Market Uncertainty and International Trade

Haichao Fan†  Guangyu Nie‡  Zhiwei Xu§

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Abstract

We study the consequences of market uncertainty for international trade dynamics. Using Chinese customs data, we find that an increase in foreign market uncertainty dampens aggregate exports on both the extensive margin (the number of exporters) and the intensive margin (the average value of trade). The adverse effects are more pronounced in industries facing tighter financial constraints than in other sectors. We propose a dynamic trade model with market uncertainty and endogenous borrowing constraints to account for the facts. Greater uncertainty depresses a firm’s expected value of exporting and borrowing capacity, leading to fewer firms entering the foreign market and a smaller production size for exporters. In a quantitative exercise, we calibrate a fully fledged dynamic general equilibrium model to the Chinese economy. Under the calibrated parameters, the uncertainty shock accounts for 25% of China’s trade collapse in the 2008 financial crisis and 60% of the decline in exports from China to the U.S. during the recent trade war.

Keywords  Market Uncertainty, Trade Collapse, Financial Constraint, Extensive Margin, Intensive Margin

JEL Classification  F10, F41, F44

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†Institute of World Economy, School of Economics, Fudan University. Email: fan_haichao@fudan.edu.cn.

‡College of Business, Shanghai University of Finance and Economics. Email: nie.guangyu@shufe.edu.cn

§Antai College of Economics and Management, Shanghai Jiao Tong University. Email: xuzhiwei09@gmail.com
1 Introduction

The global financial crisis caused substantial economic turbulence and a surge in economic uncertainty (Baker et al., 2016). Following the Great Recession, international trade experienced an unprecedented contraction known as the Great Trade Collapse. In addition, for most developing countries, we see declines in both the extensive margin (number of exporters) and the intensive margin (average export value per exporter) of trade, as shown in Figure 1.\(^1\)

The contraction of trade in the two largest economies, the United States and China, reached approximately 20% in 2008-2009, significantly above the worldwide average rate of 12%. How do exporters respond to a more uncertain global economy? What is the underlying mechanism through which market uncertainty affects firm-level exporting decisions? Is market uncertainty quantitatively important for understanding the Great Trade Collapse? The aim of this paper is to address the above questions, taking the Chinese economy as a laboratory. The reasons we focus on China are twofold. First, China, the second-largest economy in the world, experienced a much more severe trade collapse than other countries. Second, the financial crisis struck the Chinese economy directly and extensively through foreign demand (i.e., international trade) instead of its own financial system. Thereby, the Chinese economy serves as an ideal context to study the consequences of foreign market uncertainty for trade dynamics.

We first document some stylized facts regarding firms’ exporting decisions and market uncertainty. The data we use are from China’s customs database, which is comprehensive and contains detailed information for China and its trading partners. We analyze the impacts of foreign market uncertainty on the number of exporters (extensive margin) and the average value of trade (intensive margin). The results suggest that an increase in market uncertainty in the destination country significantly reduces the number of exporters and the average trade volume for individual firms. The adverse effects are more pronounced in the industries facing tighter external financing conditions than in other industries. This fact suggests that financial constraints serve as an important channel for amplifying the uncertainty shocks affecting international trade. Our empirical findings are robust to different econometric specifications and definitions of market uncertainty.

We then propose a dynamic trade theory to account for the facts. We start with a simple two-period model to illustrate the intuition. The model features heterogeneous firms that face idiosyncratic foreign demand shocks and financial constraints. The standard deviation of idiosyncratic foreign demand captures foreign market uncertainty. Each firm has two options: exporting in the current period or waiting until the next period. Entering the foreign market incurs a fixed cost. An increase in market uncertainty raises the option value of waiting, leading to a standard

\(^1\)The link for the data source is http://www.worldbank.org/en/research/brief/exporter-dynamics-database.
Figure 1: Extensive and Intensive Margins of Exports among Developing Countries

Notes: This figure plots the simple averages of the dynamics of the extensive and intensive margins of exports among developing countries from 2007 to 2012. The extensive margin is measured by the number of exporters, while the intensive margin is measured by the average value of exports per exporter. We apply an HP filter to both sequences in each country with the smoothing parameter set to 6.25. For each variable, the two series are computed as the percentage deviations from its trend. The data source is the Exporter Dynamics Database from the World Bank.
wait-and-see effect on the firm’s entry decision. In addition, in our paper, there is a novel financial channel to transmit the uncertainty shock. Specifically, the borrowing constraint distorts the production decisions of those firms with high foreign demand. This is because the firms cannot expand their production to the desired level to meet the high foreign demand due to their limited borrowing capacity. Greater market uncertainty (mean-preserved) increases both the upper and bottom tails of the demand distribution. The firms cannot benefit from an increase in the upper tail risks but suffer more from bottom tail risks. As a consequence, greater market uncertainty dampens the expected value of entering the foreign market, leading to a negative effect on the extensive margin of exports. In addition, greater uncertainty depresses the firm’s collateral value, which is proportional to expected firm value. This further dampens the scale of production for exporters, leading to a negative effect on the intensive margin of exports. Note that the standard wait-and-see channel fails to generate a negative response of the intensive margin to an uncertainty shock because of the selection effect. Therefore, the responses of the intensive margin of exports to the uncertainty shock provides a key identification for the financial constraint channel. In this sense, our model proposes a novel channel that propagates uncertainty shocks to international trade.

We generalize the two-period model to a fully fledged dynamic general equilibrium framework. We calibrate the model’s deep parameters by matching the model-implied moments with those in the Chinese data. The dynamic analysis shows that a positive uncertainty shock reduces aggregate exports on both the extensive and intensive margins. Moreover, the uncertainty shock also causes an economic recession by reducing aggregate output, and the aggregate export volume presents excessive volatility relative to the aggregate output. Thereby, our dynamic model of trade fits the observed facts quite well. In the counterfactual analysis, the model with the wait-and-see channel alone fails to generate a negative intensive margin of exports in response to an uncertainty shock because of the selection effect. When the borrowing constraint is sufficiently relaxed, the adverse impacts of uncertainty shocks on export dynamics are largely mitigated.

To evaluate the importance of market uncertainty for China’s trade collapse, we simulate the export dynamics by feeding into the model a sequence of uncertainty shocks constructed from the data. The simulation shows that the model-generated export sequence tracks the dynamics in the data counterpart quite well. In terms of magnitude, market uncertainty per se accounts for one-quarter of China’s trade collapse in the 2008 financial crisis observed in the data. We also study the consequences of market uncertainty for the Chinese economy. The simulation shows

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2 Under greater market uncertainty, the option value of waiting increases. This further increases the threshold of demand above which the firm chooses to export. Therefore, due to the selection effect, the average value of exporting for the exporters becomes larger following a positive uncertainty shock.
that market uncertainty causes a sizable decline in China’s aggregate output. Therefore, economic uncertainty is an important force that drives fluctuations in international trade and the economy overall.

**Related Literature** Our paper contributes to several strands of the literature. First, our paper is closely related to the literature on the causes of the Great Trade Collapse. Eaton et al. (2016) investigate the forces driving global trade collapse in a dynamic multi-country general equilibrium model. They find that the shift of demand away from tradable goods due to a contraction in durable investment efficiency accounts for most of the trade collapse. As the recent financial crisis extensively affects the banking sector, some research, including Amiti and Weinstein (2011), Chor and Manova (2012) and Jiao and Wen (2012), attributes the collapse in trade to the deterioration in trade credit and external financial conditions. Our paper also emphasizes the financial channel, but our departure from the above literature is that we focus on the role of the financial constraint in amplifying uncertainty shocks. Bems et al. (2011) focus on the global input-output channel and conclude that the cross-border vertical linkages played a key role in the 2008-2009 collapse of global trade. Alessandria et al. (2010) propose an inventory adjustment theory to explain the Great Trade Collapse. Their quantitative results suggest that the inventory dynamics for imported goods can account for the observed magnitude of the trade collapse during the recent Great Recession. Bems et al. (2013) provides a comprehensive survey of the above literature.

Our paper is most closely related to the recent work on uncertainty and trade dynamics. Handley (2014) and Handley and Limao (2015) study the impact of trade policy uncertainty on the firm-level exporting decisions. Under a dynamic heterogeneous-firm framework, they find that trade policy uncertainty reduces entry into export markets. Carballo et al. (2018) further examine the interaction of economic and policy uncertainty and their impacts on trade volumes in a dynamic heterogeneous-firms model. They find these two types of uncertainty and their interaction are the main contributors to the U.S. trade collapse during the 2008 financial crisis. Trade agreements can mitigate the adverse consequences caused by increased uncertainty. Novy and Taylor (2014) introduce lumpy inventory investment into a dynamic trade model and study the impact of uncertainty shocks on trade dynamics. The uncertainty shocks in their paper produce a larger contraction in international trade (imported goods) relative to domestic behavior through the wait-and-see channel. Alessandria et al. (2015) study the effect of microeconomic uncertainty shocks in a two-country DSGE model with heterogeneous producers and endogenous export participation. They find that the international reallocation channel is important in understanding microeconomic uncertainty. The above studies mainly focus on the extensive margin of international trade. The underlying mechanism relies on the standard wait-and-see channel. Complementary to the above
literature, we emphasize the financial channel in the transmission of uncertainty shocks to international trade. We argue that the intensive margin of exports is crucial to identifying the role of financial constraints in the propagation of uncertainty shocks.

As this paper bridges international trade and macroeconomic fluctuations, our work is closely related to the research on the macroeconomic consequences of economic uncertainty. Bloom et al. (2018) investigate aggregate implications of firm-level uncertainty shocks in a heterogeneous-firm model with non-convex factor adjustment costs. They show that the wait-and-see effect provides a major channel for propagating uncertainty shocks. Arellano et al. (2016) introduce demand uncertainty into a DSGE model with heterogeneous firms. Financial frictions make the idiosyncratic demand shock uninsurable, resulting in a negative effect of uncertainty shocks on firm-level production and the aggregate economy. Christiano et al. (2014) identify risk shocks through the cyclicality of the credit spread in a standard financial accelerator model. They find that risk shocks are the most important driving force for business cycles. Gilchrist et al. (2014) construct a quantitative model with both investment irreversibility and financial frictions. Their results suggest that the credit channel is more important than the wait-and-see channel for understanding the aggregate impact of uncertainty shocks. See Bloom (2014) for a more comprehensive survey of the recent research. The above literature focuses on the propagation mechanism in a closed economy.

We incorporate international trade into an otherwise standard financial accelerator model. Our quantitative model offers an alternative channel through which economic uncertainty may affect aggregate fluctuations.

The remainder of the paper proceeds as follows. Section 2 documents some stylized facts regarding the effect of uncertainty shocks on exports on both the intensive and extensive margins. Section 3 presents a simple two-period model to explain the facts and illustrate the main intuition. Section 4 generalizes the simple model to a fully fledged dynamic general equilibrium framework. Section 5 calibrates the model to the Chinese economy and conducts the quantitative analysis. Section 6 concludes the paper. All the data descriptions and the proofs of propositions are in the appendices.

2 Stylized Facts

We first empirically study the consequences of foreign market uncertainty for Chinese firms’ exporting decisions. We mainly analyze how the demand uncertainty in the destination market affects the extensive and intensive margins of exports through the financial constraint channel. The primary data we use are from China’s customs database. It provides detailed information on
each trade transaction between China and its trading partners at the HS-8 digit level from the year 2000 to 2012. To ensure the consistency of the product categorization over time, we use the conversion table from the UN Comtrade to convert the HS 2002, 2007 and 2012 codes into the HS 1996 codes at the 6-digit level. Following Chaney (2008), we define the extensive margin as the number of exporting firms and the intensive margin as the firm’s average value of exports within each HS 6-digit category to the destination market.

We construct the foreign market uncertainty faced by Chinese firms based on the trade flow data from CEPII. This database provides bilateral trade values at the HS 6-digit product level of disaggregation for more than 200 countries. The demand uncertainty in destination market \( j \) is calculated using a residual approach as follows. First, we employ the following econometric specification to estimate the residual of export volume

\[
\log \text{EX}_{ijh,t} = \alpha \log \text{EX}_{ijh,t-1} + \varphi_t + \varphi_{ijh} + \epsilon_{ijh,t},
\]

where \( \text{EX}_{ijh,t} \) denotes the export volume from country \( i \) to country \( j \) at the HS-6 product \( h \) level in year \( t \). In the estimation, we add year fixed effects \( \varphi_t \) and source-destination-product fixed effects \( \varphi_{ijh} \). The residual in the above equation \( \epsilon_{ijh,t} \) indicates the deviation from the source-destination-product average after controlling for time-specific shocks to the export value. We then calculate the demand uncertainty in destination market \( j \) as the standard deviation of the residual for each country \( j \) in year \( t \)

\[
\sigma_{jt} = \sqrt{\frac{1}{N} \sum_i \sum_h \epsilon_{ijh,t}^2},
\]

where \( N \) denotes the total number of HS 6-digit products exported by all countries to destination country \( j \) in year \( t \). We focus on Chinese export dynamics. To alleviate potential endogeneity issues, we use an alternative measure by excluding China when we calculate the uncertainty indicator \( \sigma_{jt} \). Specifically, the alternative uncertainty indicator \( \sigma_{jt}^{alt} \) is measured as

\[
\sigma_{jt}^{alt} = \sqrt{\frac{1}{N} \sum_{i \neq \text{China}} \sum_h \epsilon_{ijh,t}^{alt}}.
\]

Financial constraints are a critical factor in our empirical study. We follow Manova (2012) and use external financial dependence (FinDep) to measure the industry-level financial constraints faced by firms. Firms in an industry are financially more vulnerable and face tighter credit constraints

\[^{3}\text{It includes detailed information such as the source and destination country, import and export value, product category labeled with the eight-digit harmonized code (HS8), firm contact information (e.g., company name, telephone, zip code and contact person), types of firm (e.g., state-owned, domestic private firms, foreign-invested or joint ventures) and customs regime (i.e., “Processing and Assembling” or “Processing with Imported Materials”).}\]

\[^{4}\text{China changed HS-8 codes in 2002, 2007 and 2012, and the concordance between the old and new is available at HS-6 level.}\]
when external financial dependence is higher. Following Rajan and Zingales (1998), we use data on publicly listed companies in the United States to calculate external financial dependence for each ISIC 3-digit industry. Then, we merge this information with the HS 6-digit product category. The reasons for constructing these measures based on United States data are twofold. First, the United States is a developed country with a mature financial market, and hence, a firm’s decisions reflect industry-specific credit needs. Second, as argued by Rajan and Zingales (1998) and Kroszner, Laeven and Klingebiel (2007), the difference in the reliance on external financing across industries is attributable to technological reasons, which persist across countries.

To examine the effect of market uncertainty on export dynamics, we use the following econometric specifications:

\[
\log Y_{hjt} = \beta_1 \times \text{FinDep}_h \times \sigma_{jt} + \beta_2 \times \sigma_{jt} + \gamma \times Z_{jt} + \varphi_t + \varphi_{hj} + \varepsilon_{hjt},
\]

(3)

where \(Y_{hjt}\) denotes the extensive margin (EXT) measured by the number of exporting firms or the intensive margin (INT) measured by the average trade volume to country \(j\) within HS 6-digit product \(h\) in year \(t\); \(\text{FinDep}_h\) is the external financial dependence for the industry of HS 6-digit product \(h\); and \(\sigma_{jt}\) denotes the uncertainty in destination market \(j\) in year \(t\). To further control for the domestic country’s other characteristics, we add \(Z_{jt}\) including credit to GDP, \(\log(\text{GDP})\) and \(\log(\text{GDP per capita})\). Credit to GDP reflects the financial shocks in the destinations, while GDP and GDP per capita capture the destination country’s size and development level. We control for year fixed effects \(\varphi_t\) and product-destination pair fixed effects \(\varphi_{hj}\). As the uncertainty indicator \(\sigma_{jt}\) is constructed at the destination-country level, we cluster the error terms at the same level to address the potential correlation of errors within each destination market over time.

Table 1 reports estimation results for the extensive margin of exports. Columns 1-2 and 3-4 pertain to the estimation results using the baseline uncertainty indicator defined in (2) and the alternative uncertainty indicator, respectively. Column 1 shows that the uncertainty in the destination market has a significantly negative impact on the extensive margin of exports. When the uncertainty in a destination market increases, fewer Chinese firms choose to export to that market. Column 2 shows that the adverse effect of uncertainty on the number of exporters is more profound in industries with tighter financial constraints. These results are fairly robust to the alternative definition of the uncertainty indicator.

Table 2 reports estimation results for the intensive margin of exports. Similar to the above analysis for the extensive margin of exports, columns 1-2 and 3-4 are for the estimation results using the baseline uncertainty indicator defined in (2) and the alternative uncertainty indicator, respectively. In column 1, we test the average impact of the uncertainty in the destination market
Table 1: Impact of Uncertainty on the Extensive Margin of Exports

<table>
<thead>
<tr>
<th></th>
<th>Baseline ($\sigma_{jt}$)</th>
<th>Alternative ($\sigma_{alt}^{jt}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>log(EXT$_{hjt}$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_{jt}$</td>
<td>-0.382*** (0.084)</td>
<td>-0.353*** (0.085)</td>
</tr>
<tr>
<td></td>
<td>-0.277*** (0.099)</td>
<td>-0.273*** (0.104)</td>
</tr>
<tr>
<td>$\sigma_{jt} \times$ FinDep$_{h}$</td>
<td>-0.304*** (0.103)</td>
<td>-0.242** (0.115)</td>
</tr>
<tr>
<td>Credit to GDP</td>
<td>0.197** (0.076)</td>
<td>0.197** (0.077)</td>
</tr>
<tr>
<td></td>
<td>0.188** (0.074)</td>
<td>0.187** (0.075)</td>
</tr>
<tr>
<td>log(GDP)</td>
<td>0.535*** (0.141)</td>
<td>0.536*** (0.140)</td>
</tr>
<tr>
<td></td>
<td>0.494*** (0.134)</td>
<td>0.496*** (0.133)</td>
</tr>
<tr>
<td>log(GDP per capita)</td>
<td>-0.050 (0.139)</td>
<td>-0.036 (0.138)</td>
</tr>
<tr>
<td></td>
<td>-0.029 (0.134)</td>
<td>-0.015 (0.133)</td>
</tr>
<tr>
<td>Year fixed effects</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Product-destination fixed effects</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Observations</td>
<td>2,836,776 2,728,258</td>
<td>2,836,776 2,728,258</td>
</tr>
<tr>
<td>Adjusted R$^2$</td>
<td>0.813 0.816</td>
<td>0.813 0.816</td>
</tr>
</tbody>
</table>

Notes: The significance levels are denoted as *** $p < 0.01$, ** $p < 0.05$, and * $p < 0.1$. The numbers in parentheses are robust standard errors corrected for clustering at the destination-country level. Market uncertainty is measured by $\sigma_{jt}$ in the first two columns and measured by $\sigma_{alt}^{jt}$ in the last two columns.

on the intensive margin of exports in China. In column 2, we further add the interaction term between uncertainty $\sigma_{jt}$ and external financial dependence FinDep$_{h}$. Table 2 shows that uncertainty in the destination market significantly reduces the average value of exports for Chinese exporters. This adverse effect is more severe for those industries with tighter financial constraints. These results are fairly robust to the alternative definition of the uncertainty indicator.

In summary, the above empirical evidence suggests that demand uncertainty in a foreign market dampens exports by reducing the number of exporting firms (extensive margin) and the average trade volume (intensive margin). The adverse consequences of market uncertainty for exports are more profound in industries facing tighter borrowing constraints than in other sectors.

3 A Two-period Model

To understand the mechanism underlying the stylized facts, we build a simple two-period international trade model with market uncertainty and financial frictions. To simplify the analysis, we only consider the partial equilibrium. The economy is populated by a continuum of domestic firms.

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5 The impact of demand uncertainty in a foreign market on total exports (measured by the total trade volume to country $j$ within HS 6-digit product $h$ in year $t$) is reported in Appendix Table A1.
Table 2: Impact of Uncertainty on the Intensive Margin of Exports

<table>
<thead>
<tr>
<th>log(INT_{hjt})</th>
<th>Baseline (σ_{jt})</th>
<th>Alternative (σ^{alt}_{jt})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>σ_{jt}</td>
<td>-0.227***</td>
<td>-0.096</td>
</tr>
<tr>
<td></td>
<td>(0.059)</td>
<td>(0.074)</td>
</tr>
<tr>
<td>σ_{jt} × FinDep_{ht}</td>
<td></td>
<td>-0.337***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.103)</td>
</tr>
<tr>
<td>Credit to GDP</td>
<td>0.083</td>
<td>0.075</td>
</tr>
<tr>
<td></td>
<td>(0.087)</td>
<td>(0.088)</td>
</tr>
<tr>
<td>log(GDP)</td>
<td>-0.140</td>
<td>-0.162</td>
</tr>
<tr>
<td></td>
<td>(0.163)</td>
<td>(0.167)</td>
</tr>
<tr>
<td>log(GDP per capita)</td>
<td>0.400***</td>
<td>0.420***</td>
</tr>
<tr>
<td></td>
<td>(0.135)</td>
<td>(0.135)</td>
</tr>
<tr>
<td>Year fixed effects</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Product-destination fixed effects</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Observations</td>
<td>2,836,774</td>
<td>2,728,256</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>0.581</td>
<td>0.577</td>
</tr>
</tbody>
</table>

Notes: The significance levels are denoted as *** p < 0.01, ** p < 0.05, and * p < 0.1. The numbers in parentheses are robust standard errors corrected for clustering at the destination-country level. Market uncertainty is measured by σ_{jt} in the first two columns and measured by σ^{alt}_{jt} in the last two columns.

with a unit measure. Each firm is a monopolistic producer that employs labor to produce a final good x_t with linear technology x_t = l_t, for t = 1, 2. The marginal cost in each period is simply the wage w_t. Each firm faces a foreign demand curve x_t = ε_t p_t^θ for t = 1, 2, where p_t is the price, ε_t is the idiosyncratic demand shock, and θ > 1. At the beginning of each period, the individual firm independently draws an idiosyncratic demand ε_t from the foreign market. We assume that the idiosyncratic demand in period 1, ε_1, follows a CDF F (ε_1; σ_0) with mean 1 and standard deviation σ_0. The idiosyncratic demand in period 2, ε_2, is drawn from a CDF F (ε_2; σ) with mean 1 and standard deviation σ. Following Bloom (2009), we assume that market uncertainty is observed one period before. That is, the demand uncertainty in period 2, σ, is observed in period 1. In the analysis we devote particular emphasis to the consequences of uncertainty in period 2 on the firm’s exporting decision in period 1.

For an exporting firm, the period-by-period profit earned from the foreign market is

$$\pi_t = (p_t - w_t) ε_t p_t^{-θ}, \text{ for } t = 1, 2.$$  \hspace{1cm} (4)

To introduce the financial constraint, we assume that the production requires a working capital
constraint,
\[ w_t x_t \leq \chi, \]  
where \( \chi \) is the exogenous limit on the external funding that the firm can obtain.

The firm chooses the price and labor to maximize the discounted flow of profits in the two periods subject to the working capital constraint (5). Since the idiosyncratic demand \( \varepsilon_t \) is independently distributed over time, the firm’s optimization problem is essentially static. In the absence of the working capital constraint, the firm would set the optimal price with a constant markup, i.e., \( \bar{p}_t = \varphi w_t \), where the markup \( \varphi = \frac{\theta}{\theta - 1} \). The working capital constraint distorts the pricing behavior of an exporting firm with relatively high demand. The following proposition describes an individual firm’s optimal pricing decision.

**Proposition 1**  The exporting firm’s optimal price setting follows the trigger strategy

\[ p_t (\varepsilon_t) = \bar{p}_t \max \left\{ 1, \left( \frac{\varepsilon_t}{\bar{\varepsilon}_{bt}} \right)^{\frac{1}{\theta}} \right\}, \text{ for } t = 1, 2, \]  

where the cutoff is defined as \( \bar{\varepsilon}_{bt} = \chi \varphi^\theta w_t^{\theta - 1} \).

**Proof.** If the firm draws a sufficiently low demand such that \( \varepsilon_t < \bar{\varepsilon}_{bt} \), the working capital constraint is not binding. The firm sets the price at the first-best level \( \bar{p}_t \). However, if the firm’s demand is sufficiently high, such that \( \varepsilon_t > \bar{\varepsilon}_{bt} \), the working capital constraint is binding. In this case, the optimal price deviates from \( \bar{p}_t \). To determine the demand cutoff \( \bar{\varepsilon}_{bt} \), consider a marginal firm that sets price \( \bar{p}_t \) while the working capital constraint binds. The condition \( w_t \bar{\varepsilon}_{bt} (\varphi w_t)^{-\theta} = \chi \) pins down the cutoff \( \bar{\varepsilon}_{bt} \). It is easy to show that the constrained optimal price is \( p_t = \bar{p}_t \left( \frac{\varepsilon_t}{\bar{\varepsilon}_{bt}} \right)^{\frac{1}{\theta}} \) for \( \varepsilon_t > \bar{\varepsilon}_{bt} \).

Intuitively, the working capital constraint distorts the optimal price set by those firms with relatively high demand. This is because if a firm with high demand sets price \( \bar{p}_t \), the desired optimal output requires a larger labor input (working capital) that exceeds the firm’s borrowing capacity \( \chi \). Therefore, the financial constraint forces a firm with relatively high demand to set the optimal price above the unconstrained level \( \bar{p}_t \). The equilibrium condition for the cutoff implies that \( \bar{\varepsilon}_{bt} \) increases with the borrowing capacity \( \chi \).

Under the optimal pricing rule (6), the output produced by the exporting firm is given by

\[ x_t (\varepsilon_t) = \min \{ \bar{\varepsilon}_{bt}, \varepsilon_t \} \bar{p}_t^{-\theta}, \text{ for } t = 1, 2. \]
The profit $\pi_t(\varepsilon_t)$ is given by

$$
\pi_t(\varepsilon_t) = \begin{cases} 
\bar{\pi}_{bt} \frac{\varepsilon_t}{\bar{\varepsilon}_{bt}} & \text{if } \varepsilon_t < \bar{\varepsilon}_{bt} \\
\frac{\bar{\pi}_{bt}}{\bar{\varepsilon}_{bt}} \left[ \theta \left( \frac{\varepsilon_t}{\bar{\varepsilon}_{bt}} \right) - \theta + 1 \right] & \text{if } \varepsilon_t \geq \bar{\varepsilon}_{bt}
\end{cases}
$$

for $t = 1, 2$, \hspace{1cm} (8)

where $\bar{\pi}_{bt} = \frac{1}{\bar{\varepsilon}_{bt}^2} \bar{p}_t^1 - \theta$ is the profit for the marginal firm that draws an idiosyncratic demand of $\bar{\varepsilon}_{bt}$. It is straightforward to show that the profit $\pi_t(\varepsilon_t)$ is a concave function of the demand shock $\varepsilon_t$. Moreover, (7) and (8) imply that tightened borrowing capacity reduces output and profit for exporting firms with relatively high demand.

Figure 2 provides a graphic illustration of the profit function $\pi_t(\varepsilon_t)$. The figure shows that for sufficiently low demand satisfying $\varepsilon < \bar{\varepsilon}_b$, the profit is linear in $\varepsilon$, while for sufficiently high demand satisfying $\varepsilon \geq \bar{\varepsilon}_b$, the profit is concave in $\varepsilon$. In addition, a tightened working capital constraint (i.e., $\chi$ is smaller) reduces the cutoff $\bar{\varepsilon}_b$ and shifts the profit curve downward.

To characterize the adverse effect of market uncertainty on the intensive margin of exports (the quantity of output), we explicitly assume that $\chi$ is a decreasing function of uncertainty $\sigma$. In the
dynamic model in the later analysis, we will provide a micro-foundation for this assumption by endogenizing the borrowing capacity $\chi$.

Denote by $V (\varepsilon_1; \sigma)$ the value of exporting when the firm draws demand $\varepsilon_1$ in period 1; then, we have

$$V (\varepsilon_1; \sigma) = \pi_1 (\varepsilon_1) + \beta \int \pi_2 (\varepsilon_2) dF (\varepsilon_2; \sigma),$$

(9)

where $\sigma$, the standard deviation of the demand shock in period 2, indicates the demand uncertainty in period 2. Since the profit function $\pi_2 (\varepsilon_2)$ is concave in $\varepsilon_2$, Jensen’s inequality implies that a rise in demand uncertainty in period 2 lowers the value of exporting, i.e., $\frac{\partial V(\varepsilon_1;\sigma)}{\partial \sigma} \leq 0$.

Following Melitz (2003), we assume that exporting to the foreign market incurs a fixed entry cost $\zeta > 0$. To characterize the wait-and-see behavior discussed in the uncertainty literature, e.g., Bloom (2009), we follow the setup in Carballo et al. (2018) by allowing the potential entrants to wait and be inactive. In particular, in period 1, the firm can choose to either export or wait. Let $V^w (\sigma)$ denote the expected value of a firm that decides to wait; then, we have

$$V^w (\sigma) = \beta \int \max \{\pi_2 (\varepsilon_2) - \zeta, 0\} dF (\varepsilon_2; \sigma).$$

(10)

where $\pi_2 (\varepsilon_2) - \zeta$ is the net value for a firm that chooses to export in period 2. Note that the function inside the integral, $\max \{\pi_2 (\varepsilon_2) - \zeta, 0\}$, is generally convex in $\varepsilon_2$, so an increase in uncertainty in period 2 would raise the value of waiting, i.e., $\frac{\partial V^w (\sigma)}{\partial \sigma} \geq 0$. This property captures the wait-and-see effect of the uncertainty shock proposed by Bloom (2009).

We now discuss the firm’s optimal exporting decision in period 1. The firm’s exporting decision is determined by solving a discrete optimization problem: $\max \{V^w (\sigma), V (\varepsilon_1; \sigma) - \zeta\}$. The firm opts to enter the foreign market when the demand drawn in period 1 is sufficiently high that $\varepsilon_1 > \bar{\varepsilon}_e$. The cutoff for exporting $\bar{\varepsilon}_e$ is determined by the condition

$$V (\bar{\varepsilon}_e; \sigma) - \zeta = V^w (\sigma).$$

(11)

Since greater market uncertainty raises the value of waiting ($\frac{\partial V^w (\sigma)}{\partial \sigma} \geq 0$) and depresses the value of exporting ($\frac{\partial V(\varepsilon_1;\sigma)}{\partial \sigma} \leq 0$), the above condition implies that $\bar{\varepsilon}_e$ is increasing in the uncertainty $\sigma$. As a result, an increase in market uncertainty reduces the number of exporting firms $1 - F (\bar{\varepsilon}_e; \sigma_0)$ in period 1.

Let $EX_1 = \int ex_1 (\varepsilon_1) dF (\varepsilon_1; \sigma_0)$ denote the aggregate export volume in period 1, where $ex_1 (\varepsilon_1) = p_1 (\varepsilon_1)x_1 (\varepsilon_1)$ is the value of exporting for a firm with demand shock $\varepsilon_1$. We can further decompose
EX into

$$\log \mathbf{EX}_1 = \log [1 - F(\varepsilon; \sigma_0)] + \log \mathbf{E}[e x_1(\varepsilon_1) | \varepsilon_1 > \varepsilon; \sigma_0].$$  \hspace{1cm} (12)$$

The first term on the right-hand side of the last equation is the number of exporting firms. It reflects the extensive margin of aggregate exports. The second term on the right-hand side measures the average export volume for exporters. It indicates the intensive margin of exports.

**Extensive Margin** As we have shown that $\frac{\partial \varepsilon_e}{\partial \sigma} \geq 0$, the extensive margin is decreasing in $\sigma$. To see that the financial constraint matters for the firm’s exporting decision, we consider an extreme case in which $\chi \rightarrow \infty$. In this case, the cutoff for the binding constraint $\varepsilon_b \rightarrow \infty$. As a result, the profit function $\pi_t(\varepsilon_t)$ is linear in $\varepsilon_t$. Then, the value of exporting degenerates to

$$V(\varepsilon_1; \sigma) = \frac{1}{\theta} \left( \varepsilon_1 p_1^{1-\theta} + \beta p_2^{1-\theta} \right),$$  \hspace{1cm} (13)$$

where $\bar{p}_t$ is the unconstrained optimal price satisfying $\bar{p}_t = \varphi w_t$ for $t = 1, 2$. Obviously, in this case, $V(\varepsilon_1; \sigma)$ is immune to uncertainty $\sigma$ because of the linearity of the profit function. In this case, condition (11) implies that uncertainty affects the cutoff $\bar{\varepsilon}_e$ and the extensive margin only through the wait-and-see channel. Therefore, the extensive margin effect is weaker than that in the case with financial constraints.

**Intensive Margin** In the absence of financial constraints, the volume of exports $e x_1(\varepsilon_1)$ is linear in $\varepsilon_1$, so the intensive margin $\log \mathbf{E}[e x_1(\varepsilon_1) | \varepsilon_1 > \varepsilon; \sigma_0]$ is strictly increasing in $\sigma$ since $\frac{\partial \varepsilon_e}{\partial \sigma} > 0$. Therefore, in a model with only the wait-and-see channel, market uncertainty has a positive effect on the intensive margin because of the selection effect. However, when an endogenous financial constraint is considered (the borrowing limit satisfies $\frac{\partial \chi(\sigma)}{\partial \sigma} < 0$), it is possible that the intensive margin decreases with uncertainty. This is because a rise in uncertainty $\sigma$ depresses the borrowing capacity $\chi(\sigma)$, leading to a tighter financial constraint. As in the previous discussion, a lower $\chi(\sigma)$ reduces the individual firm’s profit and thus the export volume. When this intensive margin effect is sufficiently strong, the overall impact of uncertainty on the intensive margin can be negative. The proposition below summarizes the above analysis.

**Proposition 2** Market uncertainty has a negative impact on the extensive margin of exports, and the impact is amplified by the presence of financial constraints. The intensive margin of exports is positive when the financial constraints are absent and could be negative when an endogenous financial constraint is considered.

Proposition 2 provides a theoretical explanation for our empirical findings. It suggests that

6Recall that the mean of $\varepsilon$ is normalized to 1.
the standard wait-and-see channel per se fails to generate a negative intensive margin for exports under uncertainty shocks. Therefore, the sign of the intensive margin helps to identify the financial constraint as a key mechanism through which market uncertainty may influence trade dynamics. In the next section, we generalize the two-period model to a fully fledged dynamic general equilibrium model. Our aim is to quantitatively evaluate the consequences of market uncertainty for trade dynamics. Our quantitative exercise may shed light on the driving force for the trade collapse during the 2008 financial crisis.

4 The Fully Fledged Dynamic Model

We generalize the previous two-period model to a dynamic general equilibrium framework. The model economy is inhabited by a representative household. The household consumes final goods produced by the domestic producers, supplies labor to the producers and purchases domestic and foreign bonds as savings instruments. There are two production sectors in the economy. In the domestic sector (labeled D), final good producers purchase local intermediate goods and import foreign goods to produce. Their own demand for foreign goods forms domestic imports. The intermediate goods producers in the D sector hire labor to produce through a linear technology. In the export sector (labeled E), firms are monopolistic. They hire labor to produce export goods for the foreign market. The production is subject to the working capital constraint that captures the financial frictions. The exporting firms face idiosyncratic demand shocks from the foreign market. Similar to the two-period model, the uncertainty shocks follow an independent and identical distribution over time and across firms. The dispersion of the idiosyncratic demand shocks captures the market uncertainty in the foreign market. Exporting to the foreign market incurs a fixed entry cost. The potential entrants with relatively high demand opt to enter the foreign market, while others choose to wait and to be inactive until the next period. Our model captures the effect of uncertainty shocks on both the intensive and the extensive margins of aggregate exports. We start with the description of the representative household’s problem.

4.1 Household

A representative domestic household chooses consumption, $C_t$, hours worked $L_t$, and the holdings of a domestic nominal bond $B_t$ and foreign nominal bond $B^*_t$ to maximize its expected lifetime utility:

$$
\max_{\{C_t, L_t, B_t, B^*_t\}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left[ \log C_t - \frac{\omega}{1+\gamma} \frac{L_t^{1+\gamma}}{1+\gamma} \right],
$$

(14)
subject to the budget constraint

\[ C_t + \frac{B_t + e_t B_t^*}{P_t} \left[ 1 + \frac{\Omega}{2} \left( \frac{B_t}{B_t + e_t B_t^*} - \bar{\psi} \right)^2 \right] = w_t L_t + d_t + \frac{R_{t-1} B_{t-1} + R_{t-1}^* e_t B_{t-1}^*}{P_t}. \]  

(15)

Here, \( e_t \) is the nominal exchange rate measured by the price of foreign currency in terms of the domestic currency; \( P_t \) is the domestic aggregate price level; \( w_t \) is the real wage rate; \( R_t \) and \( R_t^* \) are the nominal interest rates for domestic and foreign bonds, respectively; and \( d_t \) is the profit distributed by firms in the whole country. The parameter \( \beta \in (0, 1) \) is the subjective discount rate, \( \varpi > 0 \) is the weight of the labor dis-utility, and \( \gamma \) is the inverse Frisch elasticity of labor supply. Following Chang et al. (2015), we introduce a quadratic cost for the portfolio adjustment that captures capital controls. The parameter \( \Omega \) describes the share of the domestic bond in the portfolio that positively correlates with the difference in the bond returns. Let \( \Lambda_t \) denote the Lagrangian multiplier for the budget constraint. The optimal labor supply is given by

\[ \varpi L_t = w_t \Lambda_t, \]  

(16)

where \( \Lambda_t = 1/C_t \). Define \( \psi_t = \frac{B_t}{B_t + e_t B_t^*} \) as the share of domestic bonds in the household’s portfolio. The optimal decisions for the holdings of domestic and foreign bonds are given by

\[ 1 + \frac{\Omega}{2} (\psi_t - \bar{\psi})^2 + \Omega (\psi_t - \bar{\psi}) (1 - \psi_t) = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \frac{P_t}{P_{t+1}} R_t, \]  

(17)

\[ 1 + \frac{\Omega}{2} (\psi_t - \bar{\psi})^2 - \Omega (\psi_t - \bar{\psi}) \psi_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \frac{P_t}{P_{t+1}} R_t^* e_{t+1}. \]  

(18)

The last two optimal conditions imply a modified version of the uncovered interest parity condition (UIP) with capital controls:

\[ \Omega (\psi_t - \bar{\psi}) = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \frac{P_t}{P_{t+1}} \left[ R_t - R_t^* \frac{e_{t+1}}{e_t} \right]. \]  

(19)

As \( \psi_t \) is the portfolio share of \( B_t \), the last equation describes the demand function of the domestic bond that describes a positive relationship between the relative bond returns and bond holdings.

### 4.2 Domestic Sector

The final goods market in the domestic country is competitive. The market is populated by a continuum of representative producers. They produce domestic final goods \( (Z_t) \) using domestic
intermediate goods \((Y_t^D)\) and imported goods \((Y_t^*)\). The production function takes CES form:

\[
Z_t = \left[ \omega \left( Y_t^D \right)^{\frac{\eta-1}{\eta}} + (1 - \omega) \left( Y_t^* \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \quad \eta > 0 \text{ and } \omega \in (0, 1).
\] (20)

Let \(P_t^D\) and \(P_t^*\) denote the prices of domestic and imported goods, respectively. We assume that the imported goods are final products; thus, \(P_t^*\) is the aggregate price level in foreign countries. The competitive final goods producer chooses demand for inputs \(Y_t^D\) and \(Y_t^*\) to solve the profit maximization problem

\[
\max_{\{Y_t^D, Y_t^*\}} \left[ \omega \left( Y_t^D \right)^{\frac{\eta-1}{\eta}} + (1 - \omega) \left( Y_t^* \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} - \frac{P_t^D}{P_t} Y_t^D - \frac{P_t^*}{P_t} e_t Y_t^*.
\] (21)

Define the real exchange rate as \(q_t = \frac{P_t^*}{P_t} e_t\). First-order conditions imply that the demands for domestic and foreign goods are given by

\[
Y_t^D = \omega^n \left[ \frac{P_t^D}{P_t} \right]^{-\eta} Z_t,
\] (22)

\[
Y_t^* = (1 - \omega)^n q_t^{-\eta} Z_t.
\] (23)

Price indexation implies

\[
1 = \omega^n \left( \frac{P_t^D}{P_t} \right)^{1-\eta} + (1 - \omega)^n (q_t)^{1-\eta}.
\] (24)

The domestic intermediate goods firms are monopolistic. They use labor to produce \(Y_t^D\) according to a linear technology \(Y_t^D = L_t^D\). The optimal price is

\[
\frac{P_t^D}{P_t} = \frac{\eta}{\eta - 1} w_t.
\] (25)

### 4.3 Export Sector

#### 4.3.1 Incumbents

The export sector is populated by monopolistic incumbents and potential entrants. We normalize the total mass of these two types of firms to be 1. Each individual firm is indexed by \(j\). The foreign demand shock faced by firm \(j\) in the export sector consists of an aggregate component \(X_t^*\) and an idiosyncratic component \(\varepsilon_{jt}\). We assume that \(\varepsilon_{jt}\) follows an independent and identical distribution with mean \(\mu\) and time-varying standard deviation \(\sigma_t\). Denote the CDF as \(F(\varepsilon_{jt}; \sigma_t)\). The time-varying standard deviation introduces an exogenous market uncertainty shock. We
assume that market uncertainty follows a Markov process with a positive serial correlation, i.e.,
\[ \text{corr}(\sigma_{t+1}, \sigma_t) > 0. \]
In the calibration, we will provide further details on the process of \( \sigma_t \). We follow Chang et al. (2015) to specify firm \( j \)'s foreign demand scheme as
\[
x_{jt} = X^*_t \varepsilon_{jt} \left( \frac{P_{jt}}{q_t P_t} \right)^{-\theta},
\]
where the elasticity parameter \( \theta > 1 \). In the above equation, \( P_{jt} \) is the price of export goods set by firm \( j \). Analogous to domestic producers, the exporting firm uses labor to produce according to a linear technology \( x_{jt} = l_{jt} \). The marginal cost is simply the wage rate \( w_t \).

The optimal price \( P_{jt} \) set by firm \( j \) solves the following profit maximization problem:
\[
\max_{P_{jt}} \left( P_{jt} - w_t \right) X^*_t \varepsilon_{jt} \left( \frac{P_{jt}}{q_t P_t} \right)^{-\theta},
\]
subject to a working capital constraint
\[
w_t x_{jt} \leq \chi_t,
\]
where \( \chi_t \) is the endogenous borrowing limit that will be defined later.

The firm’s optimal decisions are similar to those in the two-period model. Denote by \( \bar{p}_t \) the unconstrained optimal price that satisfies \( \bar{p}_t = \phi w_t \), where \( \phi = \frac{\theta}{\theta - 1} \) is the markup. Define the cutoff of the idiosyncratic demand shock as \( \varepsilon_{bt} = \chi_t q_t X^*_t \bar{p}_t - \theta q_t X^*_t \bar{p}_t \). The optimal decisions for pricing, production and profit are summarized as follows:
\[
\frac{P_{jt}(\varepsilon_{jt})}{P_t} = \bar{p}_t \max \left\{ 1, \left( \frac{\varepsilon_{jt}}{\varepsilon_{bt}} \right)^{\frac{1}{\theta}} \right\},
\]
\[
x_{jt}(\varepsilon_{jt}) = \bar{x}_{bt} \min \left\{ \frac{\varepsilon_{jt}}{\varepsilon_{bt}}, 1 \right\},
\]
\[
\pi_{jt}(\varepsilon_{jt}) = \begin{cases} 
\pi_{bt} \frac{\varepsilon_{jt}}{\varepsilon_{bt}}^{\frac{1}{\theta}} & \text{if } \varepsilon_{jt} \leq \varepsilon_{bt} \\
\pi_{bt} \left[ \theta \left( \frac{\varepsilon_{jt}}{\varepsilon_{bt}} \right)^{\frac{1}{\theta}} - \theta + 1 \right] & \text{otherwise}
\end{cases}
\]
where \( \bar{x}_{bt} = X^*_t \varepsilon_{bt} \bar{p}_t^{-\theta} q_t^\theta \) and \( \pi_{bt} = \frac{1}{\theta} \bar{p}_t \bar{x}_{bt} \). As shown in the two-period model, the optimal profit function \( \pi_{jt} \) is concave in the idiosyncratic demand shock \( \varepsilon_{jt} \).

Denote by \( V_t(\varepsilon_{jt}, \sigma_t) \) the value of the exporting firm (the incumbent) with an idiosyncratic
demand shock $\epsilon_{jt}$; then, we have

$$V_t(\epsilon_{jt}; \sigma_t) = \pi_{jt}(\epsilon_{jt}) + \bar{V}_t(\sigma_t),$$

(32)

where the discounted expected value $\bar{V}_t(\sigma_t)$ is defined recursively as

$$\bar{V}_t(\sigma_t) = \beta (1 - \delta) \mathbb{E}_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \left[ \int \pi_{jt+1}(\epsilon_{jt+1}) dF(\epsilon_{jt+1}; \sigma_{t+1}) + \bar{V}_{t+1}(\sigma_{t+1}) \right] \mid \sigma_t \right\},$$

(33)

where $\delta$ is an exogenous exit rate of the exporting firms. Note that $\sigma_t$ follows a Markov process with positive serial correlation, and the expectation operator works on the uncertainty in the next period $\sigma_{t+1}$ conditional on the current state $\sigma_t$. Therefore, $\bar{V}_t$ is a function of $\sigma_t$. We use $\sigma_t$ to capture the foreign market uncertainty faced by the individual firms. Since the profit function $\pi_t(\epsilon_{jt})$ is concave in the idiosyncratic demand shock $\epsilon_{jt}$, Jensen’s inequality implies that the expected firm value $\bar{V}_t(\sigma_t)$ is decreasing in market uncertainty $\sigma_t$. We can immediately show that $V_t(\epsilon_{jt}; \sigma_t)$ is also decreasing in market uncertainty $\sigma_t$.

Following Jermann and Quadrini (2012), we define the borrowing limit $\chi_t$ as a function of the expected discounted value of the firm in the next period

$$\chi_t = \xi \bar{V}_t(\sigma_t),$$

(34)

where $\xi \in (0, 1)$ is the parameter that indicates the tightness of the financial constraint. Due to the property of $\bar{V}_t(\sigma_t)$, the borrowing limit $\chi_t$ is decreasing in market uncertainty $\sigma_t$. As we illustrated in the two-period model, an endogenous borrowing limit that decreases with market uncertainty provides a key channel to generate a negative intensive margin effect of market uncertainty on exporting decisions.

### 4.3.2 Potential Entrants

We now discuss the exporting decisions of potential entrants. Similar to the setup in the two-period model, in each period, a potential entrant has two options: enter the foreign market in the current period or wait until the next period. In period $t$, prior to the entry decision, the potential entrant draws an idiosyncratic foreign demand, $\epsilon_{jt}$, from the distribution $F(\epsilon_{jt}; \sigma_t)$. To start exporting, an entrant must pay a fixed entry cost, $\zeta > 0$. We assume that the household that owns the firm finances the entry cost.

A potential entrant with idiosyncratic demand $\epsilon_{jt}$ compares the value of exporting $V_t(\epsilon_{jt}; \sigma_t)$ net of the entry cost $\zeta$ with the value of waiting $V^{w}_t(\sigma_t)$. Specifically, the optimal entry decision
solves the following problem:

\[
V_t(\varepsilon_{jt}; \sigma_t) \equiv \max \left\{ V_t^w(\sigma_t), V_t(\varepsilon_{jt}; \sigma_t) - \zeta \right\},
\]

in which the value of waiting \( V_t^w(\sigma_t) \) satisfies

\[
V_t^w(\sigma_t) = \beta (1 - \delta) \mathbb{E}_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \int V_{t+1}(\varepsilon_{jt+1}; \sigma_{t+1}) dF(\varepsilon_{t+1}; \sigma_{t+1}) \mid \sigma_t \right\}.
\]

Denote by \( \varepsilon_{et} \) the cutoff of the idiosyncratic demand shock that equates the value of waiting and the value of exporting net of the entry cost; then, we have

\[
V_t(\varepsilon_{et}; \sigma_t) - \zeta = V_t^w(\sigma_t).
\]

The exporting decision follows a trigger strategy. The potential entrant opts to enter the market when demand \( \varepsilon_{jt} \) is above \( \varepsilon_{et} \). Otherwise, the potential entrant chooses to wait and remain inactive. In the latter case, the inactive firm in period \( t \) receives zero profit. In period \( t+1 \), with probability \( 1 - F(\varepsilon_{et+1}; \sigma_{t+1}) \), the inactive firm may draw a relatively high demand, i.e., \( \varepsilon_{jt+1} \geq \varepsilon_{et+1} \), and become an exporter; with probability \( F(\varepsilon_{et+1}; \sigma_{t+1}) \), the firm remains inactive.

## 4.4 Aggregation and Equilibrium

Let \( M_t \) denote the end-of-period mass of exporting firms. The law of motion for \( M_t \) is

\[
M_t = (1 - \delta) M_{t-1} + [1 - F(\varepsilon_{et}; \sigma_t)] [1 - (1 - \delta) M_{t-1}] - [1 - F(\varepsilon_{et}; \sigma_t)] (1 - (1 - \delta) M_{t-1}).
\]

Aggregate labor demand \( L_t \) consists of the demand from the domestic sector \( L_t^D \) and the export sector \( L_t^E \)

\[
L_t = L_t^D + L_t^E,
\]

where \( L_t^E = \int_0^{M_t} l_{jt} dj \).
We now discuss the current account. Real exports and imports are given by

\[ \text{EX}_t = \int_0^{M_t} \frac{P_j x_{jt}}{P_t} dj = \theta \pi_{bt} \int_0^{M_t} \min \left\{ \frac{\varepsilon_{jt}}{\varepsilon_{bt}}, \left[ \frac{\varepsilon_{jt}}{\varepsilon_{bt}} \right]^\frac{1}{2} \right\} dj, \]  

(40)

\[ \text{IM}_t = q_t Y^*_t, \]  

(41)

where the real exchange rate \( q_t \) is defined as \( q_t = \frac{e_t P^*_t}{P_t} \). The second equality in the definition of exports \( \text{EX}_t \) is due to the optimal pricing equation (29) and the optimal producing decision (30).

The current account \( \text{CA}_t \) is defined as the sum of the net export and the net factor payment

\[ \text{CA}_t = \text{EX}_t - \text{IM}_t + q_t \frac{P^*_{t-1}}{P_t^*_t} (B^*_{t-1} - 1) b^*_{t-1}. \]  

(42)

The accounting identity in the balance of payments implies that the aggregate stock of foreign bonds satisfies

\[ \text{CA}_t = \frac{e_t B^*_t - B^*_t}{P_t} = q_t \left( b^*_t - \frac{P^*_{t-1}}{P^*_t} b^*_{t-1} \right). \]  

(43)

The gross domestic product (GDP) is defined as the sum of gross domestic absorption \( Z_t \) and net exports

\[ \text{GDP}_t = Z_t + \text{EX}_t - \text{IM}_t, \]  

(44)

where the gross domestic absorption is \( Z_t = \frac{P^D_t}{P_t} Y^D_t + \text{IM}_t. \)

The resource constraint is

\[ C_t + \frac{B_t + e_t B^*_t \Omega}{P_t} \left( \frac{B_t}{B_t + e_t B^*_t} - \bar{\psi} \right)^2 + \zeta [M_t - (1 - \delta) M_{t-1}] = Z_t. \]  

(45)

We assume that the domestic bond \( B_t \) is issued by the central government with a fixed supply \( B_t = B^*. \)

A competitive equilibrium consists of allocations and prices such that (i) taking the prices as given, the allocations solve the optimization problems of the household and both domestic good producers and firms in the export sector; (ii) the prices clear the markets for immediate goods, final goods, bonds and labor.\(^7\)

Appendix B summarizes the full set of dynamic equilibrium conditions. Appendix C describes our approach to solve the steady-state equilibrium. We solve the full dynamic system through the standard first-order perturbation method used in the real business cycle literature.

\(^7\)For simplicity, we assume that the purchasing power parity (PPP) condition holds in our model; therefore, the real exchange rate is constant, i.e., \( q_t = 1. \)
5 Quantitative Analysis

5.1 Calibration

We calibrate our model to the Chinese economy. One period corresponds to one quarter. For parameters commonly used in the literature, we assign their values according to the existing research. For the model-specific parameters, we calibrate their values directly by matching the model-implied moments with those in the data. Table 3 summarizes all of the parameter values.\(^8\)

For the parameters in households’ utility function, we closely follow those in the Chang et al. (2015). In particular, we set the discount factor to \(\beta = 0.995\) and the inverse Frisch elasticity to \(\gamma = 2\). We calibrate the weight of labor in utility function \(\varpi\) such that in the steady state, households allocate 40% of their time to work. For the parameters regarding the export sector, we again follow Chang et al. (2015). In particular, we set the export demand elasticity to \(\theta = 1.5\). Based on the calculation in Chang et al. (2015), we set the steady-state share of the domestic bond to \(\bar{\psi} = 0.9\). We set the portfolio adjustment cost to \(\Omega = -0.6\) to capture the fact that it is costly for Chinese households to adjust their portfolios due to tight capital controls.\(^9\) For the elasticity of substitution between domestic intermediate goods and imported foreign goods in the production of final goods, we follow Alessandria et al. (2015) and set it to \(\eta = 1.5\).

To calibrate the exit rate of exporting firms \(\delta\), we first merge China’s General Administration of Customs database with the Chinese Industrial Enterprises database from 2000 to 2013. We then compute the average annual exit rate of exporting firms. The number is 16%, corresponding to a quarterly exit rate of 4.2%. Therefore, we set \(\delta\) to be 0.042.

We normalize the mean of the idiosyncratic demand shock \(\mu\) to 1. We jointly calibrate the financial constraint parameter \(\xi\), the fixed entry cost \(\zeta\), the standard deviation of idiosyncratic demand shock \(\sigma\) (market uncertainty in the steady state) and the share of the domestic good in final good production \(\omega\) using the moment matching approach. We target four aggregate moments in the Chinese data: (i) aggregate export-to-GDP ratio of 0.268; (ii) aggregate import-to-GDP ratio of 0.234; (iii) the ratio between the 80th percentile and the 20th percentile in the distribution of the logarithmic export values for exporting firms, which is 11.25; and (iv) the quarterly entry probability of exporters of 0.078.

To pin down the process of exogenous market uncertainty, we assume that \(\sigma_t\) evolves according

\(^8\) We aggregate the foreign demand of individual exporters in equation (26) and \(\mu = 1\). The value of the aggregate component of foreign demand is set to \(X^* = \bar{q}Y^E\) in the steady state, which is 0.068.

\(^9\) Since in the steady state the bond holdings of the representative household are negative, the value of \(\Omega\) is set to be negative to make the adjustment cost convex in \(\psi_t\).
Table 3: Parameter Values

<table>
<thead>
<tr>
<th>Assigned Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$ discount factor</td>
<td>0.995</td>
</tr>
<tr>
<td>$\bar{\psi}$ portfolio share of domestic bonds</td>
<td>0.9</td>
</tr>
<tr>
<td>$\Omega$ portfolio adjustment cost</td>
<td>-0.6</td>
</tr>
<tr>
<td>$\gamma$ inverse Frisch elasticity</td>
<td>2</td>
</tr>
<tr>
<td>$\theta$ export demand elasticity</td>
<td>1.5</td>
</tr>
<tr>
<td>$\eta$ elasticity of substitution in final good production</td>
<td>1.5</td>
</tr>
<tr>
<td>$\bar{q}$ real exchange rate</td>
<td>1</td>
</tr>
<tr>
<td>$\mu$ mean of demand shock (normalized)</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Calibrated Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi$ financial constraint</td>
<td>0.02</td>
</tr>
<tr>
<td>$\zeta$ fixed entry cost</td>
<td>0.99</td>
</tr>
<tr>
<td>$\omega$ share of domestic good in final good production</td>
<td>0.61</td>
</tr>
<tr>
<td>$\sigma$ standard deviation of demand shock</td>
<td>2.89</td>
</tr>
<tr>
<td>$\delta$ exit rate of exporters</td>
<td>0.042</td>
</tr>
<tr>
<td>$\varpi$ weight of dis-utility of labor</td>
<td>5.61</td>
</tr>
<tr>
<td>$\rho_v$ persistence of uncertainty shock</td>
<td>0.96</td>
</tr>
<tr>
<td>$\Sigma_v$ standard deviation of uncertainty shock</td>
<td>0.35</td>
</tr>
</tbody>
</table>

We choose the values of persistence parameter $\rho_v$ and standard deviation of disturbances $\Sigma_v$ to match the auto-correlation and the standard deviation of the volatility of the logarithmic export values for exporting firms in the model with that in China’s General Administration of Customs database. We eventually obtain $\rho_v = 0.96$ and $\Sigma_v = 0.35$.

Appendix D provides more details about the data construction and the procedure of moment matching in the above calibration exercise.

5.2 Dynamics

We now discuss the consequences of the market uncertainty shock for aggregate export dynamics. The economy is in its steady state in the initial period. Suppose that the economy is hit by a one-standard-deviation positive uncertainty shock $\epsilon_t$ in period 1. The market uncertainty $\sigma_t$ increases and evolves according to

$$\log \sigma_t - \log \sigma = \rho_v (\log \sigma_{t-1} - \log \sigma) + \epsilon_t.$$  

(46)
Figure 3 shows the impulse responses of the extensive margin of exports (defined as the number of exporters), the intensive margins of exports (defined as the average size of exports), the export-to-GDP ratio, GDP and the volume of imports when the market uncertainty temporarily increases by 35% of the initial steady-state level. Consistent with the analytical results stated in the simple two-period model, both the extensive and the intensive margins of exports decline after a rise in market uncertainty. The adverse effect on these two margins causes a large drop in aggregate export. The uncertainty shock also depresses aggregate output, but the magnitude is much smaller than the response of exports. The excessive volatility of export dynamics is also reflected by a large drop in the export-to-GDP ratio. In addition, the import volume exhibits a moderate increase after the uncertainty shock. This is because domestic absorption (especially domestic consumption) increases after the massive drop in exports.\textsuperscript{10}

5.2.1 Why Does Wait-and-See Not Work?

In our model, market uncertainty affects export dynamics through two channels: the wait-and-see channel and the financial constraint channel. Without a financial constraint on exporters, an increase in market uncertainty raises the value of waiting, resulting in fewer firms entering the foreign market and being exporters. On the other hand, the average size of sales conditional on a firm being an exporter tends to be larger because of the selection effect. Thus, the wait-and-see channel suggests that the uncertainty shock decreases the extensive margin of exports but increases the intensive margin of exports. Figure 4 shows the dynamics of the extensive and intensive margins of exports and the export-to-GDP ratio in response to a positive uncertainty shock. The dashed lines pertain to the responses in the control model with only the wait-and-see channel. An increase in market uncertainty reduces the number of exporters but increases the average size of sales for exporters. The positive response of the intensive margin of exports largely offsets the adverse effect of the uncertainty shock on aggregate exports. The export-to-GDP ratio becomes much smoother than that in the baseline model. The above results indicate that the control model with only the wait-and-see channel fails to explain the previous empirical findings. The model cannot generate a trade collapse or excessive volatility in the aggregate export dynamics.

\textsuperscript{10}In the quantitative model proposed by Bloom et al. (2018), they also find that consumption increases with a positive uncertainty shock. They supplement their model with a negative first-moment shock to fit the consumption dynamics in the real data. We can generate a negative response of consumption in a similar way, although the focus of this paper is on the trade dynamics after an uncertainty shock.
Figure 3: Responses to an Increase in Market Uncertainty

Notes: This figure reports the impulse responses of key variables under a one-standard-deviation increase in the uncertainty shock. The x-axis is the number of periods. The y-axis indicates the deviation from the steady state (0.01=1%). The extensive margin of exports is defined as the number of exporters, i.e., $M_t$. The intensive margin of exports is defined as the average size of sales for exporters, i.e., $EX_t/M_t$. 
Figure 4: Dynamics in the Control Model with Wait-and-see Channel

Notes: This figure reports the impulse responses in the control model with only the wait-and-see channel under a one-standard-deviation increase in the uncertainty shock. The solid lines are responses in the baseline model. The dashed lines are responses in the control model. The x-axis is the number of periods. The y-axis indicates the deviation from the steady state (0.01=1%). The extensive margin of exports is defined as the number of exporters, i.e., $M_t$. The intensive margin of exports is defined as the average size of sales for exporters, i.e., $EX_t/M_t$. 
5.2.2 The Role of Financial Constraints

The second channel through which market uncertainty may affect export dynamics is the financial constraint channel. In our model, firms face working capital constraints. We follow Jermann and Quadrini (2012) and assume that the borrowing capacity increases with the expected firm value.\footnote{In a different setup, Cao et al. (2019) also show that when a firm can choose not to repay its debt and divert a certain fraction of its assets, the firm’s borrowing constraint is proportional to its continuation value.} The working capital constraint distorts the pricing and production behaviors of those firms with high demand, leaving other firms with low demand unaffected. As a result, the profit function of the exporting firm is concave in the idiosyncratic market demand. An increase in market uncertainty reduces the value of exporting in the future. This effect reinforces the reduction in the extensive margin of exports caused by the wait-and-see channel. In addition, a positive uncertainty shock also depresses the exporting firm’s borrowing capacity due to a lower expected firm value. The reduction in the borrowing capacity generates the contractionary impact of the uncertainty shock on the intensive margin of exports.

Figure 5 compares the export dynamics in the baseline model with those in the case of loose financial constraints. In the latter case, we set the parameter in the working capital constraint $\xi$ to 0.2. With sufficiently loose financial constraints, only 2% of incumbents in the export sector are financially constrained in the steady-state equilibrium. However, in the baseline model, where $\xi = 0.02$, 32% of incumbents in the export sector are financially constrained. The figure shows that having sufficiently loose financial constraints significantly mitigates the negative responses of exports to a positive uncertainty shock. In particular, both the extensive and intensive margins of exports have considerably weaker responses than those in the baseline model, resulting in a much smoother export-to-GDP ratio. The above results suggest that financial constraints play a critical role in the transmission of uncertainty shocks to international trade.

In Section 3, Proposition 2 of the two-period model highlights the role of endogenous financial constraints in explaining the negative response of the intensive margin of exports to the uncertainty shock. If the financial constraints are exogenous, then the exporters’ borrowing limit does not respond to the uncertainty shock, and response of the intensive margin of trade to an increase in market uncertainty should be positive because of the selection effect. To confirm that this intuition from the two-period model still holds in the fully fledged quantitative model, we compare the impulse response functions of the uncertainty shock in the baseline model with those in another model where the exporters’ borrowing limit is fixed at $\chi_t = \chi^*$, which is its steady-state value in the baseline model. The other components are the same as in the baseline model. In Figure 6, we see that when the borrowing limit is exogenous, the intensive margin of exports increases in response to
Figure 5: Dynamics under Loose Financial Constraints

Notes: This figure reports the impulse responses of key variables under a one-standard-deviation increase in the uncertainty shock. The x-axis is the number of periods. The y-axis indicates the deviation from the steady state (0.01=1%). The extensive margin of exports is defined as the number of exporters, i.e., $M_t$. The intensive margin of exports is defined as the average size of sales for exporters, i.e., $EX_t/M_t$. For the case of loose financial constraints, we set the parameter $\xi = 0.2$. 


Figure 6: Dynamics with an Exogenous Financial Constraint

Notes: This figure reports the impulse responses of key variables under a one-standard-deviation increase in the uncertainty shock. The x-axis is the number of periods. The y-axis indicates the deviation from the steady state (0.01=1%). The extensive margin of exports is defined as the number of exporters, i.e., $M_t$. The intensive margin of exports is defined as the average size of sales for exporters, i.e., $EX_t/M_t$. For the case of exogenous financial constraints, we keep the exporters’ borrowing limit at $\chi_t = \chi^*$, its steady-state value in the baseline model.

the uncertainty shock. The response of the extensive margin of exports to the uncertainty shock is still negative but quantitatively smaller than that in the baseline model with endogenous financial constraints. The response of the export-to-GDP ratio is also smaller in the exogenous borrowing limit case. All the results here are consistent with Proposition 2 in the two-period model.

5.3 China’s Trade Collapse in the 2008 Financial Crisis

After its accession to the World Trade Organization (WTO) in 2001, China’s real exports grew at the astonishing annual rate of approximately 20% before 2008. The global financial crisis caused an abrupt and sharp halt of this upward trend. China’s exports started to decline in the last quarter of 2008 and reached their trough in 2009 with an annual growth rate of negative 18%, which is significantly above the worldwide 12% of trade contraction in 2009 (Carballo et al., 2018). Given the enormous size of China’s exports, the contraction in its absolute value was unusually
Unlike the United States, China’s trade collapse was largely driven by the turbulence in foreign markets. To what extent does the uncertainty in foreign markets quantitatively explain the Great Trade Collapse in China? To answer this question, we simulate the export dynamics by feeding the time series of market uncertainty into our quantitative model. We first estimate the sequence of market uncertainty by selecting the values of market uncertainty $\sigma_t$ to exactly match the model-implied dispersion of the firm-level export volume with that in China’s General Administration of Customs Database. Appendix D provides additional details. We then feed the constructed uncertainty sequence into the model and compare the model-generated trade dynamics with the counterparts in the data.

The first panel in Figure 7 plots the constructed sequence of market uncertainty from 2007 to 2011. It can be seen that the uncertainty in foreign markets increased from 2007 and reached its peak in 2009, which is 30% higher than its value in 2007. Thereafter, uncertainty declines mildly in 2010 and rises again in 2011, probably driven by the European sovereign debt crisis. The surge in market uncertainty after the 2008 financial crisis leads to substantial declines in China’s exports over this period. The panel “volume of exports” in Figure 7 shows that the model-generated aggregate exports track the corresponding dynamics in the data quite well. In particular, Our quantitative model generates a 7.25% decline in exports in 2009, whereas in the data, exports decline by 29.4%. Thus, market uncertainty per se explains approximately one-quarter of China’s trade collapse in the 2008 financial crisis.

We further decompose exports into the extensive margin (the number of exporters) and the intensive margin (the average size of sales for exporters). The second and third top panels in Figure 7 show that both margins present sharp decreases over the financial crisis period. In 2009, the intensive margin falls by 4.25% in the model versus the 17.7% drop in the data, while the extensive margin falls by 3% in the model versus the 9.4% drop in the data. Thus, our quantitative model accounts for 24% and 32% of the declines in the intensive margin and the extensive margin, respectively.

To study the consequences of the uncertainty shock for aggregate output in China, we also simulate the model-implied GDP path and compare it with the data. Our quantitative model generates a sizable reduction in the aggregate economy. In particular, the GDP in the model declines by 1.4% versus the 2.1% drop in the data, accounting for 65% of the observed aggregate slowdown during the financial crisis.\textsuperscript{12} Therefore, our quantitative results propose a novel channel

\textsuperscript{12}Of course, we should not deny the effects of other shocks and mechanisms at play during the Great Recession from which our model abstracts. For example, Ouyang and Peng (2015) show that the 2008 Chinese Economic Stimulus Program, which advanced a stimulus package of four trillion RMB – equivalent to 586 billion USD – raised
Figure 7: Uncertainty and Trade Collapse in China: Financial Crisis Episode

Notes: This figure compares the model-generated dynamics with those in the real data. The solid lines represent the simulated sequences. The dashed lines represent the data sequences. For each variable, the deviations are computed as the percentage changes from their respective trends generated by a Hodrick-Prescott (HP) filter with smoothing parameter $\lambda = 6.25$. We normalize each sequence by the first observation in 2007.
for the economic uncertainty shock to influence aggregate fluctuations.

5.4 Trade Collapse During the U.S.-China Trade War

The U.S.-China trade war describes an ongoing trade conflict between the two largest economies in the world, the United States and China. The trade war was initiated by the 45th president of the United States, Donald Trump, in April 2018 to force China to abandon its “unfair trade practices” in its bilateral trade with the U.S. The trade war gradually escalated, featuring increased tariffs and other trade barriers in both countries. In addition, the trade war also significantly increased demand uncertainty in international trade. The U.S. trade policy uncertainty (TPU) index constructed by Baker, Bloom and Davis (2016) increased extraordinarily as the U.S.-China conflict intensified. The average index value of 323 between April 2018 and August 2019 is 8.2 times as high as its average between 2013 and 2015.

According to our model, such an increase in uncertainty should dramatically decrease bilateral trade. To quantify the effect of the uncertainty shock, we feed the TPU time series for the U.S. into our quantitative model and compare the model-generated exports with those in the data. The result is given in Figure 8. The uncertainty series reached its peak in February 2019 and decreased in the following several months. In response, the model-generated export series to U.S. also reached its trough in February 2019, similar to the observed pattern in the data. We find that the uncertainty shock per se explains 61 percent of the drop in the data, suggesting that the uncertainty shock plays a significant role here.

6 Conclusion

The recent global financial crisis has prompted the Great Trade Collapse and a surge in economic uncertainty. We examine the consequences of market uncertainty for firm-level export decisions and aggregate export dynamics. We propose a novel channel through which uncertainty shocks from a destination market may affect domestic export dynamics. Using Chinese customs data, we find that uncertainty in the foreign market has adverse impacts on the extensive and intensive margins of China’s exports. In particular, an increase in market uncertainty reduces the number of exporters and the average trade volume for exporters. The adverse effects are more pronounced in industries with tighter financial constraints than in other industries.

annual GDP growth by 3.2%.

13For this exercise, because we lack firm-level data for this period, we choose to measure the market uncertainty of trade using the TPU time series from Baker, Bloom and Davis (2016).
Figure 8: Uncertainty and Trade Collapse in China: Trade War Episode

Notes: This figure compares the model-generated dynamics with those in the data. The solid lines represent the simulated sequences. The dashed lines represent the data sequences. For each variable, the deviations are computed as the percentage changes from the first observation in December 2018.
We then construct a dynamic trade model to account for these facts. The model features heterogeneous firms that face idiosyncratic foreign demand and limited external financing capacity. The dispersion of idiosyncratic demand captures the market uncertainty in the destination country. Each potential entrant has options to enter the foreign market (export) in the current period or to wait until the next period. Exporting to the foreign market incurs a fixed entry cost. Thus, a rise in market uncertainty increases the option value of waiting, leading to the standard wait-and-see effect in the uncertainty literature. In addition, the financial constraints distort the optimal pricing and production decisions for the firms with high foreign demand. A positive uncertainty shock raises both the top and bottom tails of the distribution of idiosyncratic demand faced by exporters. Under financial constraints, market uncertainty reduces the expected value of exporting because exporters cannot benefit from a high demand but suffer greater bottom-tail demand risk. The uncertainty shock also has adverse financial consequences for exporters through the endogenous borrowing constraint, which in turn reduces the scale of production for incumbent exporters. Therefore, our dynamic model generates adverse effects of market uncertainty on both the extensive and intensive margins of exports.

We then calibrate our fully fledged dynamic model to the Chinese economy. We quantitatively evaluate the impact of foreign market uncertainty on China’s export dynamics. Our quantitative exercise suggests that the model is able to generate excessive volatility of exports and that market uncertainty per se accounts for 25% of the trade collapse. Moreover, an uncertainty shock can generate sizable declines in aggregate output. The above results suggest that market uncertainty is an important explanatory factor for trade dynamics and aggregate fluctuations.
References


Jermann, Urban and Vincenzo Quadrini, “Macroeconomic effects of financial shocks,” *The American Economic Review*, 2012, 102 (1), 238–271. 4.3.1, 5.2.2


Appendix

A  Impact of Uncertainty on Total Export

We evaluate the impact of foreign market uncertainty on the total value of export. The table below reports the main results. It shows that an increase in foreign market uncertainty leads to negative impact on both of the extensive and intensive margins. The adverse effect is more pronounced for those industries with more severe financial constraints. These results are consistent with those in the baseline estimation.

Table 4: Impact of Uncertainty on Total Export

<table>
<thead>
<tr>
<th></th>
<th>Baseline (σ_{jt})</th>
<th>Alternative (σ_{alt}^{jt})</th>
</tr>
</thead>
<tbody>
<tr>
<td>log(Exports_{hjt})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>σ_{jt}</td>
<td>-0.608*** (-0.132)</td>
<td>-0.583*** (-0.131)</td>
</tr>
<tr>
<td>σ_{jt} × FinDep_h</td>
<td>-0.641*** (0.200)</td>
<td>-0.529** (0.223)</td>
</tr>
<tr>
<td>Credit to GDP</td>
<td>0.280** (0.130)</td>
<td>0.279** (0.130)</td>
</tr>
<tr>
<td>log(GDP)</td>
<td>0.394* (0.230)</td>
<td>0.400* (0.228)</td>
</tr>
<tr>
<td>log(GDP per capita)</td>
<td>0.350* (0.210)</td>
<td>0.367* (0.210)</td>
</tr>
<tr>
<td>Year fixed effects</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Product-destination fixed effects</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Observations</td>
<td>2,836,774 2,728,256</td>
<td>2,836,774 2,728,256</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>0.705</td>
<td>0.705</td>
</tr>
</tbody>
</table>

Notes: The levels of significance are denoted as *** p < 0.01, ** p < 0.05, and * p < 0.1. The numbers in parentheses are robust standard errors corrected for clustering at the destination country level. The market uncertainty is measured by σ_{jt} in the first two columns and measured by σ_{alt}^{jt} in the last two columns.

B  Full Dynamic System

This appendix summarizes the full dynamic system of the quantitative model. With flexible exchange rate system and under the Purchasing-Power-Parity condition, we have q_t = 1. We also
normalize the international price level $P_t^* \equiv 1$. From the definition real exchange rate $q_t = \frac{P_t^*}{P_t}$, we have $P_t \equiv e_t$.

1. UIP condition $(B_t)$:
$$\Omega (\psi_t - \bar{\psi}) = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} (R_t^e - R_t^r).$$
(B.1)
in which $R_t^e = \frac{P_t}{P_{t+1}} R_t$ is the real interest rate.

2. Optimal condition for foreign bonds holdings $B_t^*$:
$$1 + \frac{\Omega}{2} (\psi_t - \bar{\psi})^2 - \Omega (\psi_t - \bar{\psi}) \psi_t = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} R_t^e,$$
(B.2)
where the share of domestic bonds to the total bonds holdings $\psi_t$ is defined as
$$\psi_t = \frac{b_t}{b_t + b_t^*},$$
(B.3)
in which $b_t = \frac{B_t}{P_t}$ is the real level of domestic bond holding and $b_t^* = \frac{B_t^*}{q_t} = B_t^*$.

3. Aggregate labor supply $(L_t)$:
$$\omega L_t^\gamma = w_t \Lambda_t,$$
(B.4)
where $\Lambda_t$ satisfies
$$1/C_t = \Lambda_t.$$  
(B.5)

4. Optimal demand of domestic good $Y_t^D$ and foreign good $Y_t^*$ are given by
$$Y_t^D = \omega^{\gamma} \left( \frac{P_t^D}{P_t} \right)^{-\eta} Z_t,$$
(B.6)
$$Y_t^* = (1 - \omega)^{\eta} q_t^{-\eta} Z_t.$$  
(B.7)

5. The price indexation equation is
$$1 = \left[ \omega^{\eta} \left( \frac{P_t^D}{P_t} \right)^{1-\eta} + (1 - \omega)^{\eta} q_t^{1-\eta} \right]^{\frac{1}{1-\eta}}.$$  
(B.8)

6. The optimal price of domestic goods $p_t^D$:
$$p_t^D = \frac{\eta}{\eta - 1} w_t.$$  
(B.9)
7. Definition of the expected value of exporting $\bar{V}_t(\sigma_t)$:

$$
\bar{V}_t(\sigma_t) = \beta (1 - \delta) \mathbb{E}_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \left[ \int \pi_{jt+1}(\varepsilon_{jt+1}) d\mathbf{F}(\varepsilon_{jt+1}; \sigma_{t+1}) + \bar{V}_{t+1}(\sigma_{t+1}) \right] | \sigma_t \right\}, \quad (B.10)
$$

where the profit $\pi_{jt}$ is defined as

$$
\pi_{jt} = \min \left\{ \frac{\varepsilon_{jt}}{\varepsilon_{bt}}, \theta \left[ \frac{\varepsilon_{jt}}{\varepsilon_{bt}} \right]^{\frac{1}{\theta}} - \theta + 1 \right\}. \quad (B.11)
$$

8. Output, cutoff and profit for financially constrained marginal firms are respectively given by

$$
x_{bt} = \varepsilon_{bt} \bar{p}_t - \theta \bar{q}_t X_t^*, \quad (B.12)
$$

$$
\varepsilon_{bt} = \frac{\chi_t}{q_t w_t X_t^* \bar{p}_t}, \quad (B.13)
$$

$$
\pi_{bt} = \frac{1}{\theta} \bar{p}_t x_{bt}. \quad (B.14)
$$

9. The endogenous borrowing limit $\chi_t$ for exporters is assumed to be proportional to the expected firm value:

$$
\chi_t = \xi \bar{V}_t(\sigma_t). \quad (B.15)
$$

10. Total mass of exporting firms $M_t$ evolves as

$$
M_t = (1 - \delta) M_{t-1} + [1 - \mathbf{F}(\varepsilon_{et}; \sigma_t)] [1 - (1 - \delta) M_{t-1}], \quad (B.16)
$$

where the threshold of exporting $\varepsilon_{et}$ satisfies

$$
V_t(\varepsilon_{et}; \sigma_t) = \pi_{et} + \bar{V}_t(\sigma_t) - \zeta = V_t^w(\sigma_t), \quad (B.17)
$$

where $\pi_{et} = \left\{ \begin{array}{ll} \pi_{bt} \frac{\varepsilon_{et}}{\varepsilon_{bt}} & \text{if } \varepsilon_{et} \leq \varepsilon_{bt} \\ \pi_{bt} \left( \theta \left[ \frac{\varepsilon_{et}}{\varepsilon_{bt}} \right]^{\frac{1}{\theta}} - \theta + 1 \right) & \text{otherwise} \end{array} \right. \quad (B.18)
$$

11. The value of waiting is

$$
V_t^w(\sigma_t) = \beta (1 - \delta) \mathbb{E}_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \int \forall_{t+1}(\varepsilon_{jt+1}; \sigma_{t+1}) d\mathbf{F}(\varepsilon_{t+1}; \sigma_{t+1}) | \sigma_t \right\}, \quad (B.19)
$$
where the ex ante value for a potential entrant $V_t(\varepsilon_{jt}; \sigma_t)$ satisfies

$$V_t(\varepsilon_{jt}; \sigma_t) \equiv \max \{ V_t^w(\sigma_t), V_t(\varepsilon_{jt}; \sigma_t) - \zeta \}.$$  \hspace{1cm} (B.20)

12. Aggregate labor demand $L_t$

$$L_t = Y_t^D + Y_t^E,$$ \hspace{1cm} (B.21)

where

$$Y_t^E = \int_0^{M_t} x_{jt} dj = x_{bt} J_t,$$ \hspace{1cm} (B.22)

$$J_t = (1 - \delta) M_{t-1} g_t(\varepsilon_{\min}) + [1 - (1 - \delta) M_{t-1}] g_t(\varepsilon_{et}),$$ \hspace{1cm} (B.23)

$$g_t(v) = \int_v^{\varepsilon_{\min}} \min \left\{ v \varepsilon_{bt}, 1 \right\} dF(\varepsilon; \sigma_t).$$ \hspace{1cm} (B.24)

13. Aggregate import:

$$\text{IM}_t = q_t Y_t^*.$$ \hspace{1cm} (B.25)

14. Aggregate export:

$$\text{EX}_t = \int_0^{M_t} P_t x_{jt} dj = \theta_\pi_{bt} \Delta_{et},$$ \hspace{1cm} (B.26)

where

$$\Delta_{et} = (1 - \delta) M_{t-1} h_t(\varepsilon_{\min}) + [1 - (1 - \delta) M_{t-1}] h_t(\varepsilon_{et}),$$ \hspace{1cm} (B.27)

$$h_t(v) = \int_v^{\varepsilon_{\min}} \min \left\{ v \varepsilon_{bt}, \left[ \frac{v}{\varepsilon_{bt}} \right]^2 \right\} dF(\varepsilon; \sigma_t).$$ \hspace{1cm} (B.28)

15. The GDP is

$$\text{GDP}_t = Z_t + \text{EX}_t - \text{IM}_t.$$ \hspace{1cm} (B.29)

16. Resource constraint is given by

$$C_t + \frac{B_t + e_t B_t^* \Omega}{P_t} \frac{B_t}{B_t + e_t B_t^*} = \left( \frac{B_t}{B_t + e_t B_t^*} - \psi \right)^2 + \zeta [M_t - (1 - \delta) M_{t-1}] = Z_t.$$ \hspace{1cm} (B.30)

17. BOP equation gives

$$0 = \text{EX}_t - \text{IM}_t + q_t \left( \frac{P_{t-1}}{P_t} R_{t-1} b_{t-1}^* - b_t^* \right).$$ \hspace{1cm} (B.31)
C  Solving the Steady State

From (B.1) and (B.2), the interest rates $R$ and $R^*$ are given by

$$R^e = R^* = 1/\beta,$$  \hspace{1cm} (C.1)

Given $q_t \equiv 1$, from (B.8) and (B.9), we must have

$$p^D = \left[ 1 - (1 - \omega)^\eta \right]^{1/\eta},$$ \hspace{1cm} (C.2)

$$w = \frac{\eta - 1}{\eta} p^D,$$ \hspace{1cm} (C.3)

$$\bar{p} = \frac{\theta}{\theta - 1} w.$$ \hspace{1cm} (C.4)

Given the aggregate spending $Z$, from (B.6) and (B.7), we can solve $Y^D$ and $Y^*$ as

$$Y^D = \omega^n (p^D)^{-\eta} Z,$$ \hspace{1cm} (C.5)

$$Y^* = (1 - \omega)^n Z,$$

Given $\varepsilon_b$ and $X^*$, from (B.12), we obtain

$$x_b = \varepsilon_b \bar{p}^{\theta - \theta} X^*.$$ \hspace{1cm} (C.6)

Furthermore, (B.13) and (B.14) imply

$$\chi = wx_b,$$ \hspace{1cm} (C.7)

$$\pi_b = \frac{1}{\theta} \bar{p} x_b.$$ \hspace{1cm} (C.8)

(B.15) implies $\bar{V} = \frac{1}{\xi}$. From (B.10), we obtain the implicit function that solves the cutoff $\varepsilon_b$:

$$\frac{\beta (1 - \delta)}{1 - \beta (1 - \delta)} \frac{1}{\theta - 1} \int \Psi (\varepsilon) dF (\varepsilon; \sigma) = \frac{1}{\xi},$$ \hspace{1cm} (C.9)

where

$$\Psi (\varepsilon) = \begin{cases} \frac{\varepsilon}{\bar{p}^*} & \text{if } \varepsilon < \varepsilon_b \\ \theta \left( \frac{\varepsilon}{\varepsilon_b} \right)^{\frac{1}{\beta}} - \theta + 1 & \text{otherwise} \end{cases}.$$
Since \( \int \Psi(\varepsilon) \, dF(\varepsilon;\sigma) \) is strictly decreasing in \( \varepsilon_b \), we can find a unique solution of \( \varepsilon_b \).

Given \( \varepsilon_e \), from (B.16), we can solve \( M \) as

\[
M = \frac{1 - \text{F}(\varepsilon_e;\sigma)}{1 - \text{F}(\varepsilon_e;\sigma)(1 - \delta)}.
\]

(\text{C.10})

\( Y^E \) can be expressed as

\[
Y^E = x_b J,
\]

(\text{C.11})

\[
J = (1 - \delta) Mg(\varepsilon_{\min}) + [1 - (1 - \delta) M] g(\varepsilon_e),
\]

(\text{C.12})

\[
g(v) = \int_{v}^{\infty} \min\left\{ \frac{v}{\varepsilon_b}, 1 \right\} dF(\varepsilon;\sigma).
\]

(\text{C.13})

From (26), we have

\[
q^\theta X^* = Y^E.
\]

(\text{C.14})

which determines the value of \( \varepsilon_e \). Since from (C.6), \( x_b \) is proportional to \( X^* \), and then \( X^* \) can be canceled from both sides of equation (C.14).

Given \( \varepsilon_e \), from (B.19), we can express the value of waiting as

\[
V^w = \pi_e + \bar{V} - \zeta
\]

\[
= \frac{\beta (1 - \delta)}{1 - \beta (1 - \delta)} \left[ \text{E}\pi(\varepsilon_j \geq \varepsilon_e) - (1 - \text{F}(\varepsilon_e;\sigma)) \pi_e \right]
\]

(\text{C.15})

where

\[
\pi_e = \begin{cases} 
\pi_b \varepsilon_e & \text{if } \varepsilon_e \leq \varepsilon_b \\
\pi_b \left( \frac{\varepsilon}{\varepsilon_b} \right)^{\frac{1}{\theta}} - \theta + 1 & \text{otherwise}
\end{cases}
\]

(\text{C.16})

and

\[
\text{E}\pi(\varepsilon_j \geq \varepsilon_e) = \begin{cases} 
\pi_b \left[ \frac{1}{\varepsilon_b} \int_{\varepsilon_b}^{\varepsilon_e} \varepsilon dF(\varepsilon;\sigma) + \int_{\varepsilon_e}^{\infty} \left( \theta \left( \frac{\varepsilon}{\varepsilon_b} \right)^{\frac{1}{\theta}} - \theta + 1 \right) dF(\varepsilon;\sigma) \right] & \text{if } \varepsilon_e \leq \varepsilon_b \\
\pi_b \int_{\varepsilon_e}^{\infty} \left( \theta \left( \frac{\varepsilon}{\varepsilon_b} \right)^{\frac{1}{\theta}} - \theta + 1 \right) dF(\varepsilon;\sigma) & \text{otherwise}.
\end{cases}
\]

(\text{C.17})

We can use (C.15) to pin down \( X^* \).

The aggregate labor is given by

\[
L = Y^D + Y^E.
\]

(\text{C.18})
We set the steady state labor to $L = 0.4$. Using (C.5), (C.14) and (C.18), we can solve the steady-state domestic absorption $Z$ as:

$$Z = \frac{L - Y^E}{\omega^n (p^D)^{-\eta}}.$$  \hfill (C.19)

From the resource constraint, we can solve the aggregate consumption as

$$C = Z - \zeta \delta M.$$  \hfill (C.20)

Using (B.4), we can calibrate the value of $\varpi$ in utility function (14) as $\varpi L^\gamma C - w = 0$.

From the definition of exports and imports, we have

$$\text{IM} = \bar{q} Y^*, \quad (C.21)$$
$$\text{EX} = \bar{p} b \Delta e, \quad (C.22)$$

where

$$\Delta e = (1 - \delta) M h (\varepsilon_{\min}) + [1 - (1 - \delta) M] h (\varepsilon_e), \quad (C.23)$$

$$h (v) = \int_v^\infty \min \left\{ \frac{v}{\varepsilon_b}, \left[ \frac{\varepsilon}{\varepsilon_b} \right]^{\frac{1}{\eta}} \right\} dF (\varepsilon; \sigma). \quad (C.24)$$

Given the ratio $\bar{\psi} = \frac{b}{b + \bar{b}}$ and $b$, we can solve the steady-state foreign bond holdings as

$$b^* = \frac{(1 - \bar{\psi}) b}{\psi \bar{q}}.$$  \hfill (C.25)

D Calibration

D.1 Data Description

1. **Firm-level Export Volume:** We use the individual export volume in China’s Custom Data to compute the moments and calibrate some of the structural parameters.

2. **Aggregate Data:** To calibrate the parameters, we use the aggregate data of export, import and GDP, which are from China’s Bureau of Statistics.
D.2 Estimation of the Uncertainty Shock Process

We first describe how to calibrate the AR(1) process of the uncertainty shock in equation (46). Since the uncertainty shocks cannot be directly observed in the data, we estimate the parameters for the process of uncertainty using the moments regarding the process of export volume observed in the data. Denote the standard deviation of ln \(ex_{jt}\) as \(\text{Vol}_t\), in which \(ex_{jt}\) is the exporting value of exporter \(j\) in period \(t\). In the following part, we describe the relationship between \(\text{Vol}_t\) and \(\sigma_t\).

We have assumed that the distribution of demand shock \(F(\varepsilon_j; \sigma)\) is a log-normal distribution with the mean normalized to one.\(^{14}\) For the corresponding Normal distribution, denote its mean and standard deviation as \(\hat{\mu}\) and \(\hat{\sigma}\) with \(\hat{\mu} = -\frac{1}{2}\hat{\sigma}^2\). For the ease of presentation, define \(\nu_b = \ln \varepsilon_b - \hat{\mu}, \nu_e = \ln \varepsilon_e - \hat{\mu}\).

Some truncated moments can be derived by the following equations:

\[
\begin{align*}
\mathbb{E}(\ln \varepsilon_j | \varepsilon_j \leq \varepsilon_b) &= \hat{\mu} - \hat{\sigma} \frac{\phi(\nu_e)}{\Phi(\nu_b)}, \\
\mathbb{E}(\ln \varepsilon_j | \varepsilon_j > \varepsilon_a) &= \hat{\mu} + \hat{\sigma} \frac{\phi(\nu_a) - \phi(\nu_e)}{1 - \Phi(\nu_a)}, \\
\mathbb{E}(\ln \varepsilon_j | \varepsilon_a \leq \varepsilon_j \leq \varepsilon_b) &= \hat{\mu} + \hat{\sigma} \frac{\phi(\nu_a) - \phi(\nu_b)}{1 - \Phi(\nu_a)}, \\
\mathbb{E}(\ln \varepsilon_j^2 | \varepsilon \leq \varepsilon_b) &= \hat{\sigma}^2 + \hat{\mu}^2 - (2\hat{\mu} + \hat{\sigma} \nu_b) \hat{\sigma} \frac{\phi(\nu_b)}{\Phi(\nu_b)}, \\
\mathbb{E}(\ln \varepsilon_j^2 | \varepsilon > \varepsilon_b) &= \hat{\sigma}^2 + \hat{\mu}^2 + (2\hat{\mu} + \hat{\sigma} \nu_b) \hat{\sigma} \frac{\phi(\nu_b)}{1 - \Phi(\nu_b)}, \\
\mathbb{E}(\ln \varepsilon_j^2 | \varepsilon_a \leq \varepsilon_j \leq \varepsilon_b) &= \hat{\sigma}^2 + \mu^2 + \sigma^2 \frac{\nu_a \phi(\nu_a) - \nu_b \phi(\nu_b)}{\Phi(\nu_b) - \Phi(\nu_a)} + 2\hat{\mu} \sigma \frac{\phi(\nu_a) - \phi(\nu_b)}{\Phi(\nu_b) - \Phi(\nu_a)},
\end{align*}
\]

in which \(\Phi(\cdot)\) is the CDF of the standard normal distribution while \(\phi(\cdot)\) is the PDF.

From equations (30) and (31), we can drive firm \(j\)'s exporting value \(ex_j = p_j x_j\) as

\[
ex_j = \bar{p} x_b \min \left\{ \frac{\varepsilon_j}{\varepsilon_b}, \left( \frac{\varepsilon_j}{\varepsilon_b} \right)^{\frac{1}{\vartheta}} \right\},
\]  

where \(\bar{p} = \frac{\theta}{\theta - 1} w\), and \(x_b = AX^* \varepsilon_b \bar{p}^{-\theta} q^\theta\). To be consistent with the data, we transform \(ex_j\) to its logarithmic form and get

\[
\ln ex_j = \ln (\bar{p} x_b) + \min \left\{ \ln \varepsilon_j - \ln \varepsilon_b, \frac{1}{\vartheta} (\ln \varepsilon_j - \ln \varepsilon_b) \right\}.
\]  

---

\(^{14}\)Time subscript \(t\) is omitted wherever possible.
The exporters in period $t$ consist of both incumbents and new entrants. Denote the share of incumbents in period $t$ as

$$\pi_{I,t} = \frac{(1 - \delta) M_{t-1}}{M_t},$$  \hspace{1cm} (D.9)

and the share of new entrants as

$$\pi_{E,t} = 1 - \pi_{I,t} = \frac{[1 - F(\varepsilon_e; \sigma_t)][1 - (1 - \delta) M_{t-1}]}{M_t}.$$  \hspace{1cm} (D.10)

Using (D.8), we get the mean of $\ln ex_j$ among incumbents as:

$$\mathbb{E}(\ln ex_j |\text{incumbents}) = \ln (\bar{p} x_b) + \left(1 + (\theta - 1) \frac{\Phi(\nu_b)}{\theta}\right)(\hat{\mu} - \ln \varepsilon_b) + \frac{1 - \theta}{\theta} \hat{\sigma} \phi(\nu_b).$$  \hspace{1cm} (D.11)

For the mean of $\ln ex_j$ among entrants, depending on whether $\varepsilon_e$ is larger or smaller than $\varepsilon_b$, there are two possible cases:

$$\begin{align*}
\mathbb{E}(\ln ex_j |\text{entrants}) & = \begin{cases} 
\ln (\bar{p} x_b) + \frac{1}{\theta} \left[\hat{\mu} - \ln \varepsilon_b + \hat{\sigma} \phi(\nu_b)\right], & \varepsilon_e \geq \varepsilon_b \\
\ln (\bar{p} x_b) + \frac{1}{1 - \Phi(\nu_e)} \left[1 + (\theta - 1) \frac{\Phi(\nu_b) - \theta \Phi(\nu_e)}{\theta}\right](\hat{\mu} - \ln \varepsilon_b) + \frac{1 - \theta}{\theta} \hat{\sigma} \phi(\nu_b) + \hat{\sigma} \phi(\nu_e)\right], & \varepsilon_e < \varepsilon_b.
\end{cases}
\end{align*}$$  \hspace{1cm} (D.12)

The mean of $\ln ex_j$ among all exporters is

$$\mu_{ex} = \pi_I \mathbb{E}(\ln ex_j |\text{incumbents}) + (1 - \pi_I) \mathbb{E}(\ln ex_j |\text{entrants}).$$  \hspace{1cm} (D.13)

We now compute the variance of individual export volumes.

If $\varepsilon_e \geq \varepsilon_b$, we have

$$\mathbb{V}ar(\ln ex_j) = \pi_E \mathbb{E} \left[ (\ln ex_j - \mu_{ex})^2 | \varepsilon > \varepsilon_e \right] + \pi_I \mathbb{E} \left[ (\ln ex_j - \mu_{ex})^2 \right]$$

$$= \mu_{ex}^2 + \pi_E \mathbb{E} \left[ (\ln ex_j^2) | \varepsilon > \varepsilon_e \right] - 2 \mu_{ex} \mathbb{E} (\ln ex_j |\text{entrants})$$

$$+ \pi_I \Phi(\nu_b) \mathbb{E} (\ln ex_j^2 | \varepsilon \leq \varepsilon_b) - 2 \mu_{ex} \mathbb{E} (\ln ex_j | \varepsilon \leq \varepsilon_b)$$

$$+ \pi_I [1 - \Phi(\nu_b)] \mathbb{E} (\ln ex_j^2 | \varepsilon > \varepsilon_b) - 2 \mu_{ex} \mathbb{E} (\ln s_j | \varepsilon > \varepsilon_b).$$  \hspace{1cm} (D.14)
Otherwise, if $\varepsilon_e < \varepsilon_b$, we have \(^\text{15}^\) 

\[
\text{Var} (\ln e x_j) = \pi_E E \left[ (\ln e x_j - \mu_{ex})^2 | \varepsilon > \varepsilon_e \right] + \pi_I E \left[ (\ln e x_j - \mu_{ex})^2 | \varepsilon < \varepsilon_e \leq \varepsilon_b \right] \\
= \mu_{ex}^2 + \pi_E \frac{1 - \Phi (\nu_b)}{1 - \Phi (\nu_e)} [E (\ln e x_j^2 | \varepsilon < \varepsilon \leq \varepsilon_b) - 2 \mu_{ex} E (\ln e x_j | \varepsilon < \varepsilon \leq \varepsilon_b)] \\
+ \pi_E [1 - \Phi (\nu_e)] \left[ E (\ln e x_j^2 | \varepsilon > \varepsilon_b) - 2 \mu_{ex} E (\ln e x_j | \varepsilon > \varepsilon_b) \right] \\
+ \pi_I [1 - \Phi (\nu_e)] \left[ E (\ln e x_j^2 | \varepsilon < \varepsilon_b) - 2 \mu_{ex} E (\ln e x_j | \varepsilon < \varepsilon_b) \right] ,
\] 

(D.15)

in which the values of conditional moments of $\ln e_j$ are from equations (D.1) to (D.6). Then we have $\text{Vol}_t = \sqrt{\text{Var} (\ln e x_j)}$.

The relation between $\sigma_t$ and $\text{Vol}_t$ cannot be derived analytically. However, we can numerically solve their log-linearized relationship as below:

\[
\text{Vol}_t - \text{Vol}_t^* = \rho_{Vol} (\ln \sigma_t - \ln \sigma^*),
\] 

(D.16)

in which $\text{Vol}_t^*$ and $\sigma^*$ are their respective steady-state values. Based on the last equation, we choose the sequence of $\{\sigma_t\}_{t=2011}^{2017}$ to exactly match the sequence of $\{\text{Vol}_t\}_{t=2011}^{2017}$ calculated from the data.

We can also compute the $p$-th quantile of the distribution of $\ln e x_j$ as below:

\[
\ln \varepsilon_p = \begin{cases} 
\Phi^{-1} \left( \frac{p}{\pi_I} \right), & p < \pi_I \Phi (\ln \varepsilon_e), \\
\Phi^{-1} \left( \frac{1 - \Phi (\ln \varepsilon_e)}{1 - \pi_I \Phi (\ln \varepsilon_e)} \left[ p + (1 - \pi_I) \frac{\Phi (\ln \varepsilon_e)}{1 - \Phi (\ln \varepsilon_e)} \right] \right), & p \geq \pi_I \Phi (\ln \varepsilon_e).
\end{cases}
\] 

(D.17)

D.3 Entry Rate of New Entrants

Given the empirical yearly entry rate of firms into exporting, $\lambda_a$, we need to find their corresponding quarterly entry rate $\lambda$. In the calibration process, we also consider the possibilities that the same firm might enter for several times in a given year due to the exogenous exit rate each quarter. In particular,

1. For a non-exporter in quarter 1, its probability of becoming an exporter is $\lambda$, and a non-exporter as $1 - \lambda$.

2. In quarter 2, its probability of being an exporter is $\lambda (1 - \delta + \delta \lambda) + (1 - \lambda) \lambda$. And its

\(^{15}\)For the ease of computation, the constant term $\ln (\bar{p} x_b)$ is omitted in the following expressions, since any horizontal shift of the whole distribution does not affect the variance.
probability of not being an exporter is $\lambda\delta (1 - \lambda) + (1 - \lambda)^2$.

3. In quarter 3, its probability of being an exporter is

$$\lambda_3 = \left[\lambda (1 - \delta + \delta\lambda) + (1 - \lambda) \lambda\right] (1 - \delta + \delta\lambda) + \left[\lambda\delta (1 - \lambda) + (1 - \lambda)^2\right] \lambda,$$

and the probability of not being an exporter is $1 - \lambda_3$.

4. Eventually, in quarter 4, its probability of being an exporter is

$$\lambda_4 = \lambda_3 (1 - \delta + \delta\lambda) + (1 - \lambda_3) \lambda.$$

Setting $\lambda_4 = \lambda_a$ in the data, we can solve for $\lambda$, and use it as a targeting moment for calibration.