{1+(\xi-1)(1-\alpha-\beta)}}, \tag{A.55}$$

where δ is some constant and $\hat{a}_j = a_j^{\frac{1}{1+(\xi-1)(1-\alpha-\beta)}}$ for all j. The above equation is the counterpart to equation (8) with flexible and inelastic labor. In particular, when $\xi < 1$, log aggregate output is a concave function of log sectoral shocks.

To obtain the counterpart to equation (7) for sectoral output, note that equation (A.50) implies that $y_j = z_j s_j^{\alpha+\beta}/\alpha$. Therefore, equation (A.54) implies that

$$\log y_j = \hat{\delta}_j + \frac{\xi}{1+(\xi-1)(1-\alpha-\beta)} \log z_j + \frac{1}{1+(\xi-1)(1-\alpha-\beta)} \log \text{GDP}, \tag{A.56}$$

for some constant $\hat{\delta}_j$. Therefore, as in equation (7) for our baseline model, sectoral output has a factor structure consisting of a common factor that is proportional to log aggregate output and an idiosyncratic component that is proportional to log productivity shock to that sector.

Taken together, the similarity between equation pairs (A.55)–(A.56) and (7)–(8) implies that Propositions 1–3 also hold for the economy with elastic and immobile factors of production.

A.2.4 Payments to capital

We next argue that Propositions 1–3 also apply to sectoral and aggregate payments to capital. To this end, we show that (i) log sectoral profits have a factor structure similar to equation (7) and (ii) log aggregate profits is a concave CES aggregate of log productivity shocks.

To see the first claim above, recall that in Section A.2.3, we established that the profits of sector j (which is equal to payments on its fixed capital stock) is given by $\pi_j = (1-\alpha-\beta)s_j/\alpha$, where s_j denotes sector j's demand for the intermediate input bundle. We also established that $y_j = z_j s_j^{\alpha+\beta}/\alpha$. Therefore, π_j is proportional to $(y_j/z_j)^{1/(\alpha+\beta)}$, which given (A.56), implies that (log) sectoral profits also have a factor structure similar to that of (log) sectoral output.

Similarly, $\pi_j = (1-\alpha-\beta)s_j/\alpha$ coupled with the fact that $\sum_{j=1}^{n} s_j$ is proportional to aggregate output (equation (A.51)) also implies that aggregate profits is proportional to aggregate output. Therefore, in view of (A.55), (log) aggregate profit is also a CES aggregate of sectoral shocks and is concave in those shocks when intermediate inputs are gross complements.

A.2.5 Skewness of growth rates

This section gives conditions under which the growth rates of output in the model are skewed left. We study a continuous time specification. In particular, a case where productivity follows a mean-reverting version of a finite-activity Lévy process (finite activity means that in any given time period, the number of jumps is almost surely finite). A Lévy process is a general class in which the increments are independent and stationary. There are well known results on representations for such processes. We impose one restriction, which is that the jump component of the process (from the Lévy–Itô representation) has finite activity (i.e. the Lévy measure is finite). This is a restriction to allow only certain types of jumps, which we impose because it allows the jump process to be expressed as a simple compound Poisson process, keeping the analysis relatively simple (see Cont and Tankov (2004) sections 3.4 and 4.1.1). Jump diffusions are widely studied in economics, particularly within finance.

Formally, we assume that for all i, $\varepsilon_{i,t}$ follows

$$d\varepsilon_{i,t} = -m\left(\varepsilon_{i,t}\right)dt + \sigma dW_{i,t} + \Delta\varepsilon_{i,t} \tag{A.57}$$

where $W_{i,t}$ is a standard Wiener process and $\Delta\varepsilon_{i,t}$ is a compound Poisson process, equal to a random number $k_{i,t}$ with probability λdt and zero otherwise (for a more formal definition, see Cont and Tankov (2004) section 8.3). $k_{i,t}$ is a symmetrically distributed random variable. We assume that the function m is such that ε has a well-defined unconditional distribution with finite moments. Intuitively, that requires that $m(\cdot)$ induces mean reversion in ε, for example as in an Ornstein–Uhlenbeck process.

The solution of the model is such that aggregate output is a function f of the sector productivities with the characteristics that $f_i > 0$ and $f_{ii} < 0$ $\forall i$ and $f_{ij} > 0$ $\forall i \neq j$. The question here is under what circumstances df_t is skewed left. That is, when do our results on skewness in levels also apply to growth rates?

Itô's lemma in the case of a jump diffusion (Cont and Tankov, 2004, section 8.3.2) yields

$$\begin{aligned} df_t &= \sum_i \left(-m(\varepsilon_{i,t}) f_{i,t} + \frac{\sigma^2}{2} f_{ii,t} \right) dt \\ &\quad + \sum_i f_i \sigma dW_{i,t} + \sum_i f(\ldots, \varepsilon_{i,t-} + \Delta\varepsilon_i, \ldots) - f(\ldots, \varepsilon_i, \ldots) \end{aligned} \tag{A.58}$$

Now first assume that there are no jumps, so that the final summation above is equal to zero. Then we have

$$\mathbb{E}\left[df_t^3\right] = O\left(dt^3\right) \tag{A.59}$$

$$\mathbb{E}\left[df_t^2\right] = O(dt) \tag{A.60}$$

and hence skewness is $O\left(dt^{3/2}\right) \to 0$.

Alternatively, suppose there are jumps. Then

$$\mathbb{E}\left[df_t^3\right] = \sum_i \lambda dt \mathbb{E}\left[\left(f(\ldots, \varepsilon_{i,t-} + k_t, \ldots) - f(\ldots, \varepsilon_i, \ldots)\right)^3\right] + o(dt) \tag{A.61}$$

The fact that f is globally concave implies that the expectation on the right-hand side is negative. Furthermore, $\mathbb{E}[df_t^2] = O(dt)$, so that skewness is negative and $O\left(dt^{-1/2}\right)$.

A.3 Measurement error

This section examines the effect of measurement error on the results for skewness across levels of aggregation. It first provides a simple calculation quantifying how much measurement error would be needed in order to generate the observed differences, and second it reports results for annual instead of monthly growth rates and shows that they are highly similar.

First, denote some aggregate variable by x_{agg} and a sector-level variable x_{sect}. The x's are the true variables, and assume they have identical skewness. Now suppose that while the aggregate variable is observed directly, we only see $y_{sect} = x_{sect} + \varepsilon$, where ε is symmetrically distributed measurement error. For skewness, we assume

$$\frac{skew\left(x_{sect}\right)}{skew\left(x_{agg}\right)} = 1 \tag{A.62}$$

It is then straightforward to show that

$$\frac{skew\left(y_{sect}\right)}{skew\left(x_{agg}\right)} = \left(\frac{\text{var}\left(x_{sect}\right) + \text{var}\left(\varepsilon\right)}{\text{var}\left(x_{sect}\right)}\right)^{-3/2} \tag{A.63}$$

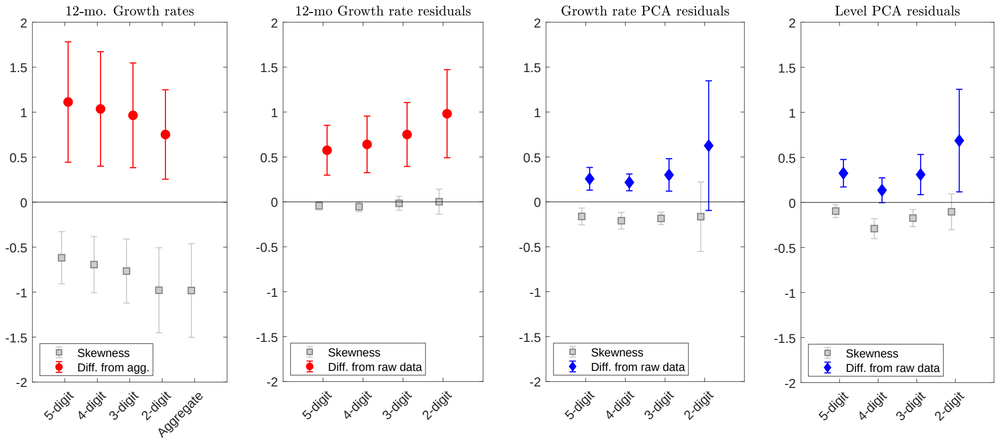
In the data, for both employment and industrial production, the ratio of skewness at the most disaggregated level to skewness at the most aggregated level is 0.35. That is, $\frac{skew(y_{sect})}{skew(x_{agg})} = 0.35$. In order for that to be due to measurement error, we must have

$$0.35 = \left(\frac{\text{var}\left(x_{sect}\right) + \text{var}\left(\varepsilon\right)}{\text{var}\left(x_{sect}\right)}\right)^{-3/2} \tag{A.64}$$

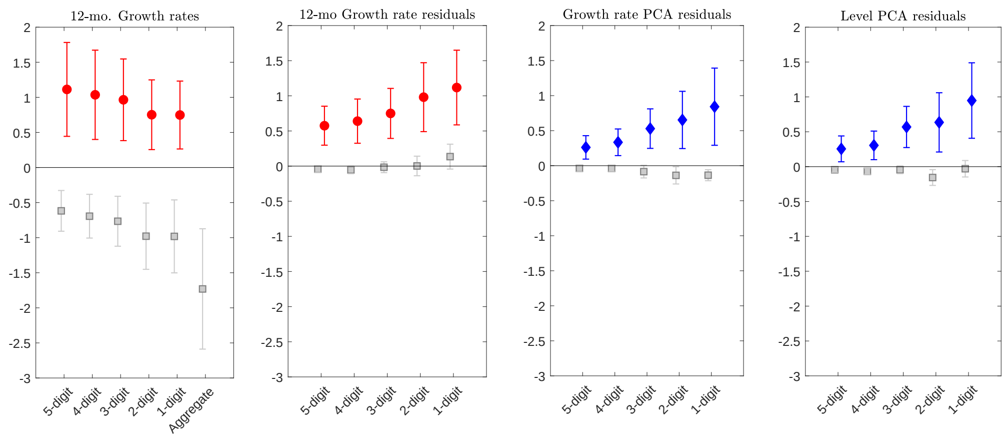
$$\Rightarrow \frac{\text{var}\left(\varepsilon\right)}{\text{var}\left(x_{sect}\right)} = 1.01 \tag{A.65}$$

In other words, as discussed in the main text, for measurement error to explain the magnitude of the decline in skewness observed with disaggregation, it would need to be the case that more than half of the variation in the observed sector-level growth rates comes from measurement error.

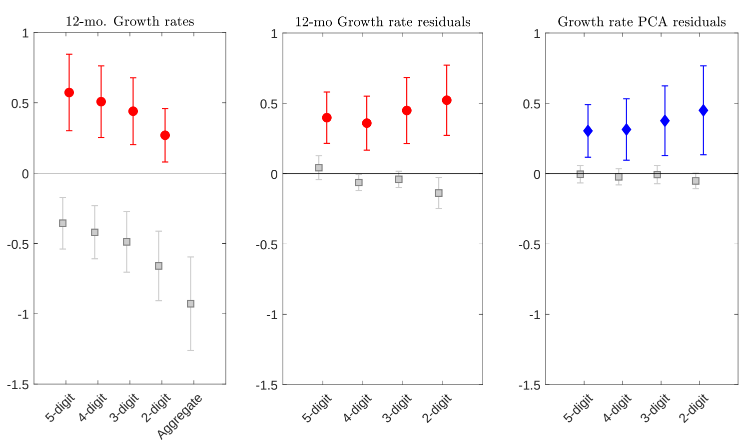
To the extent that there is measurement error, though, it is more likely to appear in monthly than annual data. For employment, in particular, annual data is based on the full universe of firms (as it comes from the full set of unemployment insurance filings). Furthermore, aggregation over time increases the strength of the signal relative to noise, since true changes in the underlying series become larger. Figure A.1 therefore reproduces the analysis of skewness of growth rates using annual rather than monthly changes. The results are highly similar to those in Figure 1, providing further evidence that the results are not driven by measurement error.



(a) Industrial Production



(b) Employment



(c) Stock Returns

Figure A.1: **Time-Series Skewness Annual Data**

Notes: This figure plots skewness for different levels of aggregation for industrial production, employment, and stock returns for growth rates (left two panels) or principal components (right two panels) together with 90-percent confidence intervals. Data is annual and starts in January 1972 and ends in December 2019 for industrial production and stock returns and starts in January 1990 and ends in December 2019 for employment.