

March 12, 2020

Presidential Address:  
Social Transmission Bias in Economics and Finance

David Hirshleifer<sup>a</sup>

I discuss a new intellectual paradigm, *social economics and finance*: the study of the social processes that shape economic thinking and behavior. This emerging field recognizes that people observe and talk to each other. A key, underexploited building block of social economics and finance is *social transmission bias*: a systematic directional shift in signals or ideas induced by social transactions. I use five “fables” (models) to illustrate the novelty and scope of the transmission bias approach, and offer several emergent themes. For example, social transmission bias compounds recursively, which can help explain booms, bubbles, return anomalies, and swings in economic sentiment.

**Key Words:** social transmission bias, social economics, social finance, behavioral economics, behavioral finance, social networks, social learning, social influence, information percolation, epidemiology, SIR model, compartmental models, visibility bias, self-enhancing transmission bias, recursive amplification, simplistic thinking, memes, folk models, cultural evolution

<sup>a</sup>Merage School of Business, UC Irvine, Irvine, CA 92617, USA

This paper was the Presidential Address of the American Finance Association at the 2020 Annual Meeting in San Diego, CA. I thank Daniel Andrei, Julien Cujean, Darrell Duffie, Ben Golub, Cam Harvey, Michael Hirshleifer, Chong Huang, Nan Ma, Robert Merton, Amit Seru, Robert Shiller, Avanidhar Subrahmanyam, Siew Hong Teoh, Sheridan Titman, Johan Walden, Ivo Welch, Jeff Zwiebel, the 2019 Retreat on Information and Social Economics (RINSE), and the FRILLS (Finance Research Informal Lunchtime Seminar) at Stanford University for extremely helpful comments, and Yushui Shi and Josh Thornton for outstanding research assistance. Presentation slides for the talk are available at SLIDES. A transcript with integrated slides is available at TRANSCRIPT. Video of the talk is available at VIDEO.

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I want to talk to you about a new intellectual paradigm, which I call *social economics and finance*—the study of how social interaction affects economic outcomes. In standard analyses of economic behavior, people interact only impersonally via trading orders and observation of market price. A missing chapter in our understanding of finance consists of the social processes that shape economic thinking and behavior.

Social economics and finance recognizes that people observe each other and talk to each other, where talking includes written text and social media. A key, underexploited, intellectual building block of social economics and finance is *social transmission bias*, the systematic directional modification of ideas or signals as they pass from person to person.

By way of comparison, some recent intellectual paradigm shifts include information economics, which recognized that some people know things that others do not; and behavioral economics and finance, which recognized that people make systematic mistakes. Of course, scholars knew these obvious facts long before each of these revolutions. The problem was that these facts were considered only informally and sporadically. They were not systematically, explicitly, and routinely incorporated in our thinking and models to generate test hypotheses. Similarly, as of now, social interaction is only starting to join the standard intellectual toolkit for finance research.

As things stand, in behavioral finance (for example), the path from assumptions to conclusions is often very direct. With respect to beliefs, when we observe that investors trade too aggressively, we conclude that they must be overconfident. When we observe that expectations become more optimistic after price run-ups than after run-downs, we conclude that they must overextrapolate. With regard to preferences, when we see individual investors tilted toward buying lottery stocks, selling winners more than losers, or saving too little—well, we have behavioral models in which investors have preferences for skewness, for realizing gains not losses, and for immediate consumption.

I am a fan of these direct approaches. They capture a large part of the truth.

#### Summary

I offer six themes about a new intellectual paradigm, *social economics and finance*, and one of its key intellectual building blocks, *social transmission bias*.

- **Compounding:** Social transmission bias compounds recursively, so small bias can have large effects.
- **Idiosyncrasy:** Social transmission bias helps explain why aggregate outcomes are often error-prone and unpredictable.
- **Dynamics:** Social transmission bias offers an endogenous *social* explanation for action booms, price bubbles, and swings in investor sentiment.
- **Emergence:** Socially emergent behavior often looks completely different from individual propensities.
- **Mimicry:** Social emergence can easily create the illusion of a direct individual propensity “for” a behavior when no such propensity exists. So the inferences drawn from empirical tests of behavioral hypotheses are often overstated.
- **Proxies:** This approach suggests *new test variables* for empirical research: (i) general social and network proxies, and (ii) proxies for transmission bias.

However, crucially, there can be *attraction* to a behavior without any *preference* for it. Moths are attracted to flame—a nearby light source. But moths are not flame-loving. There is no cognitive reward system that pays off more for approaching a flame. Instead, moths evolved under natural selection to navigate based on a distant light source, the moon. Nearby light sources fool their navigation systems. So attraction to flame is an indirect effect, not a direct preference.

Another kind of indirect effect is *social emergence*—the phenomenon that social outcomes are not just the sums of individual propensities. An example of a socially emergent effect is the phenomenon of death spirals among army ants, as seen in Figure 1. These can be up to hundreds of feet wide. The ants continue to walk in circles until they die.

Surely this could not happen to a smarter animal, such as a mammal. Except, apparently, for man’s best friend, as seen in videos of jostling dogs rotating about their shared food bowl. Are birds smarter? The most disturbing spectacle of all is seen in a video of turkeys marching in a circle around a dead cat—apparently some kind of religious ritual.

Should we conclude from these behaviors that animals have a rotative instinct? A heuristic or bias for circular motion? Of course not. In the case of ants, there are instincts for random search for food, and instincts that promote following other ants. As a result, if the ant at the front of the line by chance starts following the ant at the back of the line, a dysfunctional social outcome results—somewhat reminiscent of a fad or bubble.<sup>1</sup> These animal circles illustrate that aggregate social outcomes often look nothing like individual propensities. There is no direct instinct for a single ant, dog, or bird to move endlessly in circles. Behavior is socially emergent.

There are also socially emergent human behaviors. Consider, for example, self-enhancing transmission bias—the tendency of investors to discuss their trades with others more after experiencing strong performance than after weak performance.<sup>2</sup> If listeners fail to adjust for this bias, they will tend to be unduly attracted to the sender’s investing strategy. As we will see, this effect favors the spread of some strategies over others (Han, Hirshleifer, and Walden 2019a), but not because of any direct psychological bias for one strategy over another. The attraction to certain kinds of investment strategies is *socially emergent*.

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<sup>1</sup>This phenomenon is closely akin to information cascades (Banerjee (1992), Bikhchandani, Hirshleifer, and Welch (1992)). In information cascades, even rational social learning results in fad-like outcomes.

<sup>2</sup>Evidence for this is provided by Heimer and Simon (2015), Ammann and Schaub (2016), Huang, Hwang, and Lou (2018), Escobar and Pedraza (2019), and Lim, Ng, and Uzzi (2020).

An implication of social emergence is that it is unwarranted to slip from an empirically observed behavior to the conclusion that there is a direct psychological bias “for” that behavior. We often forget this in the finance field. Emergent *social* effects are usually neglected. A key reason for this is that *social transmission bias is missing from our standard intellectual toolkit*.

Social economics and finance seeks to expand the intellectual toolkit. To do so, it needs a firm foundation of standard ingredients: preferences, beliefs, psychological bias, optimization, and equilibrium. Furthermore, as with behavioral economics, good social economics and finance requires good assumptions about psychological bias. Such assumptions need to be based on available evidence and evolutionary plausibility. Using this foundation, the purpose is to capture, in systematic and tractable ways, socially emergent effects on economic behavior, as well as more conventional direct effects.

The main body of this talk consists of five “fables” (models) of social transmission bias in economics and finance. These fables describe research, including work-in-progress, by my coauthors and myself. My purpose is to suggest that the transmission bias approach may have novel messages to offer, and may be wide-ranging in scope of application. I then discuss some themes that emerge from these fables.

But first, in a necessarily selective way, I will mention a few milestones of the social economics and finance paradigm shift. This includes models of rational and of biased social influence in networks<sup>3</sup>; the cultural transmission of ethnic, religious, and cooperative traits (Bisin and Verdier 2000; Tabellini 2008), and the study of payoff interactions and games (Sandholm 2010). There are empirical literatures on narratives and folk models in economics and finance<sup>4</sup>; on culture, ideology, norms, and economic outcomes<sup>5</sup>; and on contagion of economic/financial behaviors, both at the level of individuals<sup>6</sup> and firms.<sup>7</sup>

Social transmission bias derives from combinations of bias in the messages that senders convey to others (or what is visible to others about “senders” as targets of

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<sup>3</sup>Rational: Banerjee (1992), Bikhchandani, Hirshleifer, and Welch (1992), and Acemoglu et al. (2011). Biased: Ellison and Fudenberg (1993), DeMarzo, Vayanos and Zwiebel (2001, 2003), Eyster and Rabin (2010), Golub and Jackson (2010), and Bohren (2016). Surveys include Jackson (2008) and Golub and Sadler (2016).

<sup>4</sup>See Shiller, Kon-Ya, and Tsutsui (1996) and Shiller (2000b, 2017, 2019).

<sup>5</sup>See, e.g., Grinblatt and Keloharju (2001), Barro and McCleary (2003) Guiso, Sapienza, and Zingales (2003), Guiso, Sapienza, and Zingales (2004), Hong and Kacperczyk (2009), Hong and Kostovetsky (2012), and Graham et al. (2019).

<sup>6</sup>See, e.g., Glaeser, Sacerdote, and Scheinkman (1996), Kelly and O’Grada (2000), Brock and Durlauf (2001), Duflo and Saez (2002), Duflo and Saez (2003), Hong, Kubik, and Stein (2004), and Bailey et al. (2019).

<sup>7</sup>See, e.g., Bizjak, Lemmon, and Whitby (2009), Chiu, Teoh, and Tian (2013), and Fracassi (2016).

observation), and bias on the part of receivers in what is absorbed and understood. There are of course other important intellectual building blocks for understanding human behavior. One such concept is social network structure, which has been the object of intense interdisciplinary study in recent years. In contrast, the complementary concept of social transmission bias is underexplored, and needs to join our standard intellectual toolkit.

Transmission bias can take at least two forms. The first is *signal distortion*: a shift in the sign or intensity of what is transmitted. For example, the owner of a stock may talk up the firm to try to persuade others to buy it. If listeners do not adequately discount for this, their beliefs will be biased.

The second is *selection*. A sender's information is not always conveyed to a potential receiver. The beliefs of a receiver who neglects this fact will be biased. When censorship by a potential sender is a function of the values of the sender's information signals, there is also selection bias, such as self-enhancing transmission bias, as discussed above.

I now turn to five fables of social transmission bias in economics and finance. My main new model in this paper is Fable 4 on biased information percolation.

## II Fable 1: Bandwidth constraints and simplistic thinking

Fable 1 is in-progress research (Hirshleifer and Tamuz 2020).

Communication bandwidth constraints, such as Twitter character limits and time constraints in conversation, are a major obstacle to conveying ideas with nuance. Another major hindrance is limited cognition. It takes considerable effort to convey and absorb message nuance. So there is usually a loss of nuance in social transmission, as is evident in TV sound-bite commentary about social issues.

Suppose that in updating their beliefs, receivers do not adjust for loss of nuance. This is consistent with standard limited attention effects in psychology and in behavioral economics and finance. Suppose further that receivers believe that senders possess meaningful information. Then receivers will tend to infer that senders have simple or extreme beliefs. So receivers will adopt beliefs that actually *are* simplistic.

As receivers become senders in turn, this loss of nuance may be iterated. So society can shift toward extremes of simplistic thinking, to the point that (conjecturally) judgments are worse than if people formed judgments in isolation. The reader can decide how realistic this is as a description of public understanding of economics and society.

Of course, often social processes improve our collective intelligence, as emphasized by Henrich (2015), and as evidenced by the rise of sophisticated arts and sciences. The conditions under which social interaction lead to these very different outcomes deserves further study.

### III Fable 2: Self-enhancing transmission bias

Fable 2 is a model of self-enhancing transmission bias (Han, Hirshleifer, and Walden 2019a). Suppose that investors adopt one of two strategies: *A* for Active and *P* for Passive. *A* refers to a strategy having either higher variance, or higher skewness. Investors of type *A* or *P* are randomly selected to meet in pairs over time. In each meeting, the Sender has a probability of reporting the Sender's strategy return to the Receiver. The Sender reports high return more often than low return.

Owing to self-enhancing transmission, there is an upward selection bias in the return reports seen by receivers. The effect of this is larger for the high-variance strategy. The high variance strategy generates a larger upper tail of high returns that are heavily reported and highly persuasive.

Receivers are subject to two standard behavioral biases: they fail to adjust for self-enhancing transmission, and they think that reported past performance is strongly indicative of future performance. In consequence, the high-variance *A* strategy tends to spread through population, even if its payoff distribution is inferior. This can explain some puzzles in trading, such as nondiversification; and pricing anomalies, such as the volatility and beta puzzles. There is also empirical support for this mechanism, as mentioned earlier.

Consider now a variation. Suppose that extreme returns are highly salient, so that receivers pay more attention to very high and very low returns. Suppose that a receiver is only converted to a strategy if the receiver attends to the sender's message. Then positive skewness strategies also tend to spread through the population—even after controlling for variance. Intuitively, positively skewed strategies generate the high return outcomes that are both heavily transmitted and attended to, and that are highly persuasive to receivers.

Fable 2 offers some important lessons. In the model, investors are *attracted* to high variance or skewness, but they do not have a *preference* for strategies based on variance or skewness. Nor do they have a *belief* that high variance or skewness is indicative of

a superior investing opportunity. Indeed, they may be *unaware* of the variances and skewnesses of the available strategies. The attraction to these strategy characteristics is socially emergent. As we show, the social nature of these effects has distinctive empirical implications about the effects of investor personality traits (e.g., degree of self-enhancing transmission bias), the position of an investor in the social network, and the effects of overall network connectivity.

## IV Fable 3: Visibility bias and overconsumption

Fable 3 is a model of visibility bias and overconsumption (Han, Hirshleifer, and Walden 2019b). *Visibility bias*, in our context, is the fact that engaging in a consumption activity is often more visible to others than refraining from doing so. For example, when the first Starbucks opened in a city in China, long lines of people stretching outdoors made it very visible to others that many were willing to spend a lot on coffee instead of drinking cheaply at home. Wearable electronics are also highly visible.

Suppose that observers do not adjust for the selection bias for seeing consumption rather than nonconsumption events. Such neglect of the features of the data-generating process is a standard behavioral bias. Then observers will tend to infer that others are consuming heavily.

Now let  $x$  denote people's need-for-saving, such as the probability of experiencing a personal wealth disaster. It is the same for everyone, but its value is uncertain. Different people have different information signals about  $x$ , so they can learn by observation of others.

Owing to visibility bias, people mistakenly "learn" from the apparent high consumption of others that there is little need to save. So in equilibrium, there is overconsumption, i.e., undersaving. Furthermore, there is a positive feedback effect, because people as targets of observation really are consuming heavily. So the total effect can be large.

This approach to understanding savings decisions has some nonobvious consequences. For example, information asymmetry about others' wealth *weakens* the overconsumption effect, consistent with some empirical findings. This is in sharp contrast to wealth signaling models (with "Veblen effects"), in which information asymmetry is the *source* of overconsumption. It also offers a novel explanation (for reasons I will not get into) for the large drop in the U.S. savings rate over a period of decades.

The moral of the story is that overconsumption is an emergent social outcome. There

is no direct bias for overconsumption. This contrasts with the standard behavioral economic theory that people find it tempting to consume immediately (present-biased preferences; (Laibson 1997)).

Owing to this difference, the social transmission bias approach has different empirical and policy implications from the behavioral economics approach. For example, in our model, overconsumption is driven by mistaken belief updating, so accurate disclosure about others' consumption can help, in sharp contrast with most behavioral approaches to undersaving. The social transmission bias approach also suggests that there can be benefits to targeting policy interventions to people with central positions in the social network. So for good policy, empirical testing is vital. As it turns out, there is empirical support for both mechanisms (see, e.g., D'Acunto, Rossi, and Weber (2019)).

## V **Fable 4:** **Biased information percolation, action booms, and price bubbles**

In the classic book, *The Psychology of Rumor*, Allport and Postman (1947) describe transmission chain experiments in which subjects sequentially recount stories to each other, much like the game of telephone or Chinese whispers. A key conclusion was that information is systematically distorted in a process of *leveling and sharpening*. Subjects emphasized the perceived essence of a story by selectively retaining details consistent with it, intensifying details, increasing magnitudes, and adding compatible new details and explanations. As a result, rumors can become inaccurate and extreme.

Like rumors, booms in social and economic activity and asset market bubbles often have a “beyond all reason” flavor, seemingly untethered from any sensible interpretation of public news arrival. Examples include sudden religious revivals; surges in vilification of political enemies, ethnic groups, and foreign countries; and price bubble episodes (as recounted in Shiller (2000a)).

In contrast, in existing models of information percolation, rational agents meet and sequentially exchange signals about the payoff of some action of common interest (e.g., Duffie, Malamud, and Manso (2009)), and decisions become efficient as signals accumulate. I consider here *biased information percolation*: distortions in information transmission. I explore whether biased information percolation can explain booms and collapses in aggregate behaviors and market prices.

I consider a setting in which each member of a continuum of agents is endowed with one private signal about the per unit payoff of an action of common interest (such as buying or selling an asset). The terminal payoff and signals are jointly Gaussian. At a sequence of Poisson arrival times, each agent is drawn in a meeting in which two agents share biased versions of the signals they have acquired up to that date. At each discrete date, public signals of growing informativeness arrive, which eventually correct the action boom or price bubble.

At each continuous date, agents myopically maximize the mean-variance expected utility of their terminal wealth. In the *Action Booms Model*, there is no fixed constraint on the aggregate action taken by agents. So as enthusiasm temporarily grows, agents jointly undertake more—an *action boom* (or negative boom). The Action Booms Model can apply to behaviors of individuals, such as political action or day trading, and to firm level behaviors, such as the adoption of organizational or governance structures, financing strategies, and projects or acquisitions.

In the *Price Bubbles Model*, the action is buying or selling an asset, so the aggregate action, demand for the asset, is constrained to be equal to supply. Supply is an exogenous random variable, as in rational expectations models of securities markets. Investors partially infer the private information of others from market price. Biased information percolation results in asset mispricing.

I introduce two types of *social transmission biases* to the information percolation setting. The first is *misreporting*—an upward or downward bias in senders' reports about their signal by some fixed amount  $b$ . I call this  $b$  bias. Each time an agent acquires a biased signal from others, an additional  $b$  is added to it, so  $b$  bias compounds recursively. Receivers interpret reports naively—without adjusting for  $b$  bias. The level of  $b$  is a random variable, the same for all meetings, and can be positive or negative, with  $E[b] > 0$ .

The second transmission bias is *meeting rate misperception*. Agents under- or over-estimate the intensity of the Poisson process that selects agents into meetings by some fixed factor  $\kappa$ , where  $\kappa = 1$  indicates no bias. I call this  $\kappa$  bias.

For  $b$  bias, I view  $b > 0$  as typical. There is evidence that consumers of media content are more likely to share positive than negative content (Berger and Milkman 2012). As discussed in Berger (2014), one possible reason for doing so is to maintain a reputation for providing useful information. As argued in marketing literature, when there is a wide range of product choices, it is often more helpful to hear about what to buy rather than what not to buy. Furthermore, being positive is often seen as an attractive personality

trait, which can be a motive for positive reporting (Tesser and Rosen 1975). Both motives may have shaped the psychology of conscious or unconscious bias in information sharing.

Alternatively,  $b$  bias can derive entirely from the message receiver. For example, receivers may be more attentive to positive than negative signals about consumer products if positive signals tend to be more useful. A related argument about net stock buying in response to news, given short-sale constraints, is made by Barber and Odean (2008). Also, if agents predominantly take long positions, then motivated reasoning (“wishful thinking”; see, e.g., the model of Bénabou and Tirole (2002)) can motivate an optimistic bias in the interpretation of messages,  $b > 0$ .

On the other hand, psychologists have documented *negativity bias* (Rozin and Royzman (2001), Baumeister et al. (2001))—a general tendency to pay more attention to negative information and to interpret information negatively. An evolutionary justification for negativity bias is that missing a threat just once can be fatal, whereas the cost of missing an opportunity is only incremental.

Receiver naivete about  $b$  bias is consistent with evidence that people often take information at face value, instead of adjusting for features of the data-generating process. There is extensive experimental and field evidence of this from several fields.

Limited attention also provides a motivation for  $\kappa$  bias, where I regard downward bias,  $\kappa < 1$ , as typical. An agent does not directly observe most of the meetings of other agents. There is much evidence of “out-of-sight, out-of-mind” salience effects, wherein people underweight information that can be inferred rather than directly observed (see, e.g., Payne, Bettman, and Johnson (1993)). The  $\kappa$  bias is also in the spirit of models of persuasion bias and of naive herding.<sup>8</sup>

In the Action Booms Model,  $\kappa$  bias does not come into play. Owing to  $b$  bias, when  $b > 0$ , on average the aggregate action follows a path that exceeds the rational benchmark (the case of  $b = 0$ ). I define the *excess action* as the percentage difference between the expected aggregate action and the expected aggregate action under the rational benchmark. At first, the smoothed path of the excess action increases slowly, and then growth accelerates. Eventually, more and more informative public information arrives, on average driving the excess action toward zero. Similarly, in the price bubbles model, the expected price path conditional on  $b > 0$ , appropriately smoothed, rises

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<sup>8</sup>See DeMarzo, Vayanos and Zwiebel (2001, 2003) and Eyster and Rabin (2010). My model shares with these the feature that agents do not properly account for how their information sources acquire signals from others. In these models, agents double-count signals that are received via multiple pathways in a social network. A difference here is that in the percolation setting, no signal is ever received from some other agent via multiple pathways.

above and then falls toward fundamental value.

So under appropriate parameter values, the smoothed excess action path and expected price path look qualitatively similar, with four phases as illustrated for the Price Bubbles model in Figure 2. For positive bubbles ( $b > 0$ ), these phases are:

- **The inception phase.** Expected price or excess action grows at an accelerating rate (convexity).
- **The boom phase.** Expected price or excess action grows at a decreasing rate (concavity), and reaches a peak.
- **The correction phase.** Starting from its peak, expected price or excess action declines at an increasing rate (concavity).
- **The terminal phase.** Expected price or excess action declines at a decreasing rate (convexity), asymptoting toward its fundamental value.

For  $b < 0$ , these phases are inverted to create a U-shape instead of a hump-shape.

The accelerating growth during the inception phase results from complementarity between rise in the per capita number of signals that agents possess, and the increase in how biased those signals are. As in previous models of information percolation, the per capita number of signals rises exponentially through time. Furthermore, I show that the average bias per signal possessed by agents increases linearly with time. So at first, price or action, on average, rises more and more quickly relative to a zero-bias benchmark. Eventually, public information arrival corrects the overoptimism. Since these boom or price bubble effects are socially driven, a cross-sectional empirical implication is that more social agents take more extreme actions.

Zooming in on the bubble trajectory without smoothing reveals oscillatory dynamics. In the  $b > 0$  case, on each of the dates of public information releases, overoptimism is on average partially corrected, inducing discrete drops in the excess action. The reverse applies when  $b < 0$ . During the inception phase of the boom, these drops are quickly reversed and followed by further steepening. As the boom peaks, under appropriate parameter values there is a period during which the amplitude of the oscillations gets large, as the downward pressure on public news dates contends with the upward pressure of biased percolation between these dates. This *peak oscillation* is a distinctive testable property of the model. Eventually, public resolution of uncertainty starts to overwhelm biased percolation, so that on average the bubble enters the terminal phase and oscillations are dampened.

Furthermore, dispersion across investors in their accumulated  $b$  biases promotes disagreement. This results in large cross-sectional differences in behavior, which, in the price bubbles model, takes the form of heavy speculative trading. So irrational trading also tends to rise and fall with the bubble. Furthermore, if  $\kappa < 1$ ,  $\kappa$  bias causes agents to perceive market price to be less informative than it really is. This also encourages irrational trading, even though there is no direct bias, such as overconfidence, “for” trading aggressively.

As in the Action Booms Model, when  $b > 0$ , bias is amplified recursively. This results in a convex growth in overvaluation that is only overcome when some very strong opposing force finally starts to operate. So the model is consistent with “beyond all reason” flavor of many bubbles episodes, such as the stupefying swings in Bitcoin prices. The implication of an accelerating initial rate of growth of bubbles is consistent with the evidence of Greenwood, Shleifer, and You (2019) that the price run-up in bubbles tends to have an explosive, i.e., convex, shape before correcting.

On average, when  $b > 0$ , there is a run-up over time during the inception and boom phases, and on average the price drops discretely at each discrete news arrival date; the reverse applies when  $b < 0$ . So as in the Action Booms model, there is peak oscillation; market valuations tend to totter before collapse.<sup>9</sup> During the correction and terminal phases of the bubble, the wiggling dampens as mispricing converges toward zero.

The Price Bubbles Model also implies event-based return predictability. One reason for this is standard: that firms may systematically undertake certain actions in response to mispricing, such as a new issue when  $b > 0$  and a repurchase when  $b < 0$ . Furthermore, even for news events that do not depend on  $b$ , under appropriate parameter conditions there is on average post-event return continuation. Intuitively, public news events only partially correct misvaluation. So variation in  $b$  generates similar variation in both corrective event-date reactions and corrective post-event returns.

The smoothed expected price path, conditional on  $b$ , is similar to the hump-shaped or U-shaped impulse response functions that generate momentum and reversal in some behavioral models (e.g., Daniel, Hirshleifer, and Subrahmanyam (1998)). The randomness of  $b$  promotes positive comovement in returns that are separated by short lags, time, and negative comovement in returns that are on opposite sides of the hump-shape or U-shape. Furthermore, the oscillatory dynamics of the model—the high frequency zigzags in price—suggest that under appropriate parameter values, serial correlations may be negative at short lags.

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<sup>9</sup>Barberis et al. (2018) offer a model of wavering of investor beliefs at the peak of bubbles.

## A The basic setting

I first lay out the assumptions common to the Action Booms and Asset Market models. Proofs are in Appendix A.

### Agents, Actions, and Payoffs

At each continuous date  $s \in [0, T)$ , each agent from a continuum  $i \in [0, 1]$  undertakes some action with intensity  $\theta_s^i$ . The common payoff per unit of the action is

$$X \sim \mathcal{N}\left(\bar{X}, \frac{1}{\tau^X}\right),$$

where  $\tau^X$  is the precision of  $X$ , defined as the reciprocal of the variance. As seen in the timeline in Figure 3, the payoff is realized and received at finite date  $T$ . In the context of security trading,  $X$  is the terminal dividend of the risky asset.

### Signals

At date 0, each agent  $i$  directly obtains one private signal about  $X$ , of the form

$$S^i = X + \epsilon^i, \quad \epsilon^i \sim \mathcal{N}\left(0, \frac{1}{\tau^S}\right),$$

where  $\tau^S$  is the precision of the private signal.

Agents also receive public signals, denoted with a  $P$  superscript, at discrete dates  $t = 0, 1, 2, \dots, T - 1$ . These signals satisfy

$$S_t^P = X + \epsilon_t^P, \quad \epsilon_t^P \sim \mathcal{N}\left(0, \frac{1}{\tau_t^P}\right),$$

where  $\tau_t^P$  is the precision of the date- $t$  public signal, and by convention unsubscripted  $\tau^P \equiv \tau_0^P$ .

The new public signals arriving at each discrete date  $t < T$  are assumed to have growing precision in such a way that the total precision of the history of public signals grows exponentially at rate  $\zeta > 0$ ,

$$\tau_s^Z = e^{\zeta \lfloor s \rfloor} \tau^P \quad 0 \leq s < T, \tag{1}$$

where the floor function indicates the greatest integer less than or equal to  $s$ . This is much like assuming that newly arriving public signals have exponentially growing precision.<sup>10</sup> An alternative to this assumption is to assume that each agent is also

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<sup>10</sup>There is full resolution of uncertainty (infinite precision) at terminal date  $T$ , so exponentially growing precision partially smooths the rate of resolution of uncertainty over time.

endowed with an unbiased but low-precision signal, and that the percolation process for unbiased signals has a greater meeting rate than for biased signals. In such a setting, unbiased signals would have little effect at first, but as in Aesop’s fable of the tortoise and the hare, they would eventually outrace biased percolation.

### Meetings and biased signal transmission

Agents are selected randomly to meet in pairs and share private signals through conversation in continuous time. In each meeting, agents exchange biased summaries of their accumulated signals. We can view signals as transmitted either by physical meetings, or in a process in which bloggers randomly see each other’s posts at different moments in time.<sup>11</sup>

### Objective and constraints

At each date  $s$ , agents choose how much of the action,  $\theta_s^i$ , to undertake. The cost per unit of the strategy is  $v_s$  units of date- $T$  consumption. In the Action Booms Model,  $v_s = v$ , an exogenous constant, where  $\bar{X} > v > 0$ .

At each date  $0 \leq s < t$  in continuous time, agent  $i$  myopically maximizes an exponential utility function,

$$\begin{aligned} \max_{\theta_s^i} \mathbb{E} \left[ -e^{-\gamma W_T^i} | \Phi_s^i \right] \quad \text{subject to} \\ W_T^i = W_s^i + \theta_s^i (X - v_s), \end{aligned} \tag{2}$$

as if this were the last action the agent would ever take.<sup>12</sup> Here  $\gamma$  is absolute risk aversion, and  $W_s^i$  is the agent’s wealth. This is defined as either the date-0 endowed claims upon date- $T$  consumption, or, in a market setting, the total market value at date  $s$  of the agent’s safe and risky claims denominated in date- $T$  consumption.<sup>13</sup> The investor’s information set  $\Phi_s^i$  includes the private and public signals the agent has received. As is standard under normality and exponential utility, there is a mean-variance conditional expected utility function. So agent  $i$ ’s optimal action is

$$\theta_s^i = \frac{\mathbb{E} [X | \Phi_s^i] - v_s}{\gamma \mathbb{V}(X | \Phi_s^i)}, \tag{3}$$

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<sup>11</sup>In a ‘blogger’ setting, each “meeting” would have only one-way information transmission, but that distinction is not important for the qualitative nature of the results.

<sup>12</sup>Myopic optimization is algebraically convenient, and is perhaps more realistic than the opposite extreme of rational dynamic optimization. Also, since rational dynamic models generate closed-form solutions with convenient properties, similar conclusions to those I derive may apply with forward-looking agents.

<sup>13</sup>Owing to the CARA (exponential) utility assumption, the value of  $W_s^i$  is irrelevant for the optimal action level, so I omit the details of its stochastic dynamics in the price bubbles model.

where  $\mathbb{V}$  denotes variance.

## B Meeting regimes, signal accumulation and average beliefs

### The meeting regimes and transmission bias

When two agents meet, they exchange messages about their accumulated private signals. With a continuum of agents and Gaussian learning, as in past percolation models, the cross-population distribution of the number of signals possessed by different agents evolves deterministically over time. As the per-capita average number of signals increases, agents can become better and better informed.

Unfortunately for the agents, in my model these signals are biased, and all agents think that the messages they receive are unbiased. Specifically, the sender of a report adds a bias of size  $b$  to the mean of the sender's signal batch each meeting. This would occur, for example, with one bias added to each of the signals. This can be called *full-batch transmission bias*. Since meetings occur repeatedly, there is iterative amplification of bias. This transmission assumption reflects the fact that sometimes there is surprising degree of credulity toward favorable reports as excitement grows. The bias  $b$  is a random variable with  $\mathbb{E}[b] > 0$  that is realized at date 0, where  $b$  is independent of all other exogenous variables.

For tractability, I further assume that at any date, only pairs of agents with the same numbers of signals are drawn to meet. This assumption is relaxed in a version of the model analyzed in the Online Appendix. A tendency to interact with similar others is called homophily.<sup>14</sup> Here, homophily by signal count causes the number of signals possessed by each agent to double with each meeting.

After percolation has proceeded for some time, so that the per capita signal count is large, adding a bias to the agent's entire batch of signals is, in a sense, a much stronger distortion than biasing a few signals. This raises the question of how robust the conclusions are to milder assumptions. I analyze an alternative possibility in the Online Appendix, that at every meeting the sender of a batch of signals adds an additional bias of  $b$  to just *one* of the signals in this batch—*single-signal transmission bias*. This severely limits recursive amplification of bias. Almost all of the qualitative conclusions derived here apply in this setting as well (with no need for the homophily assumption),

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<sup>14</sup>Cujean (2020) also models homophily in information percolation and securities trading.

except that booms and busts are no longer of potentially unlimited magnitude.<sup>15</sup>

### The information in private and public signals

Let  $n_s^i$  be the running total number of private signals possessed by agent  $i$  at date  $s$  about the value of the action,  $X$ ,  $0 \leq s < T$ , where  $n_0^i = 1$  for all  $i$ . Let  $S^{ij}$  denote the value of the  $j^{\text{th}}$  signal about  $X$  obtained by agent  $i$  through date  $s$ , exclusive of any biases that are added to signals during the sharing process,  $j = 1, \dots, n_s^i$ . Let  $S^{bij}$  be the signal inclusive of biases.

Let  $\Delta n_s^i$  be the number of signals agent  $i$  obtains about  $X$  conditional upon  $i$  having a meeting at date  $s$ . For  $s \in (0, T)$ , the average signal value, inclusive of biases, obtained by receiver  $i$  about  $X$  in a meeting at date  $s$ , if such a meeting occurs, suppressing  $s$  subscripts, is

$$Y^i \equiv \frac{1}{\Delta n^i} \sum_{k=1}^{\Delta n^i} S^{b,i,k+n^i}, \quad (4)$$

where the “ $-$ ” subscript denotes the number of signals possessed the instant (in a limiting sense) before the meeting at date  $s$ . Let the debiased mean of the signal batch be denoted  $Y^i(b=0)$ . With Gaussian distributions, a sufficient statistic for the information in a batch of signals acquired in a meeting is the total number of signals and their debiased mean,  $(\Delta n^i, Y^i(b=0))$ .

I denote the average of the private signals accumulated by agent  $i$  through date  $s$ , inclusive of biases, as

$$\bar{Y}_s^i = \frac{\sum_{j=1}^{n_s^i} S^{bij}}{n_s^i}. \quad (5)$$

The precision-weighted average of the public signals through discrete date  $t$  is

$$\bar{Z}_t \equiv \frac{\sum_{r=0}^t \tau_r^P S_r^P}{\sum_{r=0}^t \tau_r^P}. \quad (6)$$

For continuous date  $s$ , equation (6) holds with  $t$  replaced by  $s$  on the LHS, and by  $\lfloor s \rfloor$  on the RHS.

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<sup>15</sup>A technical advantage of either the homophily assumption or of single-signal transmission bias is that the number of biases an agent has is a single-valued function of the number of signals. This reduces the dimensionality of the evolution of signals and biases. With regard to recursive amplification of bias, an interesting further direction would be to model an intermediate case in which the compounding of  $b$ -bias per signal in the population is concave, but is not bounded above at one bias per signal. This would still allow for “beyond all reason” bubbles.

## C Population dynamics of signal and bias counts

Owing to the continuum of agents assumption, as in previous percolation models, the probability is zero that a given agent's meeting lineage ever overlaps with that of another given agent. This allows application of a law of large numbers to matching outcomes (see Duffie and Sun (2007)).

Let  $\mu_s(n)$  and  $\mu_s^R(n)$  respectively be the perceived and actual cross-sectional distribution of agents at date  $s$  who possess  $n$  private signals about payoff, where the  $R$  superscript denotes a rational perception. The Poisson arrival rate for meetings is  $\eta > 0$ , but agents perceive this to be  $\kappa\eta$ , where  $\kappa > 0$ , with an emphasis on the case  $\kappa \leq 1$ . Owing to the perfect homophily assumption, the support of  $\mu_s(n)$  is  $\{n : n = 2^k, k \in \mathbb{N}^0\}$ .

Let the perceived and actual per capita number of signals in the population at date  $s$  be

$$\begin{aligned}\phi_s &\equiv \sum_{n=1}^{\infty} n\mu_s(n) \\ \phi_s^R &\equiv \sum_{n=1}^{\infty} n\mu_s^R(n),\end{aligned}\tag{7}$$

where the  $R$  superscript (for ‘‘Rational’’) highlights that an expression is based upon a rational perception, and where  $\mu_s$  and  $\mu_s^R$  are the perceived and actual distributions of agent signal counts in the population.

To compare with the benchmark of unbiased updating, we count the number of the biases per signal as batches get transmitted and bias accumulates. We call a signal  $r$ -biased if its value is  $S^k + rb = X + \epsilon^k + rb$ , where  $r$  is a positive integer, and where agent  $k$  is the *original* holder of signal with value  $S^k$ .

Let  $n_s^{bi}$  be the number of  $b$  biases in an agent's signal set at date  $s$ . For example, if the agent in total has one 1-biased signal and one 2-biased signal, then  $n_s^{bi} = 1+2 = 3$ . Agent  $i$ 's number of biases per signal (the per-signal bias count) is defined as  $f_s^{bi} \equiv n_s^{bi}/n_s^i$ . Let the per-signal bias be the product  $bf_s^{bi}$ .

Let  $\nu_s(n^b)$  be the frequency across the investor population of the number of biases  $n^b$  possessed by investors. Let  $f_s^b$  be the per-signal bias count in the whole economy, defined as the ratio of per capita number of signal biases to the per capita number of signals,

$$f_s^b \equiv \frac{\sum_{n=0}^{\infty} n^b \nu_s^R(n^b)}{\phi_s^R}.\tag{8}$$

Let  $N_s$  be the number of meetings an agent has experienced by date  $s$ . Agents perceive that this has a Poisson distribution,  $N_s \sim \text{Pois}(\eta\kappa s)$ , where in fact,  $\kappa = 1$ . The number of signals and the per-signal bias of an agent turns out to be an increasing function of the number of meetings that the agent has had. Using this fact, we can also derive the population evolution of signals and biases as a function of *time*.

**Proposition 1.** *At date  $s$ :*

1. *Agents perceive the per capita number of signals in the population to be  $\phi_s = e^{\eta\kappa s}$ . The actual per capita number is  $e^{\eta s}$ .*
2. *The aggregate per-signal bias count is  $f_s^b = \eta s$ .*
3. *The perceived cross-sectional distribution for the number of signals possessed by agents in the population is*

$$\mu_s(n) = \frac{(\eta\kappa s)^{\log_2 n} e^{-\eta\kappa s}}{(\log_2 n)!}, \quad (9)$$

*$n \in \{2^N\}_{N=0,1,\dots,\infty}$ , where  $N$  is the number of meetings experienced by an agent with  $n$  signals. The actual distribution has  $\kappa = 1$ .*

From Proposition 1, we see that per-signal bias count increases linearly with time, and is therefore unlimited. With recursive amplification of bias, there is explosive growth in the total number of biases in the population that is only partly offset by the exponential growth in the total number of signals.

## D The Action Booms Model and $b$ -bias

In the Action Booms Model, for simplicity there is no learning from observing aggregate actions or their consequences. In many applications, such as acquisitions, there *is* such observation, as reflected in models in which agents learn from an aggregate summary statistic of past actions (e.g., Bikhchandani, Hirshleifer, and Welch (1992)), and in rational expectations models in which investors learn from securities prices. The basic insights I wish to provide about biases in learning from aggregate consequences ( $\kappa$  bias effects) are captured in the Price Bubbles Model of Section E. However, in some practical contexts aggregate consequences have low visibility or are observed very noisily, limiting such learning.

## D.1 Belief updating with learning from private and public signals

Under Gaussian updating, for a rational agent with  $n$  private signals and a set of public signals with total precision  $\tau^Z$ , the conditional precision of  $X$  is  $\tau^X + n\tau^S + \tau^Z$ . The conditional expectation of  $X$  given private signals with average value  $\bar{Y}$  and public signals with average value  $\bar{Z}$  places weights in proportion to the relative precisions of these signals. These precisions are identical for a biased agent and for a rational observer. So the signal weights, written as functions of dummy inputs, are

$$\begin{aligned}\lambda^X(n, \tau^S, \tau^X, \tau^Z) &\equiv \frac{\tau^X}{\tau^X + n\tau^S + \tau^Z} \\ \lambda^P(n, \tau^S, \tau^X, \tau^Z) &\equiv \frac{\tau^Z}{\tau^X + n\tau^S + \tau^Z} \\ \lambda^S(n, \tau^S, \tau^X, \tau^Z) &\equiv \frac{n\tau^S}{\tau^X + n\tau^S + \tau^Z}.\end{aligned}\tag{10}$$

Everyone correctly understand that these are the weights that everyone uses to form their updates. When applied to a biased or unbiased average of signals, these weights generates a biased or rational expectation of the fundamental. If  $\bar{Y}$  is the agent's average private signal inclusive of biases. Since the agent believes these biases are zero, the agent believes the expected fundamental to be

$$\mathbb{E}[X|\bar{Y}, n; \bar{Z}, \tau^Z] \stackrel{\mathcal{B}}{=} \left(\frac{\tau^X}{\tau^X + n\tau^S + \tau^Z}\right)\bar{X} + \left(\frac{n\tau^S}{\tau^X + n\tau^S + \tau^Z}\right)\bar{Y} + \left(\frac{\tau^Z}{\tau^X + n\tau^S + \tau^Z}\right)\bar{Z},\tag{11}$$

where  $\mathcal{B}$  (for “bias”) denotes an equality that is incorrectly *perceived* by an agent to hold.

## D.2 Optimal action and aggregate action

The average signal error of an agent  $i$  with  $n$  private signals is

$$\bar{\epsilon}^i \equiv \frac{1}{n} \sum_{j=1}^n \epsilon^{ij}.\tag{12}$$

By optimal action condition (3), and suppressing  $s$  subscripts, agents *perceive* the condition for an optimum to be

$$\begin{aligned}
\gamma (\tau^X + n\tau^S + \tau^Z)^{-1} \theta^i &= \mathbb{E} [X|\Phi^i] - v \\
&\stackrel{\mathcal{B}}{=} \lambda^X \bar{X} + \lambda^S \bar{Y}^i + \lambda^P \bar{Z} - v \\
&\stackrel{\mathcal{B}}{=} \left( \frac{\tau^X}{\tau^X + n\tau^S + \tau^Z} \right) \bar{X} + \left( \frac{n\tau^S}{\tau^X + n\tau^S + \tau^Z} \right) \underbrace{(X + \bar{\epsilon}^i)}_{\text{perceived unbiased}} \\
&\quad + \left( \frac{\tau^Z}{\tau^X + n\tau^S + \tau^Z} \right) \bar{Z} - v. \tag{13}
\end{aligned}$$

Since the signals agents obtain are biased, their optimized positions *actually* satisfy

$$\begin{aligned}
\gamma (\tau^X + n\tau^S + \tau^Z)^{-1} \theta^i &= \left( \frac{\tau^X}{\tau^X + n\tau^S + \tau^Z} \right) \bar{X} + \left( \frac{n\tau^S}{\tau^X + n\tau^S + \tau^Z} \right) \underbrace{(X + \bar{\epsilon}^i + bf^{bi})}_{\text{biased}} \\
&\quad + \left( \frac{\tau^Z}{\tau^X + n\tau^S + \tau^Z} \right) \bar{Z} - v. \tag{14}
\end{aligned}$$

Solving for agent  $i$ 's actual position gives

$$\theta^i = \frac{\tau^X(\bar{X} - v) + n\tau^S(X + \bar{\epsilon}^i + bf^{bi} - v) + \tau^Z(\bar{Z} - v)}{\gamma}. \tag{15}$$

So  $b$  bias boosts optimism and action (when  $b > 0$ ) as reflected in per-signal bias  $bf^{bi}$ , which grows recursively as ever-more biased signals are exchanged.

Aggregating across agents in the population gives the true aggregate action.

**Proposition 2.** *In the Action Booms Model, the aggregate action  $\Theta_s$  satisfies*

$$\Theta_s = \frac{\tau^X(\bar{X} - v) + \phi_s^R \tau^S (X + bf_s^b - v) + \tau_s^Z (\bar{Z}_s - v)}{\gamma}. \tag{16}$$

Taking the expectation gives the expected aggregate action path conditional on  $b$ ,

$$\mathbb{E}^R[\Theta_s|b] = \frac{(\tau^X + \tau_s^Z + \phi_s^R \tau^S)(\bar{X} - v) + \phi_s^R \tau^S bf_s^b}{\gamma}, \tag{17}$$

where  $\bar{X} - v > 0$ . This depends on the actual per capita signal count  $\phi_s^R$ , not on the perceived count  $\phi_s$  as biased by  $\kappa$ . Since there is no observation of, nor learning from, aggregate consequences in the Action Booms model, misperception of the meeting rate has no effect on beliefs and outcomes.

When  $b > 0$ , social interaction drives overoptimism, and when  $b < 0$ , overpessimism. Since these effects derive from social interaction, a testable implication is that more social agents are especially prone to bubble exuberance.

**Proposition 3.** *In the Action Booms Model, consider a deviant agent  $i$  who is more sociable than others, i.e., has a meeting rate  $\eta^i > \eta$ . Then if  $b > 0$ , agent  $i$ 's action is on average above the per capita action,  $\mathbb{E}^R[\theta_s^i|b] > \mathbb{E}^R[\Theta_s|b]$ . The reverse is the case if  $b < 0$ .*

Since action can positively or negatively boom even under full rationality, I focus on the percentage deviation of the expected aggregate action from the rational benchmark. This benchmark is the expected aggregate action if there were no transmission bias,  $b = 0$  in (17). So the *excess action* is

$$\overline{\Delta\Theta}_s(b) \equiv \frac{\mathbb{E}^R[\Theta_s|b] - \mathbb{E}^R[\Theta_s|b=0]}{\mathbb{E}^R[\Theta_s|b=0]} = \frac{\phi_s^R \tau^S b f_s^b}{(\tau^X + \phi_s^R \tau^S + \tau_s^Z)(\bar{X} - v)}. \quad (18)$$

To describe this action path, I substitute the percolation dynamics for the numbers of signals and the per-signal bias count into (16)-(18). Substituting for  $\phi_s^R$  and  $f_s^b$  from Proposition 1 and for  $\tau_s^Z$  from (1), gives

$$\overline{\Delta\Theta}_s(b) = \frac{e^{\eta s} \tau^S b \eta s}{(\tau^X + e^{\eta s} \tau^S + e^{\zeta \lfloor s \rfloor} \tau^P)(\bar{X} - v)}. \quad (19)$$

To build intuition about the excess action path, suppose that public news arrives continuously. Then relative to (19), the jumps in precision are smoothed away, with public precision path  $\tau_s^Z = e^{\zeta \lfloor s \rfloor} \tau^P$  replaced by

$$\hat{\tau}_s^Z \equiv e^{\zeta(s-0.5)} \tau^P. \quad (20)$$

A steady contributor to action booms here is the linear increase in per-signal bias as a function of time (Proposition 1). So focusing on the  $b > 0$  case, the average bias in expectations can become arbitrarily large until opposed by some countervailing force. Here that force is an initially-slow but accelerating rise in total public signal precision. Growth in per-signal-bias affects beliefs and actions slowly at first, because with only a few signals, updating is weak relative to the prior. Updating then accelerates, owing to exponential growth in the per capita number of signals,  $\phi_s$ . Together with rising per-signal bias, there can, for a time, be explosive growth in overoptimism and action.

If public signal precision takes long enough to have substantial force, action boom expectations about the fundamental come close to asymptoting to a positively-sloped straight line. This line reflects the linear increase in per-signal bias. With enough signals, the prior is swamped; expectations are dominated by the average of the biased private

signals. Even a large increase in the number of signals no longer causes much further updating per se—further updating comes mainly from the rising per-signal bias.

Eventually, public signals, as reflected in  $\bar{Z}$ , force correction, owing to the assumed exponential growth rate (with high enough exponent) in public signal precision  $\tau^Z$ . So there is a boom and bust in the excess expected aggregate action  $\overline{\Delta\Theta}_s$ .

In the benchmark for comparison, as  $s$  grows, there would be a *rational* positive (if  $X > v$ ) or negative (if  $X < v$ ) boom even when  $b = 0$ . The  $b$  bias induces large temporary swings away from this rational path. So in summary, when  $b > 0$  the trajectory of the excess action is hump-shaped, and when  $b < 0$  is  $U$ -shaped.

What about the unsmoothed excess action path? Since public info actually arrived discretely, the excess action path oscillates. Intuitively, when  $b > 0$ , at each date  $t$  on the lattice  $t = 1, 2, \dots, T - 1$ , the public signal on average partly corrects overoptimism, so that the aggregate excess expected action drops discretely. Percolation then on average causes the aggregate action to recover until the next lattice date arrives.

A figure describing the excess action in the Action Booms Model is provided in the Online Appendix. As in Figure 2 for the Price Bubbles Model, the amplitude of oscillations is very strong near the peak, as the boom loses impetus. Intuitively, for a boom to be growing quickly, the on-average drops in excess action coming from public news arrival dates need to be small relative to the percolation-driven increases. For a boom to reach its peak, the drops deriving from public news arrival must be growing to be comparable in size to the biased-percolation-driven increases, so the zigzags are large. As in Figure 2, vigorous oscillation can persist until the accumulated precision of public information starts to overwhelm biased percolation. As this occurs, the percolation-driven recoveries are dampened. This stabilizes the excess action path closer and closer to 0 in the terminal phase of the action boom. I call this pattern *peak oscillation*. (It is equally “trough oscillation,” since it occurs in mirror reflection near the trough of a negative boom,  $b < 0$ ). It will be valuable to test empirically the prediction that action booms are characterized by peak oscillation.

## E The Price Bubbles Model and $\kappa$ bias

Suppose next that there is a centralized asset market with supply noise, and that investors draw biased inferences from market price. If investors underestimate (for example) the extent to which other agents meet and share private information— $\kappa$  bias—they will underestimate the informativeness of the market price, which encourages excessive

trading. There is also an interplay between  $b$  bias, trading and mispricing.

### E.1 Belief updating with mislearning from price

Let aggregate supply of a risky asset be distributed normally,

$$\Theta \sim \mathcal{N}\left(\bar{\Theta}, \frac{1}{\tau^\Theta}\right)$$

with precision  $\tau^\Theta$ . Unlike the Action Booms model,  $\Theta$  is time-independent. Without loss of generality, for algebraic convenience I now set  $\bar{X} = 0$ .

Suppose that at each date agents forget the past history of prices, and form beliefs solely based upon the current price (together with the full history of private and other public signals).<sup>16</sup> Together with myopic optimization, this implies that at each date we can solve for a two-date trading equilibrium. It is standard in rational expectations models to conjecture and verify an equilibrium price function that is linear in the fundamental and the supply noise. In my setting price also depends on the average public signal and on the per-signal bias,  $bf_s^b$ :

$$v_s = \alpha_s^{RX} X + \alpha_s^{RP} \bar{Z}_s + \alpha_s^{R\Theta} \Theta + \alpha_s^{Rb} bf_s^b. \quad (21)$$

Here  $R$  indicates that this is the pricing function from the viewpoint of a rational observer. Since agents in the model are unaware of  $b$  bias, their *perceived* pricing function is

$$v_s \stackrel{\mathcal{B}}{=} \alpha_s^X X + \alpha_s^P \bar{Z}_s + \alpha_s^\Theta \Theta, \quad (22)$$

where the values of these alphas differ from the values of the corresponding rational alphas in (21).

I define the normalized price signal as

$$\xi_s \equiv \frac{v_s}{\alpha_s^X} - \left(\frac{\alpha_s^P}{\alpha_s^X}\right) \bar{Z}_s \stackrel{\mathcal{B}}{=} X + \left(\frac{\alpha_s^\Theta}{\alpha_s^X}\right) \Theta, \quad (23)$$

where the second equality in (23) is *perceived* by investors to hold based upon the incorrect pricing coefficients of (22). Conditional on  $X$ , agents mistakenly perceive that

$$\xi_s \stackrel{\mathcal{B}}{\sim} \mathcal{N}\left(X, \left(\frac{\alpha_s^\Theta}{\alpha_s^X}\right)^2 \frac{1}{\tau^\Theta}\right), \quad (24)$$

---

<sup>16</sup>Though unrealistic, the forgetting assumption suffices to illustrate some basic concepts simply.

so agents view  $\xi$  as another signal of the form “ $X$  plus independent noise.” By normality, the perceived variance of the fundamental,  $X$ , conditional on observing  $n$  private signals with precision  $\tau^S$ , public signals with total precision  $\tau^Z$ , and the normalized price signal with precision  $\tau^\xi$  is

$$o(n, \tau^S, \tau^X, \tau^Z, \tau^\xi) \equiv (\tau^X + n\tau^S + \tau^Z + \tau^\xi)^{-1}, \quad (25)$$

where the actual precision of the normalized price,  $\tau^{R\xi} \neq \tau^\xi$ .

By normality and the equilibrium price conjecture, an investor who updates based upon  $n$  private signals about  $X$  with average value  $\bar{Y}$ , public signals with average value  $\bar{Z}$ , and one normalized price signal  $\xi_s$  weights these signals by their perceived relative precisions. These precisions are assessed without bias, except for that of the price signal. I omit  $s$  subscripts and view these weights as functions of dummy inputs:

$$\begin{aligned} \lambda^P(n, \tau^S, \tau^X, \tau^Z) &\equiv \frac{\tau^Z}{\tau^X + n\tau^S + \tau^Z + \left(\frac{\alpha^X}{\alpha^\Theta}\right)^2 \tau^\Theta} \\ \lambda^S(n, \tau^S, \tau^X, \tau^Z) &\equiv \frac{n\tau^S}{\tau^X + n\tau^S + \tau^Z + \left(\frac{\alpha^X}{\alpha^\Theta}\right)^2 \tau^\Theta} \\ \lambda^\xi(n, \tau^S, \tau^X, \tau^Z) &\equiv \frac{\left(\frac{\alpha^X}{\alpha^\Theta}\right)^2 \tau^\Theta}{\tau^X + n\tau^S + \tau^Z + \left(\frac{\alpha^X}{\alpha^\Theta}\right)^2 \tau^\Theta}. \end{aligned} \quad (26)$$

These coefficients depend on time through their dependence on  $n$  and  $\tau^Z$ .

The above weights give an agent’s biased conditional expectation of the fundamental. Agents mistakenly assess the conditional expectation of  $X$  to be

$$\mathbb{E}[X|\bar{Y}, n; \bar{Z}, \tau^Z; v] \stackrel{B}{=} \lambda^S(\cdot)\bar{Y} + \lambda^P(\cdot)\bar{Z} + \lambda^\xi(\cdot)\xi. \quad (27)$$

Everyone correctly understand the weights that everyone uses to update their beliefs given signals and price. Substituting agents’ equilibrium price conjecture (22) into (26) and (27) gives their perception of how beliefs depend upon fundamentals  $X$ , supply shock  $\Theta$  and signal realizations.

## E.2 Optimal positions and market clearing

To pin down the equilibrium pricing parameters, I solve for optimal positions and impose market clearing. I first use these conditions to derive the perceived pricing function, which is only perceived by agents to be self-confirming. I then derive the actual pricing function, which is actually self-confirming given agents’ misperceptions.

### E.3 Agents' perceived pricing function

Since investors think they are in a rational expectations setting, we can solve for their perceived self-confirming expectations in a standard way. Investors' expectations can be substituted into the investor optimality condition (3), together with the perceived conditional precision of the fundamental from (25). Integrating over investors (based on their perceptions of the distribution across the investor population of number of private signals), the market clearing condition for the asset pins down the coefficients of the perceived pricing function.

**Proposition 4.** *In the Price Bubbles Model, the coefficients on the fundamental, the supply noise, and the public signal in the agents' **perceived** equilibrium pricing function (22) are:*

$$\alpha_s^X = \frac{\phi_s \tau^S (\gamma^2 + \phi_s \tau^S \tau^\Theta)}{(\phi_s \tau^S)^2 \tau^\Theta + \gamma^2 (\tau^X + \phi_s \tau^S + \tau_s^Z)} \quad (28)$$

$$\alpha_s^\Theta = -\frac{\gamma (\gamma^2 + \phi_s \tau^S \tau^\Theta)}{(\phi_s \tau^S)^2 \tau^\Theta + \gamma^2 (\tau^X + \phi_s \tau^S + \tau_s^Z)} \quad (29)$$

$$\alpha_s^P = \frac{\gamma^2 \tau_s^Z}{(\phi_s \tau^S)^2 \tau^\Theta + \gamma^2 (\tau^X + \phi_s \tau^S + \tau_s^Z)}. \quad (30)$$

The pricing coefficients are time-dependent, since the per capita signal count,  $\phi_s$ , and the public signal precision,  $\tau_s^Z$  as given in (1), are increasing with time. The market clearing condition aggregates over investors, as their first order conditions are linear in their numbers of private signals (see the proof of Proposition 4). So by market clearing, the price coefficients are functions of the per capita signal count in the population,  $\phi_s$ .

### E.4 Actual trading and equilibrium prices

The perceived  $\alpha$  coefficients describing the dependence of price  $v_s$  on state variables are incorrect. To pin down the coefficients of the true pricing function as given in (21), I again impose market clearing. The solution procedure takes into account that the optimized positions of investors reflects their incorrect perceptions of several variables, and of the parameters of the pricing function. So in using the first order condition for optimal investor trades, we must *not* replace the subjective  $\alpha$ 's with correct ones. We must, however, substitute out for market price in (A.9) using the *true* pricing function in (21). We must also replace the mistaken perception of each agent's average private signal value,  $X + \bar{\epsilon}_s^i$ , with its true value  $\bar{Y}_s^i = X + \bar{\epsilon}_s^i + b f_s^{bi}$ . Also, when  $\kappa \neq 1$ , the

agents' perceived aggregate signal count differs from the actual, as they misestimate the meeting rate.

**Proposition 5.** *In the Price Bubbles Model, the market price in (21) is:*

$$v_s(X, \bar{Z}_s, \Theta, b) = \frac{\tau_s^Z \gamma^2 \bar{Z}_s + (\gamma^2 + \phi_s \tau^S \tau^\Theta) [\phi_s^R \tau^S (X + b f_s^b) - \gamma \Theta]}{\gamma^2 (\tau^X + \phi_s^R \tau^S + \tau_s^Z) + \phi_s \phi_s^R (\tau^S)^2 \tau^\Theta}, \quad (31)$$

where the coefficients on  $X$ ,  $b f_s^b$ ,  $\bar{Z}_s$ , and  $\Theta$  are the  $\alpha$  coefficients in the **true** equilibrium pricing function (21).

Equating corresponding coefficients in (31) and the equilibrium pricing function (21) gives the true equilibrium pricing function in terms of exogenous parameters and the perceived and actual per capita signal counts  $\phi_s$  and  $\phi_s^R$ . Taking ratios of corresponding alpha coefficients  $\alpha^R/\alpha$ , true versus as perceived by agents in the model, reveals that if  $\kappa < 1$ , the true coefficients place greater weight on  $X$  and lower weight on the average public signal,  $\bar{Z}_s$ , and  $\Theta$ . Owing to  $\kappa$  bias, agents underestimate how much information about  $X$  is impounded into price, and correspondingly overestimate the dependence of price on public signal noise and on the supply shock.

**Proposition 6.** *In the Price Bubbles Model, if  $\kappa < 1$ ,  $\alpha_s^{RX} > \alpha_s^X$ ,  $\alpha_s^{RP} < \alpha_s^P$ , and  $\alpha_s^{R\Theta} < \alpha_s^\Theta$  for all  $0 < s < T$ . If  $\kappa > 1$  the inequalities are reversed.*

## E.5 Bubble price dynamics

The biased percolation model captures several features of asset market trading, mispricing, and bubble episodes.

### Bubble phases

Market price is stochastic owing to supply shocks and the randomness of the public signal realizations. Taking the expectation of (31) conditional upon  $b$ , the true expected price dynamic is

$$\mathbb{E}^R[v_s|b] = \frac{(\gamma^2 + \phi_s \tau^S \tau^\Theta) \phi_s^R \tau^S b f_s^b}{\gamma^2 (\tau^X + \phi_s^R \tau^S + \tau_s^Z) + \phi_s \phi_s^R (\tau^S)^2 \tau^\Theta}. \quad (32)$$

The per capita number of signals is a determinant of the market price (see also Andrei and Cujean (2017), Cujean (2020)). The biased percolation process determines the rise over time in signal counts  $\phi_s^R$  and  $\phi_s^R$  and in per-signal bias  $b f_s^b$  in (32).

Over time, percolation shifts the distribution toward higher per capita signal counts, both perceived and actual,  $\phi_s$  and  $\phi_s^R$ , in the sense of First Order Stochastic Dominance.

Intuitively, when  $b > 0$ , higher  $\phi_s^R$  increases the market price, because a greater number of signals, as reflected in  $\phi_s^R$ , causes heavier impounding of the growing per-signal bias, as reflected in  $f_s^b$ , into price. This amplification of bias over time is reflected in the  $b$  term in the numerator.

It is evident that expected price is increasing with the size of the bias,  $b$ . Indeed, when  $b = 0$  the expected price is zero, so unconditionally,  $\kappa$  bias does not on average induce price bubbles. For  $b > 0$ , it is evident that expected price is at first increasing with time, since  $v_0 = 0$ , whereas  $\mathbb{E}^R[v_s|b] > 0$  when  $s > 0$  (since  $\phi_s, \phi_s^R, f_s^b > 0$  when  $s > 0$ ).

Focusing on  $b > 0$ , to describe curvature of the expected price path, consider the smoothed version of the path of precision of public signals as in (20). Under appropriate parameter values, as depicted in Figure 2, the expected price dynamic in (32) has four phases, as described earlier, though of course realizations are very different from expectations. If, as is realistic for many asset markets, there are costs of or constraints on short-selling, in practice positive bubbles (the  $b > 0$  case) will tend to be more pronounced than negative ones.

What causes the rise and fall in expected price? During the inception phase, iterative amplification of bias causes growth in expected price to accelerate. To build intuition from equation (32), first treat  $\tau_s^Z$  as approximately constant. The slow initial growth in expected price is reflected in the numerator by the fact that in the term in parentheses,  $\phi_s \tau^S \tau^\Theta$  is at first small relative to  $\gamma^2$ , so that there is only a modest effect of an increase in  $\phi_s$  on the overall ratio (bearing in mind that the coefficient on this term and the denominator are also increasing with time). As  $\phi_s$  grows exponentially, its effect intensifies.

As  $s$  grows (still treating  $\tau_s^Z$  as approximately constant), the terms in the numerator and denominator containing the product  $\phi_s^R \phi_s$  in (32) dominate. This reflects a high degree of certainty of investors who have accumulated many private signals, which causes them to incorporate per-signal bias  $b f_s^b$  almost completely into price. Owing to biased information percolation, this is linearly increasing with time (Proposition 1, Part 2). So the expected price is convex at first, but this acceleration eventually ends. Indeed, since  $\tau_s^Z$  grows exponentially (with a sufficiently high exponent), eventually public information arrival overwhelms mispricing, and correction ensues.

Ceteris paribus, higher  $\kappa$  (meaning lower  $\kappa$  bias) increases perceived per capita signal count  $\theta_s$ . The partial derivative of the RHS of equation (32) with respect to  $\theta_s$  is negative, so greater  $\kappa$  bias tends to attenuate (but cannot eliminate) the price effects of  $b$  bias.

Intuitively, the effects of  $b$  bias come from investors updating from their priors both toward their own  $b$ -biased signals, and toward the  $b$ -biased market price. A lower  $\kappa$  bias reduces the updating toward price.

#### Market reactions to public news events

Owing to mispricing, during bubbles, the returns on the dates of foreseeable news arrival are predictable, even without conditioning upon the value of the event realization.

**Proposition 7.** *In the Price Bubbles Model, when  $b > 0$ , on average the price drops discretely when the date- $t$  public signal arrives,  $t \geq 1$ . The opposite is the case when  $b < 0$ .*

Intuitively,  $b > 0$  temporarily results in growing overpricing over time as signals are shared with bias. Public news arrival partly corrects overpricing at each date. When  $b < 0$ , public news on average partly corrects underpricing. (The  $b > 0$  implication would be modified if there were a positive expected supply of the asset, which would incrementally induce a positive announcement date risk premium.) To empirically test these predictions, the sign of  $b$ , i.e., whether the asset is in an upward or downward bubble, can be proxied by price run-ups or run-downs not justified by fundamentals, i.e., run-ups in value proxies.

This predictability does *not* derive from a general propensity to misinterpret value-relevant information signals, as in models of investor overconfidence or limited attention (Hirshleifer and Teoh 2003; Peng and Xiong 2006). This predictability instead derives from investor misunderstanding of the social transmission process. This is also true of the return anomaly predictions of Section E.6.

When  $b > 0$ , the price drops in the model suggest that there is some truth to the Wall Street adage, “Buy on the rumor, sell on the news.” This reflects the idea that favorable rumors about a stock cause its price to rise, but that when reliable information arrives it tends to disconfirm the rumor, correcting price downward. Evidence suggesting that earning news has a downward corrective effect on stocks that have experienced run-ups is provided by Aboody, Lehavy, and Trueman (2010).<sup>17</sup>

#### Peak oscillation and dead cat bounce

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<sup>17</sup>They find that on average price rises in the lead-up to earnings announcement dates and drops immediately following them, after conditioning on longer-term past price run-ups. Average run-ups are indeed a feature of the  $b > 0$  case, which precedes a more rapid run-up near the public news event and subsequent reversal. However, the walkdown by analysts toward beatable forecasts also induces other return patterns through the earnings cycle (Richardson, Teoh, and Wysocki 2004; Linnainmaa and Zhang 2019).

As illustrated in Figure 2, the model can generate predictable oscillations which, on average, intensify near the peak of the bubble, and which augur its collapse. There is a natural tottering of market valuations at the peak of a bubble before corrective forces come to dominate. Even in the terminal phase of the bubble, there is zigzagging, albeit less extreme, along the journey to zero—“dead cat bounce.”

Intuitively, as discussed earlier, in the inception phase of the bubble, there is accelerating expected price growth. In the limit as time becomes large, if there were no public information arrival, the system would asymptote toward an increasing straight line. That line reflects certainty on the part of investors, based on sharing many signals, that the value of the fundamental is  $X + bf_s^b$ , the sum of the true fundamental and the per capita per-signal bias. Expected valuations would in principle increase without limit.

Public signals arrive with exponentially growing precision. If, as I assume, the growth rate for public precision is high enough, investors’ optimistic faith in their biased private information is eventually challenged. There is, on average, a downward jump on each discrete news arrival date, followed by recovery toward the peak owing to biased percolation and new meetings. As public information becomes more and more conclusive, the recovery curves successively flatten, and the bubble collapses. The bubbly effects of biased percolation never completely vanish, however, so small oscillations continue along the expected price path as it approaches zero.

#### Trading statics and dynamics

Both  $\kappa$  bias and  $b$  bias introduce irrational motives for trading. This provides a new perspective on empirical findings that investors trade actively and lose money thereby (Barber and Odean 2000). Even when there is no direct behavioral bias, such as overconfidence, “for” excessive trading, social processes can *attract* investors to this behavior (see also Fable 2).

If  $\kappa < 1$  (my focal case), investors underestimate the per capita number of private signals,  $\phi_s$ , that are reflected in price. This results in underweighting of price in forming beliefs, which favors extreme expectations and willingness to take bets against the market. At date 0 there is no such underestimation (since there have been no meetings). So underweighting of the private information in price becomes more severe over time. At least for a time, as a bubble grows, speculative trading increases. Eventually, the bubble corrects, and disagreement dissipates, so eventually speculative investor trading declines toward zero (so that investors just absorb the supply shock).

The underweighting of the information in market price is akin to cursedness as in Eyster, Rabin, and Vayanos (2018), meaning investor neglect of the fact that the actions of others reflect private information. Here the underweighting of others' information is endogenous. Investors underestimate how informative price is, because they don't realize how often others are meeting to exchange signals. The model therefore describes the conditions under which people impute too little information to price versus too much (which would occur when the meeting rate is overestimated,  $\kappa > 1$ ). Furthermore, the biased percolation approach suggests that such effects are modulated by the other parameters of the social interaction process, such as the intensity of social interaction (as reflected in the meeting rate  $\eta$ ).

The  $b$  bias further contributes to investor disagreement. Over time there is a tendency for signal counts  $n_s^i$  and per-signal biases,  $b_s^{bi}$  to become more dispersed as investors randomly experience different meetings and acquire different numbers of private signals. This is reflected in the true cross-population signal-count distribution (with  $\kappa = 1$ ) in equation (9). By Part 3 of Proposition 1, the distribution of number of meetings  $N$  across investors is  $\text{Pois}(\eta s)$ , so the mean and variance is proportional to time,  $s$ . So higher  $s$  also tends to spread the distribution of the number of signals,  $n = 2^N$ .

Greater dispersion across investors in their signal counts also tends to increase the cross-investor dispersion in per-signal bias, since an investor's per-signal bias is linearly increasing in the investor's signal count (Proposition 1). As dispersion in per-signal bias accumulates, highly biased investors are motivated to buy (if, e.g.,  $b > 0$ ) from low-bias investors. Furthermore, the growing number of signals over time increases an investor's confidence in the investor's (biased) average signal value, further encouraging speculative trading.

However, owing to public news arrival and the impounding of private information into market price, eventually disagreement must decline. So eventually, as the bubble loses impetus, open interest and trading volume decline. So there is a rise-and-fall path for trading in a bubble analogous to the path for expected price. An interesting further direction would be to develop and contrast the implications for volume and price volatility of the biased information percolation approach to dynamic models of asymmetric information under rational updating (He and Wang (1995)).

## E.6 Return anomalies

Biased information percolation induces other patterns of abnormal return predictability, trading, and expectations based upon both public news events and on past returns.

### Event-based return predictability

Public news events positively predict future abnormal security returns. At the discrete arrival date  $t$  of a public signal, when  $b > 0$ , the stock is on average overpriced, and the public news on average corrects it downward (Proposition 7). But on average the correction is only partial, resulting in a negative post-event long-run expected return. Symmetrically, when  $b < 0$ , on average the public news exceeds expectations, and is followed by a positive post-event expected return. I describe this effect in terms of price reactions. Let the event-date price change be  $\Delta v_t \equiv v_t - v_{t-}$ , where  $t-$  denotes a time just before date  $t$ .

**Proposition 8.** *Conditional on  $b$ , after the arrival of the date- $t$  public signal, the sign of the expected post-event price change through the terminal date  $T$  is the same as the sign of the expected event-date price reaction,  $\text{Sign}(\mathbb{E}^R[X - v_t|b]) = \text{Sign}(\mathbb{E}^R[\Delta v_t|b])$ . Both expectations are negatively proportional to  $b$ .*

This implication of long-horizon return continuation after public corporate news events—i.e., that when  $b > 0$ , on average the price falls on the event date and then has long-horizon downward drift, and that when  $b < 0$ , on average the price jumps on the event date and has a long-run upward return drift—is consistent with the new issues puzzle (Loughran and Ritter 1995) and the repurchase anomaly (Ikenberry, Lakonishok and Vermaelen (1995, 2000)). These effects may be reinforced if, as evidence suggests, firms deliberately take informative actions in response to preexisting mispricing (Dong, Hirshleifer, and Teoh 2012), as reflected in  $b$  here.

The model further predicts that *short horizon* post-event drift varies conditionally depending upon the phase of the bubble. For example, when  $b > 0$ , during the bubble upswing, a public news event is on average met with a price drop, but is followed, on average, by price increases through the peak of the bubble before eventual correction.

So an untested implication here is that conditional upon being in a bubble upswing, post-earnings announcement drift (PEAD) is reversed (i.e., there is Post-Earnings Announcement Reversal), and that PEAD is especially strong during the downward correction phase.

Arguably, firms will time IPOs specifically when  $b > 0$  and during the inception

or boom phases rather than the correction or terminal phases (“striking while the iron is hot” to ensure a successful offering). So it is intriguing that after IPOs there are, on average, positive mean abnormal returns for several quarters before the eventual long-run underperformance (Ritter and Welch 2002).

The return predictability pattern is subtler during the correction phase of a bubble. When  $b > 0$ , between the price drops on discrete public news dates, percolation still promotes price increases. So there can still be reversal at very short horizons. However, returns over an only-moderately short horizon—one that is long enough to include the next discrete news arrival date, and the associated price drop—will exhibit post-event downward correction.

We have seen that since higher  $b$  induces pre-event overvaluation, on average it induces a lower (less positive or more negative) event-date price reaction and post-event price drift. This suggests that variability of  $b$  induces comovement between event-date corrective price reactions and subsequent corrective drift. For simplicity I show this for public news at date 1.

**Proposition 9.** *Consider a random sample of public news events in which different possible realizations of  $b$  are possible. If  $\mathbb{V}(b)$  is sufficiently large, there is long-horizon return continuation after public news events,  $\text{cov}^R(\Delta v_t, X - v_t) > 0$ .*

Proposition 9 is consistent with post-earnings announcement drift, the tendency for earnings surprises to positively predict future returns up to 3 quarters forward, with surprise measured by the market return reaction at the announcement date (or by earnings relative to analyst forecasts, which is highly correlated).

Turning to post-event drift over a short horizon, the next proposition relies on public announcements occurring at regular intervals, as with quarterly earnings announcements.

**Proposition 10.** *Conditional on  $b$ , in a sample of public announcements randomly selected from among the announcements at dates  $t = 1, \dots, T - 1$ , on average there is **short-horizon** price change continuation,  $\text{Sign}(\mathbb{E}^R[v_{t+1} - v_t|b]) = \text{Sign}(\mathbb{E}^R[\Delta v_t|b])$ . Both expectations are negatively proportional to  $b$ .*

A favorable earnings surprise is associated with high profits, since the current earnings level includes its latest change. So the sluggish response to earnings news in the model is also consistent with the profitability anomaly—the finding that profits are a positive predictor of future abnormal returns (Novy-Marx 2013; Wahal 2019), and with

evidence that the profitability anomaly is associated with sluggish updating in expectations of fundamentals (Bouchaud et al. (2019)).

### Serial correlation of returns

The average price trajectories after  $b > 0$ , as in Figure 2, or  $b < 0$ , are reminiscent of the hump-shaped and U-shaped impulse response functions that can generate momentum and reversal in behavioral models (see, e.g., Figure 1 of Daniel, Hirshleifer, and Subrahmanyam (1998)). Intuitively, conditional on  $b$ , the shapes of the expected price paths suggest that at short horizons high returns tend to be followed by high returns, and low returns by low returns, i.e., there is momentum. In contrast, with long lags, positive price changes during the boom phase tend to be followed by negative price changes during the correction (in the hump-shaped case), and negative price changes tend to be followed by positive ones (in the U-shaped case).

The randomness of  $b$  tends to further promote positive autocorrelation at short lags, and negative at long lags. By (31), a higher positive value of  $b$ , for given realizations of other random variables, increases the date 0 to 1 price change, since  $f_s^b$  increases from 0 to a positive quantity. During the early phase of the bubble, a similar effect promotes successive price increases at short lags, which contributes positively to positive autocorrelation. Similarly, during bubble correction, a higher positive value of  $b$  promotes successive price decreases at short lags. As time increases and public precision grows, the  $\gamma^2 \tau_s^Z$  term in the denominator discounts the  $b$  term more and more heavily. This also contributes to positive autocorrelation. Similar points apply to lower negative values of  $b$  during negative bubbles.

Similar reasoning further suggests that  $b$  realizations have opposite effects upon price changes separated by long lags which cross between the upswing versus downswing of the bubble. This suggests that randomness of  $b$  contributes negatively to long-lag return autocorrelation.<sup>18</sup> I verify formally the contribution of  $b$  to short-lag momentum, where for simplicity I assume that public information arrives smoothly.<sup>19</sup>

**Proposition 11.** *Suppose that public information arrives smoothly, such that precision satisfies  $\hat{\tau}_s^Z \equiv e^{\zeta(s-0.5)} \tau^P$ ,  $s \in [0, T)$ , and that  $\mathbb{V}(b)$  is sufficiently large. Then at a*

<sup>18</sup> Andrei and Cujean (2017) derive momentum and reversal in a fully rational information percolation setting. The imperfectly rational approach considered here addresses return predictability as an *anomaly*, in the sense of generating Sharpe ratios that are extreme relative to a rational assessment of risk. Biased percolation offers a new explanation for this quantitative puzzle.

<sup>19</sup>For simplicity, Proposition 11 is premised on public information arrives smoothly in continuous time. Conjecturally, this is much like assuming that the time horizon for the price-change autocorrelation, while short, is not too short.

*sufficiently short time horizon, successive price changes are positively autocorrelated.*

Without smoothing, as we have seen, there is oscillation in short-horizon conditional expected returns. The frequency of oscillation should depend on how often public news arrives (e.g., in the form of press releases, manager interviews, communications between investor relations staff with analysts, and investigative reports by financial media and commentators). So an interesting topic for exploration is whether the model is consistent with the short-term reversal anomaly of Jegadeesh (1990), along with medium term momentum and long-term reversal. If so, the approach may unify these seemingly-disparate serial correlation effects.

Several behavioral finance models offer explanations for return momentum and/or reversal when agents do not talk to each other (see, e.g., Barberis, Shleifer, and Vishny (1998), Daniel, Hirshleifer, and Subrahmanyam (1998), Hong and Stein (1999), and Duffie (2010)). I offer here a type of possible explanation that is seldom considered. Rather than individual-level biases for over- or underreaction, such as overextrapolation, overconfidence, or limited attention, return predictability derives from *biases in the social transmission process*.

The social nature of this explanation implies that return predictability depends on features of the social interaction process, such as how heavily investors interact, and how biased are these communications. This suggests looking to proxies for the extent to which people communicate through personal meetings versus electronically, one-to-one versus in blogs or social media, how socially connected they are, and the extent to which investors bias or censor the reports they make to others. The implication that social structure affects bubbles and return anomalies deserves empirical testing.

Let the precision-weighted expectation of the fundamental in the investor population be defined as in equation (B.5) in the Appendix. The next proposition verifies that after a recent price run-up, investor expectations become more optimistic.

**Proposition 12.** *Suppose that public information arrives smoothly, such that precision satisfies  $\hat{\tau}_s^Z \equiv e^{\zeta(s-0.5)}\tau^P$ ,  $s \in [0, T)$ , and that  $\mathbb{V}(b)$  is sufficiently large. Then at a sufficiently short time horizon, the change in the precision-weighted expectation of the fundamental  $X$  is positively correlated with the immediately preceding price change.*

(I am grateful to Yushui Shi for help with the proof, which is in the Online Appendix.) Intuitively, the rise-and-fall pattern of expected price suggests that beliefs become more optimistic after price run-ups than after run-downs. Again, the smoothness assumptions seems much like assuming the horizon is short but not “too short.” Proposition 12 is

consistent with the evidence that forecasts are positively associated with past returns (Greenwood and Shleifer 2014). Here, this correlation is an emergent, non-causal effect. Agents do not extrapolate past returns in forming their expectations. The illusion of extrapolation is created by the biased percolation process. During the rise of a bubble, percolation increases prices, and subsequently makes investors more optimistic. Similarly, during the correction phase of the bubble, correction of overpricing on average foreshadows further downward correction in investor beliefs.

This highlights one of my key themes, which I call *mimicry*. We cannot, in general, infer from an individual behavior that there is a direct bias “for” that behavior. Social transmission bias indirectly generates outcomes that can look very similar to direct individual-level behavioral biases.

## F Caveats and extensions of Fable 4

We have seen that biased information percolation offers an explanation for a wide set of trading and return patterns which are almost always attributed to relatively direct psychology biases “for” certain investor beliefs and behaviors. Missing from the model are no rational arbitrageurs who understand  $b$  bias and  $\kappa$  bias. As in other behavioral asset pricing models, introducing rational arbitrageurs would diminish but not eliminate mispricing.

How plausible is the assumption of  $b$  bias? Why would someone add  $b$  as a sender, and yet, as a receiver, neglect the fact that others do so? One reason, as mentioned earlier, is that  $b$ -bias may be introduced unconsciously. A second reason is that in reality people do indeed often fail to put themselves into others’ shoes, i.e., to think through how others use information or behave strategically.<sup>20</sup>

More broadly, if  $b$  were stochastic across agents (or they think that it is), then knowledge that one personally has a value  $b_i > 0$  would not conclusively refute, in the agent’s mind, the possibility that in the population  $\mathbb{E}[b_j] = 0$ . In a related vein, there are plausible errors in higher order beliefs that would effectively induce neglect of  $b$  bias.<sup>21</sup>

Since even small biases in the model are amplified recursively, outcomes are highly sensitive to small variations in bias  $b$ , making the sign and scale of action booms and

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<sup>20</sup>This is reflected in the cognitive hierarchy model in game theory (level- $k$  thinking; Camerer, Ho, and Chong (2004)), cursed equilibrium (Eyster and Rabin 2005), and the extensive experimental evidence that motivates these approaches.

<sup>21</sup>Suppose most people understand  $b$  bias, and wrongly believe that everyone does. If some fools fail to adjust the signals they receive downward by  $b > 0$ , the signals they pass on are upward biased, and such bias will be amplified through the meeting process.

price bubbles hard to predict. For example, an arbitrarily small difference in  $b$  between positive and negative values eventually makes the difference between a large positive or negative bubble.

Importantly, the conclusions of the model apply even when  $b$  is very close to zero. Owing to recursive amplification of bias in sequential meetings, even a small bias ultimately has a very large effect. The  $b \approx 0$  possibility suggests another reason why people would ignore  $b$ , i.e., act as if  $b = 0$ . A heuristic of ignoring parameters that are very close to zero is very effective in many decision domains. So treating  $b = 0$  may be a misapplication of a generally-effective heuristic. Furthermore, people may be fooled by early experience, since, when  $b \approx 0$ , the model implies that for a long time the behavior of the system will be very close to the behavior when  $b = 0$ .

The model takes the distribution of  $b$  bias as exogenous. What causes the level of transmission bias to be high or low? One likely driver is general public mood. Good mood tends to induce more positive evaluations and recall (Wright and Bower 1992). This suggests more positive transmission of evaluations of possible actions, such as buying an asset, to others as well. This raises the question of whether episodes that induce good versus bad mood, such as the US defeat in the Vietnam War during the mid-1970s, or the victory of the West in the Cold War, affected social transmission dynamics.

The millennial high-tech stock market boom is consistent with a high realization of  $b$  in reporting one's opinion about high-tech start-ups. There was general positive mood about a strong economy, and the world felt much safer to many after the end of the Cold War. There was excitement about the rise of the internet, and a sudden rise in status to being tech-savvy. This episode highlights the need for empirical study of whether and how general mood affects transmission bias, action booms, and mispricing.

Mood is also likely to shift *endogenously* during a boom period, resulting in positive feedback to transmission bias. Growing excitement may cause people to bias their signals reports favorably during the inception phase of an action boom or market bubble, with downward bias during the correction phase. There is evidence that people tend to pass on exaggeratedly bad news when the topic is emotionally negative, and exaggeratedly good news when the topic is positive Heath (1996). Such behaviors may exacerbate booms and their collapse. So the effect of mood on social transmission deserves exploration.

In pioneering empirical tests of the effects of investor sentiment (Lee, Shleifer, and Thaler (1991), Baker and Wurgler (2006, 2007)), fluctuations in sentiment are taken as exogenous. The shifts in optimism or pessimism about an asset in the Price Bubbles Model endogenize swings in investors sentiment as an emergent *social phenomenon*.

Behavioral finance has offered theories of sentiment as a direct reflection of individual-level psychological biases. So empirical testing is needed of the effects of conditioning variables motivated by the social approach to investor sentiment. These include proxies for sociability, network structure, and shifts in communication technologies (such as the rise of the internet, blogs, and social media).

## VI Fable 5: Biased transmission of folk models

My final fable is about biased transmission of folk models. I suggest why folk models important, and briefly sketch some work-in-progress to try to suggest how contagion of economic folk models can be analyzed.

A *folk model* is an understanding of how the world works—an internal representation of external reality.<sup>22</sup> Examples include belief in Heaven and Hell, or in the payback criterion for capital budgeting. (After all, Hell is a kind of payback.) Distinguished scholars have argued that what I call folk models are key drivers of human behavior. As lenses for seeing the world, different folk models induce different biases in the social transmission of information signals.

Folk models are often shaped by vivid narratives—stories (descriptions of sequences of events), often with emotional and moral overtones. However, folk models are also often mundane beliefs about how the world works. In either case, since thinking is costly and effortful, folk models are often oversimplified representations of reality.

Folk models spread from person to person through conversation, and *compete* for human attention and belief. This competition is greatest for directly conflicting folk models. For example, the folk model that holds that past returns tend to continue (“the trend is your friend”) is opposed by the folk model that holds that they tend to reverse (“buy on the dips”). Shifts in frequency of folk models in a population over time are examples of *cultural evolution* (see, e.g., the survey of Mesoudi, Whiten, and Laland (2006)). Behavioral finance takes investor extrapolative versus contrarian beliefs as *given*—perhaps even hard-wired. In contrast, in social finance, such beliefs can potentially be in competition, and spread contagiously.

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<sup>22</sup>An equivalent term from cognitive science and psychology is *mental model* (Craik 1943; Johnson 1983). Folk economic models have been discussed under a variety of names, such as popular models (Shiller 1990), mental constructs (Denzau and North 1994), folk economics, models or beliefs (Rubin (2003), Hirshleifer and Teoh (2009, 2018), Boyer and Petersen (2018)), mental models (Hoff and Stiglitz 2016), and others (Rubin (2002), Caplan (2007), Hirshleifer (2008)).

Several authors propose that compartmental models from epidemiology can capture the spread of folk models and financial behavior. In such models, an infection with a financial belief, such as enthusiasm for a technical strategy or for Bitcoin, spreads through a population over time via random contacts. Some versions of compartmental models, such as the SIR model, have an epidemic rise-and-fall time path. So the SIR Model has been applied to help explain trading and price bubbles.<sup>23</sup>

In the SIR model, there are three types of agents:  $\mathcal{S}$  (Susceptible),  $\mathcal{I}$  (Infected), and  $\mathcal{R}$  (Recovered/Resistant/Removed). Under appropriate parameter values, the fraction of  $\mathcal{I}$  agents can rise in a bubble-like trajectory before declining toward zero. At the start, almost everyone is susceptible. In a random meeting, there is a probability that an  $\mathcal{S}$  becomes infected. This probability is a measure of transmission bias toward infection. The infected randomly recover over time. Having joined the  $\mathcal{R}$  compartment, an agent never changes again, so in the long run the fraction of infecteds declines to zero.

The rise of infection is at first self-reinforcing. When the infected fraction increases, meetings more often include an  $\mathcal{I}$  instead of a pair of  $\mathcal{S}$ 's. So there is a convex upswing in the fraction of infected. Eventually, however, as the susceptible fraction declines, so does the rate of new infection. As a result, recovery draws the fraction infected toward zero.

How should we adapt the compartmental approach to biased transmission of a novel folk model, and its effects on economic behaviors? Consider a folk model that induces overestimation of the expected payoff (e.g., terminal dividend) from the relevant action, and three modifications to the SIR model.

First is to move to the standard SIRS model, in which an  $\mathcal{R}$  has a probability of becoming susceptible again. This replenishes the  $\mathcal{S}$  compartment, so the infected fraction asymptotes toward a positive, endemic long-run steady state. This is often realistic. For example, after a religious awakening fades, the fraction of the population that is religious seldom declines to zero.

Second, I introduce investors and an asset market. As in Fable 4, there is again a single risky asset, a random terminal-date payoff, mean-variance preferences, and myopically optimal trading decisions. However, here there is no private information nor supply noise, so (unrealistically) investor behavior and the price path is deterministic. Also, here there are two types of investor beliefs, overoptimistic beliefs, which are held by the infecteds; and rational beliefs, which are held by the  $\mathcal{S}$  and  $\mathcal{R}$  agents.

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<sup>23</sup>See Shive (2010), Burnside, Eichenbaum, and Rebelo (2016), Shiller (2017, 2019), and Chincio (2019).

Third, I modify the transmission bias by introducing *buzz*, the level of excitement about the folk model. I argue that this makes the folk model more contagious; rapid growth in the number of adherents of a folk model generates excitement. When many people are adopting the pro-Bitcoin folk model, for example, others may hear that Bitcoin is “hot,” making a meeting with a Bitcoin enthusiast may be more persuasive. I capture buzz by the time derivative of the fraction of infecteds,  $I$ , in the population, so buzz is proportional to  $\dot{I}$ .

When  $\dot{I} < 0$ , there is *negative* buzz, which promotes recovery from the folk model. In the language of the SIRS model, I am generalizing to allow the contagion parameter of the model to vary endogenously, and to take negative values. This is not permitted in models of disease spread, where meetings can spread an infection, but cannot cure it. In contrast, it is realistic that meetings can sometimes cure agents of belief in a folk model.

In equilibrium prices reflect a weighted average of the beliefs of the three investor types in the population. Figure 4 shows a possible price trajectory in my SIRS model with Buzz. It shares with the standard SIRS model the possibility that correction of the initial epidemic itself overshoots, resulting in damped oscillations. Under appropriate parameter values, there is an initial rise and fall in price. This is similar to some behavioral models in which a hump-shaped impulse response function induces return momentum at short lags and reversal at long lags. Trading also rises and falls with bubble, because disagreement comes from the infecteds, whose frequency starts small, rises, and ends small.

As the bubble collapses, the model implies that correction can overshoot. The fraction of infecteds declines below its long-run equilibrium value, so after the crash, prices correct upward again, consistent with evidence of Goetzmann and Kim (2018). As in Fable 4, there is possible dead cat bounce.

The effect of buzz is to exaggerate the boom on way up, and the collapse on the way down. So it intensifies the positive feedback feature of bubbles. The damped oscillation in stock price suggests rich patterns in return autocorrelations at different lags.<sup>24</sup>

Buzz effects are somewhat somewhat akin to return extrapolation as modelled in behavioral finance (DeLong et al. (1990)), since shifts in the frequencies of types induce

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<sup>24</sup>At short horizons, price changes tend to continue, suggesting positive autocorrelation. At somewhat longer lags, a return on the rising side of a hump or the declining side of a trough tends to be paired with a return on the opposite side, which suggests reversal. At even longer lags, a return on the rising side of a hump or the declining side of a trough tends to be paired with a return in a similar position on the next hump or trough, suggesting positive autocorrelation.

price changes. Empirically, one of the key differences is that in a social contagion model, the average effect of buzz on investor beliefs and trading depends on the frequencies of different investor types in the population. These determine the probability of a mixed meeting in which an agent can potentially be infected or cured.

My main points in Fable 5 are about the general compartmental approach to folk models rather than the SIRS model in particular. The SIRS model is useful example, since it is similar to the SIR model, the compartmental model most familiar to economists. However, similar points can be made, for example, in compartmental models without resistants, and where there are exposed agents who take time to become infectious.

More broadly, my focus has been on shifts in the frequencies of competing folk models. But ultimately, social economics and finance needs to analyze the *content* of folk models. Where do folk models come from, and what determines how they evolve, sometimes into elaborate forms? Pioneering steps along these lines have been taken, e.g., by Benabou and Tirole (2006, 2016) and Shiller (2017, 2019).

## VII Emergent themes

I now sum up with some themes about social transmission bias, and about social economics and finance, that emerge from my five fables.

1. **Compounding: Social transmission bias compounds recursively.**  
So small bias has large effects.
2. **Idiosyncrasy Social transmission bias helps explain the idiosyncrasy of aggregate outcomes—why they are often error-prone and unpredictable.**  
This derives from the sensitivity of outcomes to small biases (Theme 1), and from external shocks that interact with these biases.
3. **Dynamics: Social transmission bias can explain action booms, price bubbles, and swings in investor sentiment.**  
This explanation can accommodate, but does not require, standard ingredients for bubbles from behavioral finance, such as overconfidence and overextrapolation.
4. **Emergence: Socially emergent behavior often looks completely different from individual propensities.**  
Groups of army ants form death spirals, even though individual ants have no

propensity to walk in circles. Self-enhancing transmission is a conversational behavior, which looks nothing like the emergent behavior of aggressive trading. Biased reporting of signals in the biased percolation model is a conversational behavior; the outcome of price bubbles and collapses is very different.

5. **Mimicry: Social emergence can easily create the illusion of a direct individual propensity “for” a behavior when no such propensity exists.**

Examples include the illusory appearance of lottery preferences in the self-enhancing transmission model, of present-biased consumption preference in the visibility bias model, and of extrapolative beliefs in the biased percolation model.

Owing to the possibility of illusory effects, we often overstate the inferences from empirical tests.

6. **Proxies: The social transmission bias approach, and social economics and finance in general suggests *new test variables* for empirical research.**

The first category of test variables consists of general proxies for social interaction and network position.<sup>25</sup> The second category consists of proxies for the sources of social transmission bias.<sup>26</sup>

In closing, we academics, like the turkeys marching around a dead cat, sometimes get caught in closed loops which, at worst, degenerate into ritualistic cycles. Even at best, they act as blinders that separate us from part of reality. But we humans are smarter than turkeys—though it does not always seem that way. We can break out of our closed loops.

Such breakout moments are rare and precious. I believe that social economics and finance is a once-in-a-generation research opportunity. The poet John Keats, on experiencing a life-changing revelation, said that he felt

. . . like some watcher of the skies  
When a new planet swims into his ken . . .

Like that astronomer, we have a new world to explore—a new way to understand human behavior. The outline of an emerging field of inquiry is now discernable: social economics

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<sup>25</sup>These include proxies for sociability, communication technologies and media, individual social network position, and overall social network connectivity. Also, since both social interaction and stock trading are associated with spatial proximity, firm location can proxy for features of the relevant social network.

<sup>26</sup>These include individual psychological traits, environmental cues, the content of folk models, and the textual characteristics of folk models and transmitted signals.

and finance; and some of its key building blocks: social networks, folk models, and, as I have emphasized tonight, *social transmission bias*.

# Appendix

This Appendix provides proofs of Propositions 1-11. The Online Appendix analyzes biased information percolation with single-signal transmission bias, and provides the proof of Proposition 12 and a figure illustrating the Action Booms model.

## A Proofs

**Proof of Proposition 1:** I use the following lemma.

**Lemma 1.** *For an agent who has had  $N^i \geq 0$  meetings and who possesses  $n^i$  signals, in the Full-Batch Transmission Bias regime:*

1. *The number of signals the agent possesses is a function of the number of meetings,*

$$n^i = 2^{N^i}. \quad (\text{A.1})$$

2. *The agent's per-signal bias count as a function of the number of meetings is*

$$f^{bi} = \frac{N^i}{2}. \quad (\text{A.2})$$

3. *An agent's per-signal bias count is an increasing function of the number of signals the agent has,*

$$f^{bi} = \frac{\log_2 n^i}{2}. \quad (\text{A.3})$$

### Proof of Lemma 1:

I suppress  $i$  superscripts.

Part 1: Let  $B(N)$  be the function that gives any agent's number of biases as a function of an agent's number of meetings  $N$ , and let  $n(N)$  be the function that gives any agent's number of signals. For  $N = 0$ ,  $n(0) = 1$  as claimed. Suppose as inductive hypothesis that the claim holds through some value  $N$ . Then the next meeting is with a partner who also has  $2^N$  signals. So  $n(N + 1) = 2^{N+1}$ . So by induction the result holds.

Part 2: At agent  $i$ 's  $N$ 'th meeting,  $N \geq 0$ , agent  $i$  has  $n(N) = 2^N$  signals. Owing to homophily, agent  $i$  is paired with another agent who has the same number of signals.

Let  $g^b(N)$  be the function that gives any agent's per-signal bias as a function of  $N$ . After 0 meetings, an agent has 0 biases, so  $B(0) = g^b(0) = 0$ , as claimed. As inductive

hypothesis, suppose that for all  $N' \leq N$ ,  $g^b(N') = N'/2$ , so that the number of biases possessed by an agent with  $N'$  signals satisfies  $B(N') = 2^{N'-1}N'$ . I will show that these relations also hold for  $N + 1$ .

Coming into an agent's  $(N + 1)^{th}$  meeting, the total number of biases possessed by the agent and by the agent's sending partner is  $B(N)$ , and the sender adds a new  $b$  increment to his average signal, so the receiver receives a total of  $B(N) + n(N)$  additional biases. As of meeting  $N$ , the receiver's per-signal bias count is  $g^b(N) \equiv B(N)/n(N)$ . After meeting  $N + 1$ , it is

$$\begin{aligned} g^b(N + 1) &= \frac{2B(N) + n(N)}{2n(N)} \\ &= g^b(N) + \frac{1}{2} \\ &= \frac{N + 1}{2}. \end{aligned}$$

Furthermore, the number of biases the agent possesses is  $2^{N+1}g^b(N + 1) = 2^N(N + 1)$  as was to be proved. So the claim follows by induction.

Part 3 follows immediately from the other parts.  $\parallel$

I now apply Lemma 1 to complete the proof.

Part 1: The perceived per-capita number of signals at date  $s$  is

$$\begin{aligned} \phi_s &\stackrel{\mathcal{B}}{=} \int_{i \in [0,1], \mu} n_t^i di \\ &= \sum_{N=0}^{\infty} \mu_s(n(N))n(N) \\ &= \sum_{N=0}^{\infty} \frac{(\eta\kappa s)^N e^{-\eta\kappa s}}{N!} 2^N \\ &= e^{\eta\kappa s} \sum_{N=0}^{\infty} \frac{(2\eta\kappa s)^N e^{-2\eta\kappa s}}{N!} \\ &= e^{\eta\kappa s}, \end{aligned} \tag{A.4}$$

where the last summation is the sum of the probabilities of a  $\text{Pois}(2\eta\kappa s)$  distribution. The rational assessment sets  $\kappa = 1$ .

Part 2: To calculate the population-wide per-signal bias count  $f_s^b$ , recall that for an agent with  $N^i$  meetings, by Proposition 1 the number of signals is  $n(N^i) = 2^{N^i}$ , and that the per-signal bias count is  $g^b(N^i) = N^i/2$ , so the agent's total number of biases is  $2^{N^i-1}N^i$ .

So letting  $n^{bi}$  denote the number of biases of agent  $i$  at date  $s$ , the per capita average number of biases at date  $s$  is

$$\begin{aligned}
\bar{B}_s &\equiv \int_{i \in [0,1]} n_s^{bi} di \\
&= \sum_{N=0}^{\infty} \mu_s(n^i(N)) g^b(N) 2^N \\
&= \frac{e^{\eta\kappa s}}{2} \sum_{N=0}^{\infty} N \frac{(2\eta\kappa s)^N e^{-2\eta\kappa s}}{N!} \\
&= e^{\eta\kappa s} \eta\kappa s, \tag{A.5}
\end{aligned}$$

since the mean of  $\text{Pois}(2\eta\kappa s)$  is  $2\eta\kappa s$ . Dividing by  $\phi_t$  as given in Part 1 gives population per-signal bias count  $f_s^b = \eta\kappa s$  as asserted.

Part 3: The perceived frequency of an agent with  $N^i$  meetings by date  $s$  is  $\text{Pois}(\kappa\eta s)$ . But  $n^i = 2^{N^i}$  by Proposition 1 Part 1, so the result follows immediately. ||

**Proof of Proposition 2:** Under Gaussian updating, for an agent with  $n$  private signals and a set of public signals with total precision  $\tau^Z$ , the variance of  $X$  given these signals is  $\tau^X + n\tau^S + \tau^Z$ . The private signal errors (i.e., the  $\epsilon$  terms) average to zero across any infinite set of agents with a given number of signals  $n$ , i.e.,  $\int_{i:n^i=n} \bar{\epsilon}^i di = 0$ . When  $\kappa < 1$ , agents misperceive the per capita signal count to be  $\phi_s$  instead of  $\phi_s^R$ . This derives from their mistaken belief that  $\kappa = 1$ . An agent's per-signal bias count  $f_s^{bi}$  depends only on  $n_s^i$ , so I denote this function as  $h^b(n_s^i)$ . Restoring  $s$  subscripts, integrating across agents in (15) gives

$$\begin{aligned}
\Theta_s &= \int_{i \in [0,1], \mu^R} \theta_s^i di = \frac{1}{\gamma} \int_{i, \mu^R} [\tau^X \bar{X} + n_s^i \tau^S (X + \bar{\epsilon}_s^i + b f_s^{bi}) + \tau_s^Z \bar{Z}_s - (\tau^X + n_s^i \tau^S + \tau_s^Z) v] di \\
&= \frac{1}{\gamma} \sum_n \int_{i:n^i=n, \mu^R} [\tau^X \bar{X} + n \tau^S (X + \bar{\epsilon}_s^i + b f_s^{bi}) + \tau_s^Z \bar{Z}_s - (\tau^X + n \tau^S + \tau_s^Z) v] di \\
&= \frac{(\tau_s^Z \bar{Z}_s + \tau^X \bar{X})}{\gamma} + \frac{1}{\gamma} \sum_n \mu_s^R(n) [n \tau^S (X + b h^b(n)) - (\tau^X + n \tau^S + \tau_s^Z) v] \\
&= \frac{1}{\gamma} [\tau_s^Z \bar{Z}_s + \tau^X \bar{X} + \tau^S (\phi_s^R X + b \bar{B}_s) - (\tau^X + \phi_s^R \tau^S + \tau_s^Z) v] \\
&= \frac{1}{\gamma} [\tau_s^Z \bar{Z}_s + \tau^X \bar{X} + \phi_s^R \tau^S (X + b f_s^b) - (\tau^X + \phi_s^R \tau^S + \tau_s^Z) v], \tag{A.6}
\end{aligned}$$

where as a reminder,  $\bar{B}_s$  is the per capita number of biases per agent, and  $f_s^b = \bar{B}_s / \phi_s^R$  is the per-signal bias count. So the aggregate amount of the activity is linear in the product of the per capita number of signals and the per-signal bias.

The second equation follows directly by setting  $b = 0$ , subtracting, and dividing.  $\parallel$

**Proof of Proposition 3:** The agent has measure zero, and so does not affect the equilibrium. The expected aggregate action is given by equation (17).

For the deviant agent, by (15), bearing in mind the independence of number of signals  $n_s^i$ , fundamental  $X$ , and signal noise,

$$\begin{aligned}\gamma \mathbb{E}^R[\theta_s^i] &= \mathbb{E}^R[\tau^X(\bar{X} - v) + n_s^i \tau^S (X + \bar{\epsilon}^i + b f_s^{bi} - v) + \tau^Z(\bar{Z} - v)] \\ &= \tau^X(\bar{X} - v) + \tau^S b \mathbb{E}^R[n_s^i f_s^{bi}] + \tau^S \mathbb{E}^R[n_s^i](\bar{X} - v) + \tau^Z(\bar{Z} - v). \quad (\text{A.7})\end{aligned}$$

The number of signals possessed by  $i$  is greater than that of other agents in a first order stochastic dominance sense, owing to the higher meeting arrival rate. Furthermore,  $f^{bi}$  is increasing in  $n^i$  by equation (A.3). So the product  $n_s^i f_s^{bi}$ , the number of biases, is also higher in a FOSD sense for  $i$  than for other agents. It follows that  $\mathbb{E}[n_s^i f_s^{bi}] > \phi_s^R f_s^b$ , and  $\mathbb{E}^R[n_s^i] > \phi_s^R$  so  $\mathbb{E}^R[\Theta_s] > \mathbb{E}^R[\theta_s^i]$ .  $\parallel$

**Proof of Proposition 4:** For visual clarity I omit  $s$  subscripts. Recalling that  $\xi$  is the normalized price, substituting into optimality condition (3), using the conditional precision as in (25) and the definitions of  $\lambda$ 's in (26), an agent's perceived optimal position satisfies

$$\begin{aligned}\gamma \left[ \tau^X + n\tau^S + \tau^Z + \left( \frac{\alpha^X}{\alpha^\Theta} \right)^2 \tau^\Theta \right]^{-1} \theta^i &= \mathbb{E}[X|\Phi^i] - v \\ &\stackrel{B}{=} \lambda^S \bar{Y}^i + \lambda^P \bar{Z} + \lambda^\xi \xi - v \\ &= \lambda^S (X + \bar{\epsilon}^i) + \lambda^P \bar{Z} + \lambda^\xi \xi - v \\ &= \left[ \frac{n^i \tau^S}{\tau^X + n^i \tau^S + \tau^Z + \left( \frac{\alpha^X}{\alpha^\Theta} \right)^2 \tau^\Theta} \right] (X + \bar{\epsilon}^i) + \\ &\quad + \left[ \frac{\tau^Z}{\tau^X + n^i \tau^S + \tau^Z + \left( \frac{\alpha^X}{\alpha^\Theta} \right)^2 \tau^\Theta} \right] \bar{Z} \\ &\quad + \left[ \frac{\left( \frac{\alpha^X}{\alpha^\Theta} \right)^2 \tau^\Theta}{\tau^X + n^i \tau^S + \tau^Z + \left( \frac{\alpha^X}{\alpha^\Theta} \right)^2 \tau^\Theta} \right] \left[ \frac{v}{\alpha^X} - \left( \frac{\alpha^P}{\alpha^X} \right) \bar{Z} \right] - v.\end{aligned} \quad (\text{A.8})$$

Multiplying by the LHS bracketed expression gives the condition for agent  $i$ 's position,

$$\gamma\theta^i = n^i\tau^S (X + \bar{\epsilon}^i) + \left[ \tau^Z - \frac{\alpha^P \alpha^X \tau^\Theta}{(\alpha^\Theta)^2} \right] \bar{Z} + \left[ \left( \frac{\alpha^X}{(\alpha^\Theta)^2} \right) \tau^\Theta (1 - \alpha^X) - \tau^X - n^i \tau^S - \tau^Z \right] v. \quad (\text{A.9})$$

To solve for the perceived pricing coefficients, I impose market clearing by integrating demands over agents  $i$ . For any given number of private signals, signal noise errors (i.e., the  $\epsilon$  terms) average to zero across infinite groups of agents i.e.,  $\int_{i:n^i=n} \bar{\epsilon}^i di = 0$ , where  $\bar{\epsilon}^i$  is as was defined in (12). So agents think that in equilibrium, at each date,

$$\begin{aligned} \gamma\Theta \stackrel{\mathcal{B}}{=} \int_{i \in [0,1], \mu} \gamma\theta^i di &= \int_{i \in [0,1], \mu} \left\{ n^i \tau^S (X + \bar{\epsilon}^i) + \left[ \tau^Z - \frac{\alpha^P \alpha^X \tau^\Theta}{(\alpha^\Theta)^2} \right] \bar{Z} \right. \\ &\quad \left. + \left[ \left( \frac{\alpha^X}{(\alpha^\Theta)^2} \right) \tau^\Theta (1 - \alpha^X) - \tau^X - n\tau^S - \tau^Z \right] v \right\} di \\ &= \left[ \tau^Z - \frac{\alpha^P \alpha^X \tau^\Theta}{(\alpha^\Theta)^2} \right] \bar{Z} \\ &\quad + \sum_n \mu(n) \left\{ n\tau^S X + \left[ \left( \frac{\alpha^X}{(\alpha^\Theta)^2} \right) \tau^\Theta (1 - \alpha^X) - \tau^X - n\tau^S - \tau^Z \right] v \right\} \\ &= \left[ \tau^Z - \frac{\alpha^P \alpha^X \tau^\Theta}{(\alpha^\Theta)^2} \right] \bar{Z} + \phi\tau^S X + \left[ \left( \frac{\alpha^X}{(\alpha^\Theta)^2} \right) \tau^\Theta (1 - \alpha^X) - \tau^X - \phi\tau^S - \tau^Z \right] v. \end{aligned} \quad (\text{A.10})$$

Solving for price,

$$v \stackrel{\mathcal{B}}{=} \frac{\gamma\Theta - \left[ \tau^Z - \frac{\alpha^P \alpha^X \tau^\Theta}{(\alpha^\Theta)^2} \right] \bar{Z} - \phi\tau^S X}{\left( \frac{\alpha^X}{(\alpha^\Theta)^2} \right) \tau^\Theta (1 - \alpha^X) - \tau^X - \phi\tau^S - \tau^Z}. \quad (\text{A.11})$$

Equating corresponding coefficients on the shocks  $\Theta$ ,  $X$  and  $\bar{Z}$  in the agents' price conjecture (22) gives a system of equations that can be solved for the coefficients of the perceived equilibrium price,

$$\alpha^\Theta = \frac{\gamma}{\left( \frac{\alpha^X}{(\alpha^\Theta)^2} \right) \tau^\Theta (1 - \alpha^X) - (\tau^X + \phi\tau^S + \tau^Z)} \quad (\text{A.12})$$

$$\alpha^P = \frac{\frac{\alpha^P \alpha^X \tau^\Theta}{(\alpha^\Theta)^2} - \tau^Z}{\left( \frac{\alpha^X}{(\alpha^\Theta)^2} \right) \tau^\Theta (1 - \alpha^X) - (\tau^X + \phi\tau^S + \tau^Z)} \quad (\text{A.13})$$

$$\alpha^X = -\frac{\phi\tau^S}{\left( \frac{\alpha^X}{(\alpha^\Theta)^2} \right) \tau^\Theta (1 - \alpha^X) - (\tau^X + \phi\tau^S + \tau^Z)}. \quad (\text{A.14})$$

Dividing (A.12) by (A.14) gives

$$\alpha^\Theta = -\frac{\gamma}{\phi\tau^S}\alpha^X. \quad (\text{A.15})$$

Substituting this expression for  $\alpha^\Theta$  into (A.14) and solving for  $\alpha^X$  verifies (28). It follows immediately by (A.15) that  $\alpha^\Theta$  satisfies (29).

To solve for  $\alpha^P$ , replace  $\alpha^X/\alpha^\Theta$  in the numerator of (A.13) with  $-\phi\tau^S/\gamma$  (by (A.15)) to obtain

$$\alpha^P = \frac{-\frac{\phi\tau^S\alpha^P\tau^\Theta}{\gamma\alpha^\Theta} - \tau^Z}{\left(\frac{\alpha^X}{(\alpha^\Theta)^2}\right)\tau^\Theta(1 - \alpha^X) - (\tau^X + \phi\tau^S + \tau^Z)}. \quad (\text{A.16})$$

Dividing this equation by (A.12) gives

$$\frac{\alpha^P}{\alpha^\Theta} = -\frac{\alpha^P\phi\tau^S\tau^\Theta}{\alpha^\Theta\gamma^2} - \frac{\tau^Z}{\gamma}. \quad (\text{A.17})$$

Solving for  $\alpha^P/\alpha^\Theta$  gives

$$\frac{\alpha^P}{\alpha^\Theta} = -\frac{\gamma\tau^Z}{\gamma^2 + \phi\tau^S\tau^\Theta}. \quad (\text{A.18})$$

So using (29) we obtain (30).  $\parallel$

**Proof of Proposition 5:** Suppressing  $s$  subscripts, in first order condition (A.8), we replace the mistaken perception of the agent's average private signal value,  $X + \bar{\epsilon}^i$ , with  $X + \bar{\epsilon}^i + bf^{bi}$  to obtain

$$\begin{aligned} \gamma\theta^i &= n^i\tau^S(X + \bar{\epsilon}^i + bf^{bi}) + \tau^Z\bar{Z} + \left(\frac{\alpha^X}{\alpha^\Theta}\right)^2\tau^\Theta\left(\frac{v - \alpha^P\bar{Z}}{\alpha^X}\right) \\ &\quad - v\left[\tau^X + n^i\tau^S + \tau^Z + \left(\frac{\alpha^X}{\alpha^\Theta}\right)^2\tau^\Theta\right] \\ &= n^i\tau^S(X + \bar{\epsilon}^i + bf^{bi}) + \left[\tau^Z - \left(\frac{\alpha^X\alpha^P}{(\alpha^\Theta)^2}\right)\tau^\Theta\right]\bar{Z} \\ &\quad - \left\{\tau^X + n^i\tau^S + \tau^Z + \left(\frac{\alpha^X}{(\alpha^\Theta)^2}\right)(\alpha^X - 1)\tau^\Theta\right\}v. \end{aligned} \quad (\text{A.19})$$

Integrating across agents introduces the rational assessment of per capita signal count

and per-signal bias. Bearing in mind that  $f^{bi}$  depends only on  $n^i$ ,

$$\begin{aligned}
\gamma^\Theta &= \int_{i \in [0,1], \mu^R} \gamma \theta^i di \\
&= \int_{i \in [0,1], \mu^R} \left[ n^i \tau^S (X + \bar{\epsilon}^i + b f^{bi}) + \left[ \tau^Z - \left( \frac{\alpha^X \alpha^P}{(\alpha^\Theta)^2} \right) \tau^\Theta \right] \bar{Z} \right. \\
&\quad \left. - \left\{ \tau^X + n^i \tau^S + \tau^Z + \left( \frac{\alpha^X}{(\alpha^\Theta)^2} \right) (\alpha^X - 1) \tau^\Theta \right\} v \right] di \\
&= \left[ \tau^Z - \left( \frac{\alpha^X \alpha^P}{(\alpha^\Theta)^2} \right) \tau^\Theta \right] \bar{Z} \\
&\quad + \sum_n \mu^R(n) \left( n \tau^S (X + b h^b(n)) - \left\{ \tau^X + n \tau^S + \tau^Z + \left( \frac{\alpha^X}{(\alpha^\Theta)^2} \right) (\alpha^X - 1) \tau^\Theta \right\} v \right) \\
&= \left[ \tau^Z - \left( \frac{\alpha^X \alpha^P}{(\alpha^\Theta)^2} \right) \tau^\Theta \right] \bar{Z} + \tau^S (\phi^R X + b \bar{B}) \\
&\quad - \left\{ \tau^X + \phi^R \tau^S + \tau^Z + \left( \frac{\alpha^X}{(\alpha^\Theta)^2} \right) (\alpha^X - 1) \tau^\Theta \right\} v \\
&= \left[ \tau^Z - \left( \frac{\alpha^X \alpha^P}{(\alpha^\Theta)^2} \right) \tau^\Theta \right] \bar{Z} + \phi^R \tau^S (X + b f^b) \\
&\quad - \left\{ \tau^X + \phi^R \tau^S + \tau^Z + \left( \frac{\alpha^X}{(\alpha^\Theta)^2} \right) (\alpha^X - 1) \tau^\Theta \right\} v, \tag{A.20}
\end{aligned}$$

where as a reminder,  $\bar{B}$  is the per capita number of biases per agent, and  $f^b = \bar{B}/\phi^R$  is the per-signal bias.

Solving for the equilibrium market price gives

$$v = \frac{\left[ \tau^Z - \left( \frac{\alpha^X \alpha^P}{(\alpha^\Theta)^2} \right) \tau^\Theta \right] \bar{Z} + \phi^R \tau^S (X + b f^b) - \gamma^\Theta}{\tau^X + \phi^R \tau^S + \tau^Z + \left( \frac{\alpha^X}{(\alpha^\Theta)^2} \right) (\alpha^X - 1) \tau^\Theta}. \tag{A.21}$$

Equation (A.21) contains  $\alpha^X/\alpha^\Theta$  and  $\alpha^P/\alpha^\Theta$ , ratios of mistaken coefficients that agents have in their price conjectures. The first of these, by (A.15) is  $-\phi \tau^S/\gamma$ . The second of these is given by (A.18). Also, by (28) and (29),

$$\frac{\alpha^X - 1}{\alpha^\Theta} = \frac{\gamma (\tau^X + \tau^Z)}{\gamma^2 + \phi \tau^S \tau^\Theta}. \tag{A.22}$$

So we can substitute out all  $\alpha$  ratios in (A.21), which yields (31). ||

**Proof of Proposition 6:** By inspection of (31) in Proposition 5, the actual  $\alpha$  coeffi-

cients are

$$\alpha_s^{RX} = \alpha_s^{Rb} = \frac{\phi_s^R \tau^S (\gamma^2 + \phi_s \tau^S \tau^\Theta)}{\gamma^2 (\tau^X + \phi_s^R \tau^S + \tau_s^Z) + \phi_s \phi_s^R (\tau^S)^2 \tau^\Theta} \quad (\text{A.23})$$

$$\alpha_s^{RP} = \frac{\tau_s^Z \gamma^2}{\gamma^2 (\tau^X + \phi_s^R \tau^S + \tau_s^Z) + \phi_s \phi_s^R (\tau^S)^2 \tau^\Theta} \quad (\text{A.24})$$

$$\alpha_s^{R\Theta} = -\frac{\gamma (\gamma^2 + \phi_s \tau^S \tau^\Theta)}{\gamma^2 (\tau^X + \phi_s^R \tau^S + \tau_s^Z) + \phi_s \phi_s^R (\tau^S)^2 \tau^\Theta}. \quad (\text{A.25})$$

So

$$\begin{aligned} \frac{\alpha_s^{RX}}{\alpha_s^X} &= \left[ \frac{\phi_s^R \tau^S (\gamma^2 + \phi_s \tau^S \tau^\Theta)}{\phi_s \tau^S (\gamma^2 + \phi_s \tau^S \tau^\Theta)} \right] \left[ \frac{\phi_s^2 (\tau^S)^2 \tau^\Theta + \gamma^2 (\tau^X + \phi_s \tau^S + \tau_s^Z)}{\phi_s^R \phi_s (\tau^S)^2 \tau^\Theta + \gamma^2 (\tau^X + \phi_s^R \tau^S + \tau_s^Z)} \right] \\ &= \frac{\phi_s^R}{\phi_s} \left[ \frac{\phi_s^2 (\tau^S)^2 \tau^\Theta + \gamma^2 (\tau^X + \phi_s \tau^S + \tau_s^Z)}{\phi_s^R \phi_s (\tau^S)^2 \tau^\Theta + \gamma^2 (\tau^X + \phi_s^R \tau^S + \tau_s^Z)} \right] \\ &= 1 + \frac{\phi_s^R [\phi_s^2 (\tau^S)^2 \tau^\Theta + \gamma^2 (\tau^X + \phi_s \tau^S + \tau_s^Z)] - \phi_s [\phi_s^R \phi_s (\tau^S)^2 \tau^\Theta + \gamma^2 (\tau^X + \phi_s^R \tau^S + \tau_s^Z)]}{\phi_s [\phi_s^R \phi_s (\tau^S)^2 \tau^\Theta + \gamma^2 (\tau^X + \phi_s^R \tau^S + \tau_s^Z)]} \\ &= 1 + \frac{(\phi_s^R - \phi_s) \gamma^2 (\tau^X + \tau_s^Z)}{\phi_s [\phi_s^R \phi_s (\tau^S)^2 \tau^\Theta + \gamma^2 (\tau^X + \phi_s^R \tau^S + \tau_s^Z)]}. \end{aligned} \quad (\text{A.26})$$

The term  $\phi_s^R - \phi_s \gtrless 0$  iff  $\kappa \gtrless 1$ . All other terms are also positive, so  $\alpha_s^{RX}/\alpha_s^X \gtrless 1$  iff  $\kappa \gtrless 1$ . Similarly,

$$\begin{aligned} \frac{\alpha_s^{RP}}{\alpha_s^P} &= \left( \frac{\tau_s^Z \gamma^2}{\tau_s^Z \gamma^2} \right) \left[ \frac{\phi_s^2 (\tau^S)^2 \tau^\Theta + \gamma^2 (\tau^X + \phi_s \tau^S + \tau_s^Z)}{\phi_s^R \phi_s (\tau^S)^2 \tau^\Theta + \gamma^2 (\tau^X + \phi_s^R \tau^S + \tau_s^Z)} \right] \\ &= \frac{\phi_s^2 (\tau^S)^2 \tau^\Theta + \gamma^2 (\tau^X + \phi_s \tau^S + \tau_s^Z)}{\phi_s^R \phi_s (\tau^S)^2 \tau^\Theta + \gamma^2 (\tau^X + \phi_s^R \tau^S + \tau_s^Z)} \\ &= 1 + \frac{[\phi_s^2 (\tau^S)^2 \tau^\Theta + \gamma^2 (\tau^X + \phi_s \tau^S + \tau_s^Z)] - [\phi_s^R \phi_s (\tau^S)^2 \tau^\Theta + \gamma^2 (\tau^X + \phi_s^R \tau^S + \tau_s^Z)]}{\phi_s^R \phi_s (\tau^S)^2 \tau^\Theta + \gamma^2 (\tau^X + \phi_s^R \tau^S + \tau_s^Z)} \\ &= 1 + \frac{(\phi_s - \phi_s^R) \tau^S (\gamma^2 + \phi_s \tau^S \tau^\Theta)}{\phi_s^R \phi_s (\tau^S)^2 \tau^\Theta + \gamma^2 (\tau^X + \phi_s^R \tau^S + \tau_s^Z)}. \end{aligned} \quad (\text{A.27})$$

So  $\alpha_s^{RP}/\alpha_s^P \gtrless 1$  iff  $\kappa \gtrless 1$ .

Finally, it is easy to see that  $\alpha_s^{R\Theta}/\alpha_s^\Theta = \alpha_s^{RP}/\alpha_s^P$ , since  $\alpha_s^{R\Theta}$  and  $\alpha_s^\Theta$  also have the same numerators. ||

**Proof of Proposition 7:** Let “ $t-$ ” denote a date just before discrete date  $t$ , and recall

$\tau_s^Z$  as given in (1). By (31), the expected date- $t$  price change conditional only upon  $b$  is

$$\begin{aligned}\mathbb{E}^R[v_t - v_{t-}|b] &= \frac{(\gamma^2 + \phi_t \tau^S \tau^\Theta) \phi_t^R \tau^S b f_t^b}{\gamma^2(\tau^X + \phi_t^R \tau^S + e^{\zeta t} \tau^P) + \phi_t \phi_t^R (\tau^S)^2 \tau^\Theta} - \frac{(\gamma^2 + \phi_t \tau^S \tau^\Theta) \phi_t^R \tau^S b f_t^b}{\gamma^2(\tau^X + \phi_t^R \tau^S + e^{\zeta(t-1)} \tau^P) + \phi_t \phi_t^R (\tau^S)^2 \tau^\Theta} \\ &= \frac{(\gamma^2 + \phi_t \tau^S \tau^\Theta) \phi_t^R \tau^S b f_t^b \gamma^2 (e^{\zeta(t-1)} - e^{\zeta t}) \tau^P}{[\gamma^2(\tau^X + \phi_t^R \tau^S + e^{\zeta t} \tau^P) + \phi_t \phi_t^R (\tau^S)^2 \tau^\Theta][\gamma^2(\tau^X + \phi_t^R \tau^S + e^{\zeta(t-1)} \tau^P) + \phi_t \phi_t^R (\tau^S)^2 \tau^\Theta]}.\end{aligned}\tag{A.28}$$

Since  $\zeta > 0$ , this expression is negatively proportional to  $b$ .  $\parallel$

**Proof of Proposition 8:** By Proposition 7,  $\text{Sign}(\mathbb{E}^R[\Delta v_t|b]) = -\text{Sign}(b)$ . By (32), the expected long-run post-event price change conditional on  $b$  at any date  $s$  is

$$\begin{aligned}\mathbb{E}^R[X - v_s|b] &= -\mathbb{E}^R[v_s|b] \\ &= -\frac{(\gamma^2 + \phi_s \tau^S \tau^\Theta) \phi_s^R \tau^S b f_s^b}{\gamma^2(\tau^X + \phi_s^R \tau^S + \tau_s^Z) + \phi_s \phi_s^R (\tau^S)^2 \tau^\Theta},\end{aligned}\tag{A.29}$$

so  $\text{Sign}(\mathbb{E}^R[X - v_s|b]) = -\text{Sign}(b) = \text{Sign}(\mathbb{E}^R[\Delta v_t|b])$ . Inspection of (A.28) and (A.29) shows that both expectations are negatively proportional to  $b$ .  $\parallel$

**Proof of Proposition 9:** By (31), both  $v_t$  and  $v_{t-}$  contain  $b$  in the numerator. Since  $b$  is independent of all other exogenous variables, the contribution of  $\mathbb{V}(b)$  to the covariance comes from the  $b$  terms in the numerators, with the denominators as multiplied constants. Letting  $K > 0$  denote the denominator of  $v_{t-}$ , the denominator of  $v_t$  is  $K + \gamma^2 \tau^P$ . Then the coefficient on  $b$  in  $v_t$  is

$$\frac{(\gamma^2 + \phi_t \tau^S \tau^\Theta) \phi_t^R \tau^S f_t^b}{K + \gamma^2 \tau^P}\tag{A.30}$$

and in  $v_{t-}$  is

$$\frac{(\gamma^2 + \phi_t \tau^S \tau^\Theta) \phi_t^R \tau^S f_t^b}{K}.\tag{A.31}$$

Since the numerators are identical, the coefficient on  $b$  in  $v_t - v_{t-}$  is negative. Since the coefficient on  $b$  is negative in  $X - v_t$ , when  $\mathbb{V}(b)$  is sufficiently large, the covariance is positive.  $\parallel$

**Proof of Proposition 10:** Consider a random sample of public news dates. Consider first the case  $b > 0$ . Let the terminal price be defined as  $v_T \equiv X$ . By date 1, the stock is, on average, overpriced,  $\mathbb{E}^R[X|b] < \mathbb{E}^R[v_1|b]$ , which implies that

$$0 > \frac{\mathbb{E}^R[X - v_1|b]}{T - 1} = \frac{\mathbb{E}^R \left[ \sum_{t=1}^{T-1} (v_{t+1} - v_t) \middle| b \right]}{T - 1}.\tag{A.32}$$

The RHS is the expected value of  $v_{t+1} - v_t$  in a random sample of public event dates from  $t = 1 \dots T - 1$ , i.e., the expected short horizon drift after these dates. The sign of the expectations in (A.32) is reversed when  $b < 0$ . Furthermore, by Proposition 8, the LHS is negatively proportional to  $b$ , so the RHS is as well.

The expected average event-date reaction in such a sample is  $\mathbb{E}^R[\Delta v_t | b] < 0$ , with opposite sign when  $b < 0$ ; and is negatively proportional to  $b$ , by Proposition 7. So conditional on  $b$ , the expected event-date and short-horizon reactions have the same sign. ||

**Proof of Proposition 11:** Let  $\Delta$  be a short time interval. We seek to sign

$$\text{cov}^R(v_{s+\Delta} - v_s, v_{s+2\Delta} - v_{s+\Delta}) = \mathbb{E}^R[(v_{s+\Delta} - v_s)(v_{s+2\Delta} - v_{s+\Delta})], \quad (\text{A.33})$$

where the equality holds because  $\mathbb{E}[v_s] = 0$  for all  $s$ . Since  $\mathbb{V}(b)$  is large, to an arbitrarily good approximation, in substituting for dated  $v$  values from (31) into (A.33), we can consider only the coefficients on  $b$ . Let the relevant terms from the numerator and the denominator, from (31) be denoted

$$\begin{aligned} g(s) &\equiv (\gamma^2 + \phi_s \tau^S \tau^\Theta) \tau^S \phi_s^R f_s^b \\ h(s) &\equiv \gamma^2 (\tau^X + \phi_s^R \tau^S + \tau_s^Z) + \phi_s \phi_s^R (\tau^S)^2 \tau^\Theta, \end{aligned} \quad (\text{A.34})$$

When  $\mathbb{V}(b)$  is large and  $\Delta$  small, two-term Taylor expansions applied to the factors of the expectation give the approximation

$$\mathbb{E}^R[(v_{s+\Delta} - v_s)(v_{s+2\Delta} - v_{s+\Delta})] \approx \left( \frac{g+g'\Delta+\frac{1}{2}g''\Delta^2}{h+h'\Delta+\frac{1}{2}h''\Delta^2} - \frac{g}{h} \right) \left( \frac{g+g'2\Delta+\frac{1}{2}g''4\Delta^2}{h+h'2\Delta+\frac{1}{2}h''4\Delta^2} - \frac{g+g'\Delta+\frac{1}{2}g''\Delta^2}{h+h'\Delta+\frac{1}{2}h''\Delta^2} \right) \mathbb{V}(b).$$

I abbreviate this covariance as  $\text{cov}^R$ , suppress  $s$  subscripts. Since the terms of order  $\Delta$  cancel, I remove terms of order higher than  $\Delta^2$ , to obtain

$$\begin{aligned} \text{cov}^R &\approx \frac{(g'h - h'g)^2 \Delta^2}{h^4 + 4h^3 h' \Delta + 5h^2 h'^2 \Delta^2 + 3h^3 h'' \Delta^2} \mathbb{V}(b) \\ &\approx \frac{(g'h - h'g)^2 \Delta^2}{h^4} \mathbb{V}(b) > 0. \end{aligned} \quad (\text{A.35})$$

(This covariance of price changes of course approaches zero as  $\Delta \rightarrow 0$ , but the autocorrelation, which is scaled by standard deviations, does not.) ||

## References

- Aboudy, D., R. Lehavy, and B. Trueman (2010, June). Limited attention and the earnings announcement returns of past stock market winners. *Review of Accounting Studies* 15(2), 317–344.
- Acemoglu, D., M. Dahleh, I. Lobel, and A. E. Ozdaglar (2011, October). Bayesian learning in social networks. *Review of Economic Studies* 78(4), 1201–1236.
- Allport, G. W. and L. Postman (1947). *The Psychology of Rumor*. New York, NY: Henry Holt.
- Ammann, M. and N. Schaub (2016, June). Social interaction and investing: Evidence from an online social trading network. Working paper, University of St. Gallen.
- Andrei, D. and J. Cujean (2017, March). Information percolation, momentum and reversal. *Journal of Financial Economics* 123(3), 617–645.
- Bailey, M., E. Dávila, T. Kuchler, and J. Stroebel (2019, November). House price beliefs and mortgage leverage choice. *Review of Economic Studies* 86(6), 2403–2452.
- Baker, M. and J. Wurgler (2006, August). Investor sentiment and the cross-section of stock returns. *Journal of Finance* 61(4), 1645–1680.
- Baker, M. and J. Wurgler (2007, Spring). Investor sentiment in the stock market. *Journal of Economic Perspectives* 21(2), 129–151.
- Banerjee, A. (1992, September). A simple model of herd behavior. *Quarterly Journal of Economics* 107(3), 797–817.
- Barber, B. and T. Odean (2000, April). Trading is hazardous to your wealth: The common stock investment performance of individual investors. *Journal of Finance* 55(2), 773–806.
- Barber, B. M. and T. Odean (2008, April). All that glitters: The effect of attention and news on the buying behavior of individual and institutional investors. *Review of Financial Studies* 21(2), 785–818.
- Barberis, N., R. Greenwood, L. Jin, and A. Shleifer (2018, August). Extrapolation and bubbles. *Journal of Financial Economics* 129(2), 203–227.
- Barberis, N., A. Shleifer, and R. Vishny (1998, September). A model of investor sentiment. *Journal of Financial Economics* 49(3), 307–343.
- Barro, R. J. and R. M. McCleary (2003, October). Religion and economic growth across countries. *American Sociological Review* 68(5), 760–781.
- Baumeister, R. F., E. Bratslavsky, C. Finkenauer, and K. D. Vohs (2001, December). Bad is stronger than good. *Review of General Psychology* 5(4), 323–370.
- Bénabou, R. and J. Tirole (2002, August). Self-confidence and personal motivation. *Quarterly Journal of Economics* 117(3), 871–915.
- Bénabou, R. and J. Tirole (2006, May). Belief in a just world and redistributive politics. *Quarterly Journal of Economics* 121(2), 699–746.

- Bénabou, R. and J. Tirole (2016, Summer). Mindful economics: The production, consumption, and value of beliefs. *Journal of Economic Perspectives* 30(3), 141–164.
- Berger, J. (2014, October). Word of mouth and interpersonal communication: A review and directions for future research. *Journal of Consumer Psychology* 24(4), 586–607.
- Berger, J. and K. L. Milkman (2012, April). What makes online content viral? *Journal of Marketing Research* 49(2), 192–205.
- Bikhchandani, S., D. Hirshleifer, and I. Welch (1992, October). A theory of fads, fashion, custom, and cultural change as informational cascades. *Journal of Political Economy* 100(5), 992–1026.
- Bisin, A. and T. Verdier (2000). Beyond the melting pot: Cultural transmission, marriage, and the evolution of ethnic and religious traits. *Quarterly Journal of Economics* 115(3), 955–988.
- Bizjak, J. M., M. L. Lemmon, and R. J. Whitby (2009, November). Option backdating and board interlocks. *Review of Financial Studies* 22(11), 4821–4847.
- Bohren, J. A. (2016, May). Informational herding with model misspecification. *Journal of Economic Theory* 163, 222–247.
- Bouchaud, J.-P., P. Krüger, A. Landier, and D. Thesmar (2019, April). Sticky expectations and the profitability anomaly. *Journal of Finance* 74(2), 639–674.
- Boyer, P. and M. B. Petersen (2018). Folk-economic beliefs: An evolutionary cognitive model. *Behavioral and Brain Sciences* 41, e158.
- Brock, W. A. and S. N. Durlauf (2001, April). Discrete choice with social interactions. *Review of Economic Studies* 68(2), 235–260.
- Burnside, C., M. Eichenbaum, and S. T. Rebelo (2016, August). Understanding booms and busts in housing markets. *Journal of Political Economy* 124(4), 1088–1147.
- Camerer, C. F., T.-H. Ho, and J.-K. Chong (2004, August). A cognitive hierarchy model of games. *Quarterly Journal of Economics* 119(3), 861–898.
- Caplan, B. (2007). *The Myth of the Rational Voter: Why Democracies Choose Bad Policies*. Princeton, NJ: Princeton University Press.
- Chinco, A. (2019, September). The madness of crowds and the likelihood of bubbles. Working paper, University of Chicago.
- Chiu, P.-C., S. H. Teoh, and F. Tian (2013, May). Board interlocks and earnings management contagion. *The Accounting Review* 88(3), 915–944.
- Craik, K. J. W. (1943). *The Nature of Explanation*. Cambridge, UK: Cambridge University Press.
- Cujean, J. (2020, January). Idea sharing and the performance of mutual funds. *Journal of Financial Economics* 135(1), 88–119.

- D'Acunto, F., A. Rossi, and M. Weber (2019, February). Crowdsourcing financial information to change spending behavior. Working paper, Boston College.
- Daniel, K., D. Hirshleifer, and A. Subrahmanyam (1998, December). Investor psychology and security market under- and overreactions. *Journal of Finance* 53(6), 1839–1885.
- DeLong, J. B., A. Shleifer, L. Summers, and R. J. Waldmann (1990). Positive feedback investment strategies and destabilizing rational speculation. *Journal of Finance* 45(2), 375–395.
- DeMarzo, P., D. Vayanos, and J. Zwiebel (2001). Social networks and financial markets. Working paper, MIT and Stanford University.
- DeMarzo, P., D. Vayanos, and J. Zwiebel (2003). Persuasion bias, social influence, and uni-dimensional opinions. *Quarterly Journal of Economics* 118, 909–968.
- Denzau, A. T. and D. C. North (1994, February). Shared mental models: ideologies and institutions. *Kyklos* 47(1), 3–31.
- Dong, M., D. A. Hirshleifer, and S. H. Teh (2012, December). Overvalued equity and financing decisions. *Review of Financial Studies* 25(12), 3645–3683.
- Duffie, D. (2010, August). Presidential address: Asset price dynamics with slow-moving capital. *Journal of Finance* 65(4), 1237–1267.
- Duffie, D., S. Malamud, and G. Manso (2009, October). Information percolation with equilibrium search dynamics. *Econometrica* 77(5), 1513–1574.
- Duffie, D. and Y. Sun (2007, February). Existence of independent random matching. *Annals of Applied Probability* 17(1), 386–419.
- Duflo, E. and E. Saez (2002, July). Participation and investment decisions in a retirement plan: The influence of colleagues' choices. *Journal of Public Economics* 85(1), 121–148.
- Duflo, E. and E. Saez (2003, August). The role of information and social interactions in retirement plan decisions: Evidence from a randomized experiment. *Quarterly Journal of Economics* 118(3), 815–842.
- Ellison, G. and D. Fudenberg (1993, August). Rules of thumb for social learning. *Journal of Political Economy* 101(4), 612–643.
- Escobar, L. and A. Pedraza (2019). Active trading and (poor) performance. Working paper, World Bank.
- Eyster, E. and M. Rabin (2005, September). Cursed equilibrium. *Econometrica* 73(5), 1623–1672.
- Eyster, E. and M. Rabin (2010, November). Naïve herding in rich-information settings. *American Economic Journal: Microeconomics* 2(4), 221–243.
- Eyster, E., M. Rabin, and D. Vayanos (2018, October). Financial markets where traders neglect the informational content of prices. *Journal of Finance* 74(1), 371–399.

- Fracassi, C. (2016, May). Corporate finance policies and social networks. *Management Science* 63(8), 2420–2438.
- Glaeser, E. L., B. Sacerdote, and J. A. Scheinkman (1996, May). Crime and social interactions. *Quarterly Journal of Economics* 111(2), 507–548.
- Goetzmann, W. N. and D. Kim (2018, January). Negative bubbles: What happens after a crash. Working paper, Yale School of Management.
- Golub, B. and M. O. Jackson (2010, February). Naive learning in social networks and the wisdom of crowds. *American Economic Journal: Microeconomics* 2(1), 112–149.
- Golub, B. and E. Sadler (2016). Learning in social networks. In Y. Bramoullé, A. Galeotti, and B. W. Rogers (Eds.), *The Oxford Handbook of the Economics of Networks*, Chapter 19. New York, NY: Oxford University Press.
- Graham, J. R., J. Grennan, C. R. Harvey, and S. Rajgopal (2019, June). Corporate culture: Evidence from the field. Working paper, Duke University.
- Greenwood, R. and A. Shleifer (2014, March). Expectations of returns and expected returns. *Review of Financial Studies* 27(3), 714–746.
- Greenwood, R., A. Shleifer, and Y. You (2019, January). Bubbles for Fama. *Journal of Financial Economics* 131(1), 20–43.
- Grinblatt, M. and M. Keloharju (2001, June). How distance, language and culture influence stockholdings and trades. *Journal of Finance* 56(3), 1053–1073.
- Guiso, L., P. Sapienza, and L. Zingales (2003, January). People’s opium? Religion and economic attitudes. *Journal of Monetary Economics* 50(1), 225–282.
- Guiso, L., P. Sapienza, and L. Zingales (2004, June). The role of social capital in financial development. *American Economic Review* 94(3), 526–556.
- Han, B., D. Hirshleifer, and J. Walden (2019a, June). Social transmission bias and investor behavior. NBER Working Paper No. 24281.
- Han, B., D. Hirshleifer, and J. Walden (2019b, June). Visibility bias in the transmission of consumption beliefs and undersaving. Working paper, University of Toronto.
- He, H. and J. Wang (1995, October). Differential information and dynamic behavior of stock trading volume. *Review of Financial Studies* 8(4), 919–972.
- Heath, C. (1996). Do people prefer to pass along good or bad news? Valence and relevance as predictors of transmission propensity. *Organizational Behavior and Human Decision Process* 68, 79–94.
- Heimer, R. and D. Simon (2015, October). Facebook finance: How social interaction propagates active investing. Working paper, Brandeis University.
- Henrich, J. (2015). *The Secret of Our Success: How Culture Is Driving Human Evolution, Domesticating Our Species, and Making Us Smarter*. Princeton, NJ: Princeton University Press.

- Hirshleifer, D. (2008, November). Psychological bias as a driver of financial regulation. *European Financial Management* 14(5), 856–874.
- Hirshleifer, D. and O. Tamuz (2020). Sound bites: Communication bandwidth constraints and simplistic thinking. Work in progress.
- Hirshleifer, D. and S. H. Teoh (2003, March). Herd behavior and cascading in capital markets: A review and synthesis. *European Financial Management* 9(1), 25–66.
- Hirshleifer, D. and S. H. Teoh (2009, Winter). The psychological attraction approach to accounting and disclosure policy. *Contemporary Accounting Research* 26(4), 1067–1090.
- Hirshleifer, D. and S. H. Teoh (2018). Social transmission bias and the cultural evolution of folk economic beliefs. *Behavioral and Brain Sciences* 41, e170.
- Hoff, K. and J. E. Stiglitz (2016). Striving for balance in economics: Towards a theory of the social determination of behavior. *Journal of Economic Behavior & Organization* 126(PB), 25–57.
- Hong, H. G. and M. T. Kacperczyk (2009, July). The price of sin: The effects of social norms on markets. *Journal of Financial Economics* 93(1), 15–36.
- Hong, H. G. and L. Kostovetsky (2012, January). Red and blue investing: Values and finance. *Journal of Financial Economics* 103(1), 1–19.
- Hong, H. G., J. D. Kubik, and J. C. Stein (2004, February). Social interaction and stock market participation. *Journal of Finance* 59(1), 137–163.
- Hong, H. G. and J. C. Stein (1999, December). A unified theory of underreaction, momentum trading and overreaction in asset markets. *Journal of Finance* 54(6), 2143–2184.
- Huang, S., B.-H. Hwang, and D. Lou (2018, July). The speed of communication. Working paper, University of Hong Kong.
- Ikenberry, D., J. Lakonishok, and T. Vermaelen (1995, October). Market underreaction to open market share repurchases. *Journal of Financial Economics* 39(2-3), 181–208.
- Ikenberry, D., J. Lakonishok, and T. Vermaelen (2000). Stock repurchases in Canada: Performance and strategic trading. *Journal of Finance* 55(5), 2373–2397.
- Jackson, M. O. (2008). *Social and Economic Networks*. Princeton, NJ: Princeton University Press.
- Jegadeesh, N. (1990, July). Evidence of predictable behavior of security returns. *Journal of Finance* 45(3), 881–898.
- Johnson, P. N. (1983). *Mental Models*. Cambridge, UK: Cambridge University Press.
- Kelly, M. and C. O. O’Grada (2000). Market contagion: Evidence from the panics of 1854 and 1857. *American Economic Review* 90(5), 1110–1124.
- Laibson, D. (1997, May). Golden eggs and hyperbolic discounting. *Quarterly Journal of Economics* 112(2), 443–477.

- Lee, C., A. Shleifer, and R. Thaler (1991). Investor sentiment and the closed-end fund puzzle. *Journal of Finance* 46, 75–109.
- Lim, S. S., J. Ng, and B. Uzzi (2020, January). Biased information transmission in investor social network: Evidence from professional traders. Working paper, DePaul University.
- Linnainmaa, J. T. and C. Zhang (2019, February). The earnings announcement return cycle. Working paper, Dartmouth.
- Loughran, T. and J. Ritter (1995, March). The new issues puzzle. *Journal of Finance* 50(1), 23–52.
- Mesoudi, A., A. Whiten, and K. N. Laland (2006, November). Towards a unified science of cultural evolution. *Behavioral and Brain Sciences* 29(4), 329–347.
- Novy-Marx, R. (2013, April). The other side of value: The gross profitability premium. *Journal of Financial Economics* 108(1), 1–28.
- Payne, J., J. Bettman, and E. Johnson (1993). *The Adaptive Decisionmaker* (1st ed.). Cambridge, U.K.: Cambridge University Press.
- Peng, L. and W. Xiong (2006). Investor attention, overconfidence and category learning. *Journal of Financial Economics* 80(3), 563–602.
- Richardson, S., S. H. Teoh, and P. D. Wysocki (2004, Winter). The walk-down to beatable analyst forecasts: The role of equity issuance and insider trading incentives. *Contemporary Accounting Research* 21(4), 885–924.
- Ritter, J. R. and I. Welch (2002, August). A review of IPO activity, pricing, and allocations. *Journal of Finance* 57(4), 1795–1828.
- Rozin, P. and E. B. Royzman (2001, November). Negativity bias, negativity dominance, and contagion. *Personality and Social Psychology Review* 5(4), 296–320.
- Rubin, P. H. (2002). *Darwinian Politics: The Evolutionary Origin of Freedom*. New Brunswick, NJ: Rutgers University Press.
- Rubin, P. H. (2003, July). Folk economics. *Southern Economic Journal* 70(1), 157–171.
- Sandholm, W. H. (2010, December). *Population Games and Evolutionary Dynamics*. Cambridge, MA: MIT Press.
- Shiller, R. J. (1990, Spring). Speculative prices and popular models. *Journal of Economic Perspectives* 4(2), 55–65.
- Shiller, R. J. (2000a). *Irrational exuberance*. Princeton, NJ: Princeton University Press.
- Shiller, R. J. (2000b). Measuring bubble expectations and investor confidence. *Journal of Psychology and Financial Markets* 1(1), 49–60.
- Shiller, R. J. (2017, April). Narrative economics. *American Economic Review* 107(4), 967–1004.

- Shiller, R. J. (2019). *Narrative Economics: How Stories Go Viral and Drive Major Economic Events*. Princeton, NJ: Princeton University Press.
- Shiller, R. J., F. Kon-Ya, and Y. Tsutsui (1996, February). Why did the Nikkei crash? Expanding the scope of expectations data collection. *Review of Economics and Statistics* 78(1), 156–164.
- Shive, S. (2010, February). An epidemic model of investor behavior. *Journal of Financial and Quantitative Analysis* 45(1), 169–198.
- Tabellini, G. (2008, August). The scope of cooperation: Values and incentives. *Quarterly Journal of Economics* 123(3), 905–950.
- Tesser, A. and S. Rosen (1975). The reluctance to transmit bad news. In L. Berkowitz (Ed.), *Advances in Experimental Social Psychology*, Volume 8. New York, NY: Academic Press.
- Wahal, S. (2019, February). The profitability and investment premium: Pre-1963 evidence. *Journal of Financial Economics* 131(2), 362–377.
- Wright, W. F. and G. H. Bower (1992, July). Mood effects on subjective probability assessment. *Organizational Behavior and Human Decision Processes* 52(2), 276–291.

Figure 1: **Army ant death spiral.** Example of a dysfunctional emergent social outcome. Army ants continue to walk in circles until they die. Source: George Lin, <https://www.youtube.com/watch?v=N0HoqjxfvJ4>.

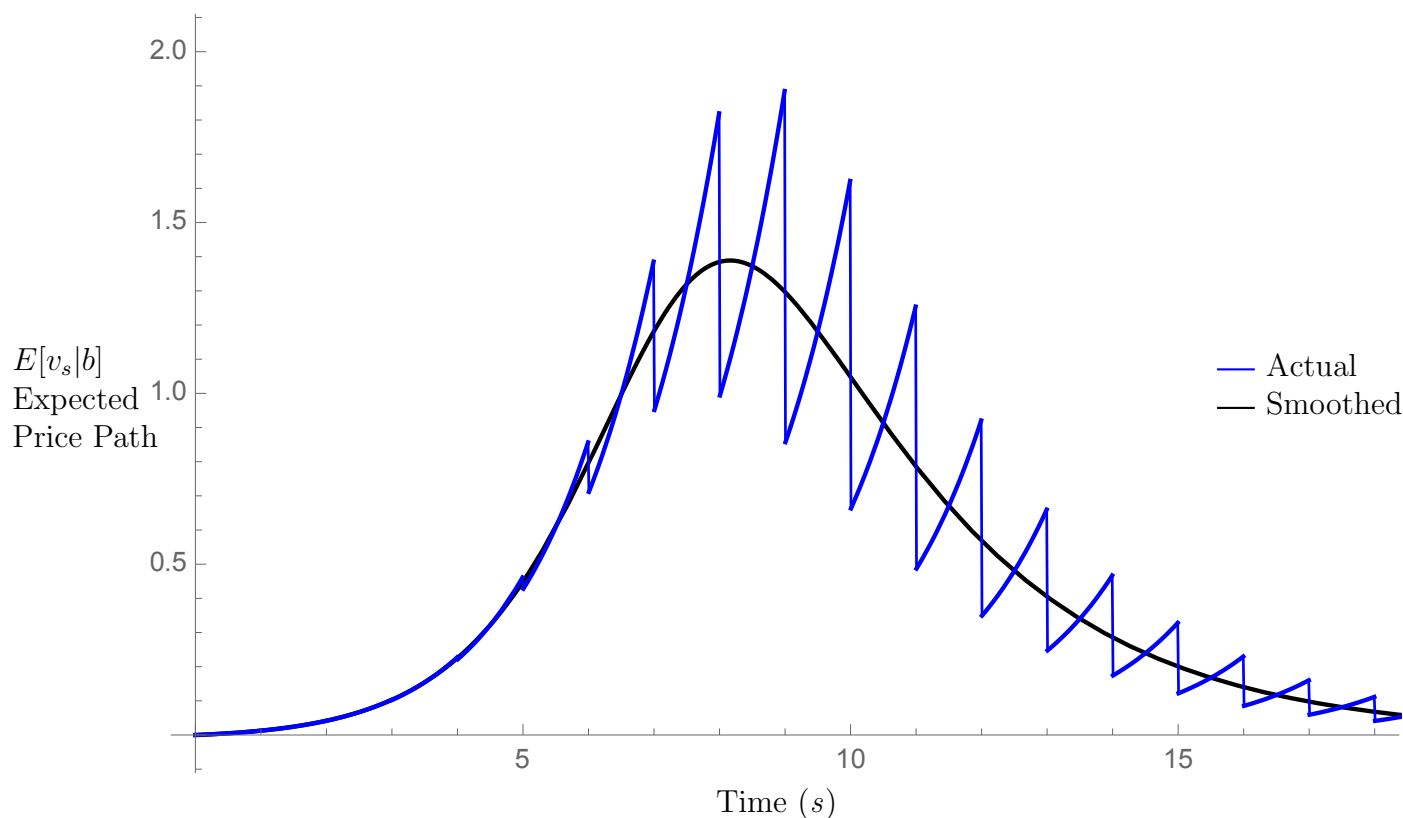


Figure 2: **Expected price path in the Price Bubbles Model.** The figure plots (32) and shows the smoothed and unsmoothed expected price paths for positive bubbles ( $b > 0$ ), where  $b$  is the transmission bias. These paths illustrate the following four phases: (i) the inception phase, (ii) the boom phase, (iii) the correction phase, and (iv) the terminal phase. As in (20), smoothed public precision is  $\hat{\tau}_s^Z \equiv e^{\zeta(s-0.5)}\tau^P$ , where  $\zeta$  parametrizes the growth rate in the smoothed precision,  $s$  is time, and  $\tau^P$  is initial unsmoothed public precision. Parameter values:  $\tau^P = 0.1, \tau^\theta = 0.1, \tau^S = 0.1, \tau^X = 100, \eta = 0.39, \gamma = 0.1, \kappa = 0.5, \zeta = 1$ , and  $b = 10$ , where  $\tau^\theta$  is the supply shock precision,  $\tau^S$  is private signal precision,  $\tau^X$  is precision of asset payoff,  $\eta$  is the percolation meeting rate,  $\gamma$  is the coefficient of absolute risk aversion,  $\kappa$  is the coefficient for the misperception of the meeting rate.

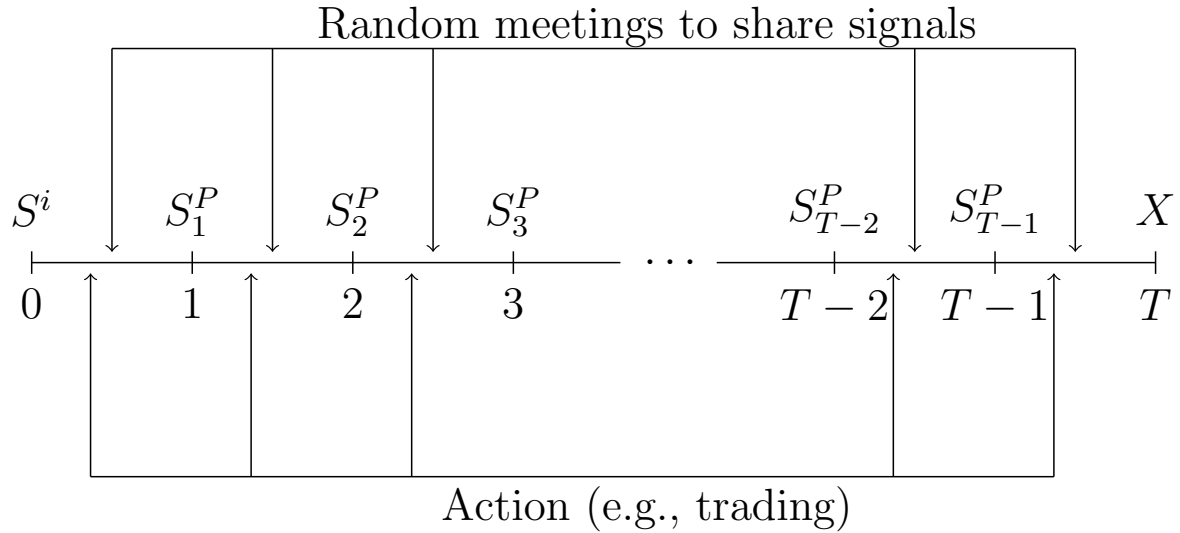


Figure 3: **Timeline of payoffs for the basic setting of Action Booms and Price Bubbles Models.** At each continuous date  $s \in [0, T)$ , each agent  $i$  chooses the level of an action with intensity  $\theta_s^i$ . Each agent is endowed with one private signal  $S^i$  at date 0. Agents are randomly drawn in pairs in continuous time to meet and share private signals. At each discrete date agents receive public signals  $S_t^P$ . The payoff  $X$  per unit of the action is realized and received at finite date  $T$ . In the context of security trading,  $X$  is the terminal dividend of the risky asset.

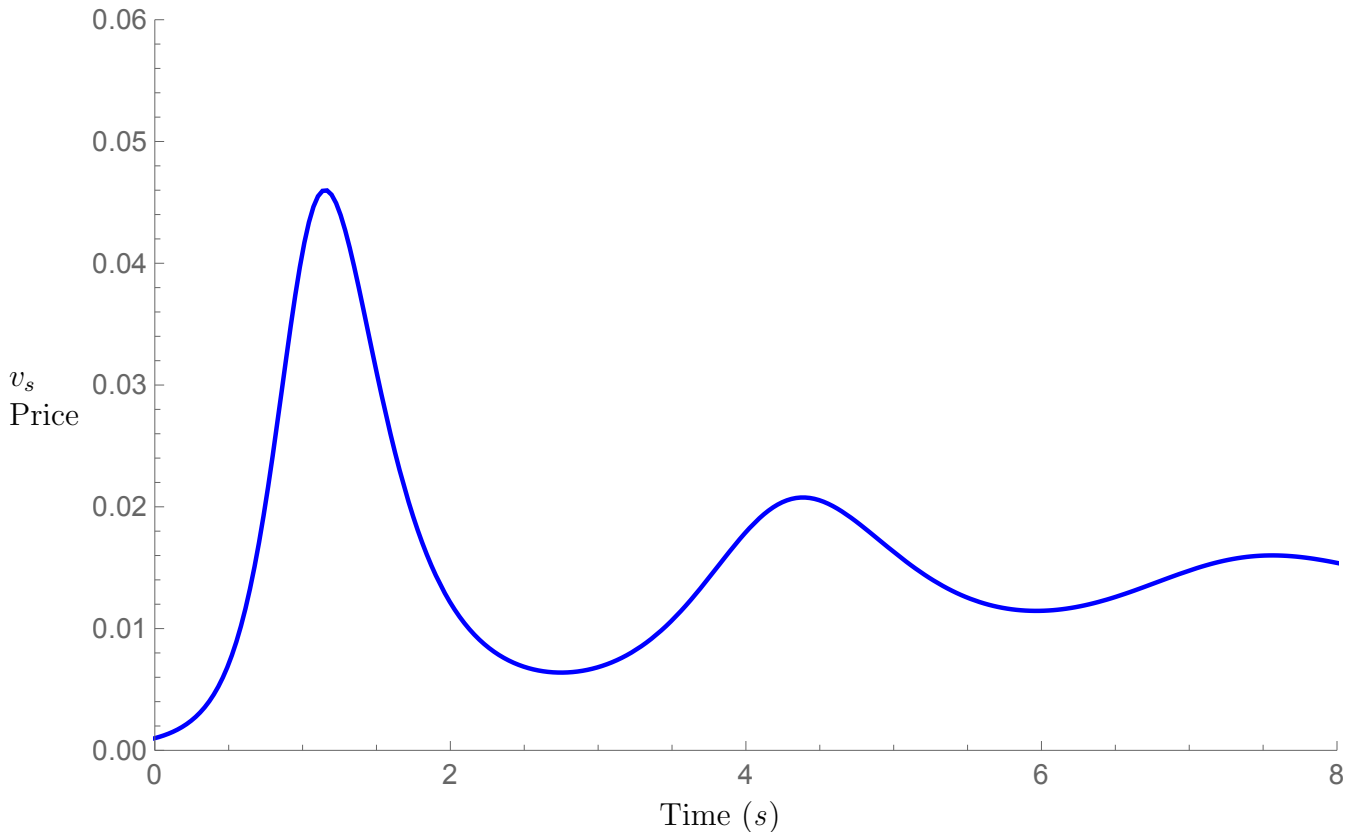


Figure 4: **Price trajectory in SIRS model with asset market and buzz.** This figure plots the price path over time in a SIRS model with asset market and buzz, with damped oscillations owing to overreaction and overcorrection. Effects are exacerbated by buzz, suggesting rich patterns in return autocorrelations at different lags. The figure plots the equilibrium price based on the following system of differential equations:  $R'(s) = rI(s) - kR(s)$  and  $I'(s) = -rI(s) + m(a + bDI'(s))I(s)(1 - I(s))$ , where  $R(s)$  is the fraction of the population that is resistant at date  $s$ ,  $r$  is the recovery rate,  $I(s)$  is the fraction of the population that is infected with the folk model,  $k$  is the rate of loss of resistance,  $m$  is the meeting frequency parameter,  $a$  is the inherent contagiousness parameter,  $b$  is buzz parameter, and  $D$  is an equilibrium pricing parameter that reflects agents' risk aversion. Price is  $v_s = DI(s)$  and  $D > 0$  is a constant. Parameter values:  $a = 80, b = 5, r = 6.0, m = 0.58, k = 0.1$ , and  $D = 1$ . Initial conditions:  $I(0) = 0.001$  and  $R(0) = 0.8$ .

# Online Appendix

Presidential Address:

Social Transmission Bias in Economics and Finance

David Hirshleifer

The Online Appendix provides a graph of the excess action path in the Action Booms Model, provides the proof of Proposition 12, and analyzes biased information percolation with single-signal transmission bias.

## A The excess action path in the Action Booms Model

In the Action Booms Model, the excess action path under a specific set of parameter values is depicted in Figure 5.

## B Proof of Proposition 12

**Proof of Proposition 12:** In the first equality in (A.8), we now interpret the expectation as a rational observer's correct understanding of the biased expectation used by agent  $i$  in forming agent  $i$ 's incorrectly-optimized position. Continuing to suppress the subscript  $s$ , agent  $i$ 's perceived-optimal position and the biased expectation of the fundamental satisfy

$$\begin{aligned} \gamma \left[ \tau^X + n^i \tau^S + \tau^Z + \left( \frac{\alpha^X}{\alpha^\Theta} \right)^2 \tau^\Theta \right]^{-1} \theta^i &= \mathbb{E} [X | \Phi^i] - v \\ \gamma \theta^i &= (\mathbb{E} [X | \Phi^i] - v) \left[ \tau^X + n^i \tau^S + \tau^Z + \left( \frac{\alpha^X}{\alpha^\Theta} \right)^2 \tau^\Theta \right]. \end{aligned} \tag{B.1}$$

Integrating both sides over agent  $i$  based on the true distribution of agents in the population, gives

$$\begin{aligned} \gamma^\Theta &= \int_{i \in [0,1], \mu^R} \gamma \theta^i di = \int_{i \in [0,1], \mu^R} \left\{ (\mathbb{E} [X | \Phi^i] - v) \left[ \tau^X + n^i \tau^S + \tau^Z + \left( \frac{\alpha^X}{\alpha^\Theta} \right)^2 \tau^\Theta \right] \right\} di \\ &= \int_{i \in [0,1], \mu^R} \left\{ \mathbb{E} [X | \Phi^i] \left[ \tau^X + n^i \tau^S + \tau^Z + \left( \frac{\alpha^X}{\alpha^\Theta} \right)^2 \tau^\Theta \right] \right\} di \\ &\quad - v \int_{i \in [0,1], \mu^R} \left\{ \tau^X + n^i \tau^S + \tau^Z + \left( \frac{\alpha^X}{\alpha^\Theta} \right)^2 \tau^\Theta \right\} di. \end{aligned} \tag{B.2}$$

Let  $k(n) \equiv \tau^X + n\tau^S + \tau^Z + \left(\frac{\alpha^X}{\alpha^\Theta}\right)^2 \tau^\Theta$ . Then

$$\begin{aligned} \int_{i \in [0,1], \mu^R} k(n^i) di &= \sum_n \mu^R(n) k(n) \\ &= \tau^X + \phi^R \tau^S + \tau^Z + \left(\frac{\alpha^X}{\alpha^\Theta}\right)^2 \tau^\Theta = k(\phi^R) \end{aligned} \quad (\text{B.3})$$

and

$$\int_{i \in [0,1], \mu^R} \{\mathbb{E}[X|\Phi^i] k(n^i)\} di = \sum_n \mu^R(n) k(n) \frac{\int_{i:n^i=n} \mathbb{E}[X|\Phi^i]}{\mu^R(n)}. \quad (\text{B.4})$$

Let  $\bar{E}^n$  be the average expectation among agents who have  $n$  private signals,

$$\begin{aligned} \bar{E}^n &\equiv \frac{\int_{i:n^i=n} \mathbb{E}[X|\Phi^i]}{\mu^R(n)} \\ &= \lambda^S(n, \tau^Z) \left( X + b f^b(n) + \frac{\int_{i:n^i=n} \bar{e}^i di}{\mu^R(n)} \right) + \lambda^P(n, \tau^Z) \bar{Z} + \lambda^\xi(n, \tau^Z) \xi \\ &= \lambda^S(n, \tau^Z) (X + b f^b(n)) + \lambda^P(n, \tau^Z) \bar{Z} + \lambda^\xi(n, \tau^Z) \xi, \end{aligned}$$

and let

$$\bar{E} \equiv \frac{\sum_n \mu^R(n) k(n) \bar{E}^n}{\sum_n \mu^R(n) k(n)} \quad (\text{B.5})$$

be the precision-weighted-average expectation over all agents. So (B.2) can be rewritten as

$$\begin{aligned} \gamma\Theta &= \bar{E} \sum_n \mu^R(n) k(n) - v \sum_n \mu^R(n) k(n) \\ &= (\bar{E} - v) \sum_n \mu^R(n) k(n) \\ &= (\bar{E} - v) \left( \tau^X + \phi^R \tau^S + \tau^Z + \left(\frac{\alpha^X}{\alpha^\Theta}\right)^2 \tau^\Theta \right). \end{aligned} \quad (\text{B.6})$$

Solving for  $\bar{E}$  in terms of price  $v$  and supply  $\Theta$  gives

$$\begin{aligned} \bar{E} &= v + \left[ \tau^X + \phi^R \tau^S + \tau^Z + \left(\frac{\alpha^X}{\alpha^\Theta}\right)^2 \tau^\Theta \right]^{-1} \gamma\Theta \\ &= v + [k(\phi^R)]^{-1} \gamma\Theta. \end{aligned} \quad (\text{B.7})$$

So the covariance between the changes in precision-weighted expectation and previous price change is

$$\begin{aligned} \text{cov}^R(v_{s+\Delta} - v_s, \bar{E}_{s+2\Delta} - \bar{E}_{s+\Delta}) &= \text{cov}^R(v_{s+\Delta} - v_s, v_{s+2\Delta} - v_{s+\Delta}) \\ &\quad + \text{cov}^R(v_{s+\Delta} - v_s, \gamma\Theta([k_{s+2\Delta}(\phi^R)]^{-1} - [k_{s+\Delta}(\phi^R)]^{-1})) \\ &\approx \text{cov}^R(v_{s+\Delta} - v_s, v_{s+2\Delta} - v_{s+\Delta}) > 0. \end{aligned} \quad (\text{B.8})$$

The approximate equality comes from the fact that  $\mathbb{V}(b)$  is large, making longer covariance negligible. I.e., the expression for  $v$  in (31) contains  $b$  terms, whereas  $k(\cdot)$  does not depend on  $b$ . The inequality then follows by Proposition 11.  $\parallel$

## C The Single-Signal Transmission Bias Regime

Under the single-signal transmission bias percolation regime, one new bias is added to just a single signal (not the mean signal) at each batch transmission. Owing to normality, which implies linear updating, it does not matter which signal the bias is added to. So we are free to view the bias as always being added to the sender's original unbiased signal. It follows that any signal possessed by any agent that was acquired socially has one bias, and that the single originally-held signal of an agent has none. So as the number of signals increases, the per-signal bias count increase asymptotically toward 1.

**Proposition 13.** *Under Single-Signal Transmission Bias, the total number of biases in the  $n^i \geq 1$  signals possessed by an agent  $i$  is a linear function of the agent's number of signals,  $C(n^i) = n^i - 1$ . So the per-signal bias count of agent  $i$  is*

$$f^{bi} = \frac{n^i - 1}{n_i}. \quad (\text{C.1})$$

Furthermore, at any date  $0 \leq s \leq T$ , the aggregate per-signal bias count is

$$f_s^b = 1 - \frac{1}{\phi_s^R} \quad (\text{C.2})$$

**Proof of Proposition 13:** As inductive hypothesis, suppose that the claim is true for all  $m \leq n$ . We need to show that  $C(n+1) = n$ . Consider an agent  $i$  who has  $n+1$  signals. All but one of these signals were acquired through meetings with agents whom we index with  $k$ , where  $0 < n^k < n$  and  $\sum_k n^k = n$ . By hypothesis,  $C(n^k) = n^k - 1$ . Furthermore, in transmitting a batch of signals, each agent  $k$  adds one new bias, so each contributes  $C(n^k) + 1 = n^k$  biases to agent  $i$ . So in total,

$$C(n+1) = \sum_k n^k = n,$$

which confirms the first claim by induction. Equation (C.1) follows immediately.

The per-signal bias count of any agent with  $n$  signals can therefore be denoted  $h^b(n)$ . By (8), since  $C(n) = n - 1$  and  $\nu(n^b) = \mu(n^b + 1) = \mu(n)$ , aggregate per-signal bias count at date  $s$  is

$$\begin{aligned} f_s^b &= \frac{\sum_{n=1}^{\infty} \mu_s(n)(n-1)}{\phi_s^R} \\ &= 1 - \frac{1}{\phi_s^R}. \end{aligned}$$

||

When agents are drawn in pairs independently, the information percolation model of Andrei and Cujean (2017) provides expressions for the per capita signal count,  $\phi_s$ , and the distribution of signal count across agents,  $\mu_s(n)$ , in terms of the Poisson meeting rate. In my model, the perceived meeting rate is  $\kappa\eta$ , so

$$\begin{aligned} \mu_s(n) &= e^{-\kappa\eta n s} (e^{\kappa\eta s} - 1)^{n-1} \\ \phi_s &= e^{\kappa\eta s}, \quad 0 \leq s < T, \end{aligned} \tag{C.3}$$

where for a rational observer,  $\kappa = 1$ . Here  $\mu$  has support  $\{1, 2, 3, \dots\}$ .

By (C.3), we have

$$f_s^b = 1 - e^{-\eta s}. \tag{C.4}$$

Proposition 13 shows that per-signal bias approaches a horizontal asymptote over time, in contrast with the unlimited increase under full-batch transmission bias in the main text.

Propositions 2-11 and their proofs are not based upon the details of the percolation regime, and therefore also apply under single-signal transmission bias. I next turn to results that are specific to the single-signal transmission bias regime.

## C.1 Action booms under single-signal transmission bias

Substituting the expressions for  $\phi_s^R$  and  $f_s^b$  from equation (C.3) and Propositions 13, and  $\tau_s^Z$  from (1) into (A.6), we obtain

$$\Theta_s^{SS}(b) = \frac{\tau^X(\bar{X} - v) + e^{\eta s} \tau^S [X + b(1 - e^{-\eta s}) - v] + e^{\zeta|s|}(\bar{Z}_s - v)}{\gamma} \tag{C.5}$$

and for the excess action,

$$\overline{\Delta\Theta}_s^{SS}(b) = \frac{e^{\eta s} \tau^S b(1 - e^{-\eta s})}{\tau^X(\bar{X} - v) + e^{\eta s} \tau^S (X - v) + e^{\zeta|s|}(\bar{Z}_s - v)}, \tag{C.6}$$

where the  $SS$  superscript indicates the Single-Signal transmission bias regime.

## C.2 Price bubbles under single-signal transmission bias

The rising evolution of  $\phi_s$  and  $\phi_s^R$  through time depends on the transmission regime. So I next substitute the solutions for the deterministic per capita signal count  $\phi_s^R$  and per-signal bias count  $f_s^b$ , as derived earlier, into the pricing equations to describe the dynamics of valuations.

Substituting (C.3) and (C.2) into (31) gives

$$v_s(X, \bar{Z}_s, \Theta, b) = \frac{e^{\zeta[s]}\tau^P\gamma^2\bar{Z}_s + (\gamma^2 + e^{\kappa\eta s}\tau^S\tau^\Theta)\{e^{\eta s}\tau^S[X + b(1 - e^{-\eta s})] - \gamma\Theta\}}{\gamma^2(\tau^X + e^{\eta s}\tau^S + e^{\zeta[s]}\tau^P) + e^{(\kappa+1)\eta s}(\tau^S)^2\tau^\Theta}, \quad (\text{C.7})$$

which depends on the random public signals and supply shock.

Taking the conditional expectation of the above gives the expected price path for a rational observer,

$$\mathbb{E}^R[v_s|b] = \frac{(\gamma^2 + e^{\kappa\eta s}\tau^S\tau^\Theta)e^{\eta s}\tau^S b(1 - e^{-\eta s})}{\gamma^2(\tau^X + e^{\eta s}\tau^S + e^{\zeta[s]}\tau^P) + e^{(\kappa+1)\eta s}(\tau^S)^2\tau^\Theta}. \quad (\text{C.8})$$

Price behavior in this setting is generally qualitatively similar to that of the full-batch transmission regime of the main text, as reflected in the applicability of most of the propositions to both regimes. Under appropriate parameter values, when  $b > 0$ , prices still increase for a time as signals and biases accumulate, and then correct downward as public information arrives. However, under single-signal transmission, booms or bubbles are less extreme, because iterative amplification of bias is more limited.

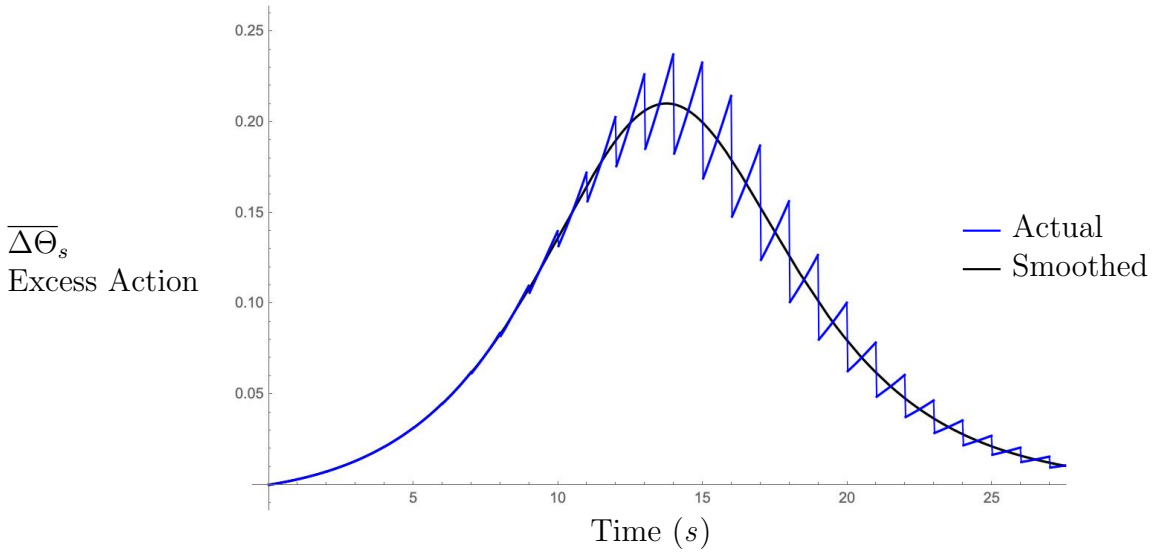


Figure 5: **Dynamics of action booms.** The excess action is the percentage deviation of the expected aggregate action conditional on  $b$  from the rational benchmark ( $b = 0$ ). The figure plots (19), and shows the smoothed and unsmoothed excess action paths for positive action booms ( $b > 0$ ). As in (20), smoothed public precision is  $\hat{\tau}_s^Z \equiv e^{\zeta(s-0.5)}\tau^P$ . Parameter values:  $\tau^P = 0.1, \tau^\theta = 0.1, \tau^S = 0.1, \tau^X = 71.6, \eta = 0.183, \gamma = 0.5, \zeta = 1, \nu = 0.5$ , and  $b = 5$ .