Economic Slowdown and Housing Dynamics in China: A Tale of Two Investments by Firms

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Abstract

In the past decade, the Chinese economy has witnessed a great housing boom, accompanied by a slowdown in economic growth and an increase in firms’ financial investment. The waning economic prospects are shown to lead to a surge in housing prices by stimulating firms’ demand for financial (especially housing) assets. Motivated by these facts, we take an off-the-shelf dynamic New Keynesian model with a novel modeling of firms’ dynamic portfolio choice between physical and financial investment. Housing assets earn a positive return and can be used as collateral for the firm’s external finances. A negative productivity shock decreases the relative return of production capital, which translates into a housing boom by increasing the firm’s housing demand. A rise in house prices then generates competing effects on real investment: it not only raises the firm’s leverage due to the collateral effect but also depresses the firm’s demand for physical capital because of the crowding-out effect. After calibrating the model for the Chinese economy, our quantitative exercise suggests the former effect is dominated by the latter, which implies counter-cyclical housing prices. Among the policies used to stabilize the aggregate economy and housing markets, our counterfactual analysis implies that the capital-subsidization policy targeting house prices performs better than monetary and deleveraging policies.

Keywords: Counter-cyclical Housing Boom; Chinese Business Cycles; Collateral Effect; Crowding-out Effect; Stabilization Policies.

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1 Introduction

The Chinese financial sector, especially the shadow banking system, has experienced substantial expansion in the past decade. Meanwhile, the transaction cost for investing in financial assets has decreased substantially. Figure 1 presents the time series for the fee rates of the wealth management products issued by commercial banks in China. The figure shows that the costs for wealth management products (WMPs) have greatly declined over time.\(^1\) Therefore, financial assets have become an important investment instrument in Chinese firms’ portfolio decisions.\(^2\) The shadow banking system plays an important role in intermediating the market liquidity of the housing market (Chen, Ren, and Zha, 2018; Allen et al., 2018). Thereby, nonfinancial firms’ financial investment behavior may provide an important mechanism to understand China’s housing boom during the recent economic slowdown. To this end, we incorporate individual firm’s portfolio choice regarding financial assets (in particular, real estate assets) and production capital into an otherwise standard dynamic general equilibrium framework. We then use the model to quantitatively evaluate the aggregate implications of the housing boom for the Chinese macroeconomy through the lens of firm-level investment decision.

The empirical evidence suggests that Chinese firms’ financial investment is strongly counter-cyclical and positively comoves with housing prices. Specifically, the upper-left panel in Figure 2 shows that the holding of financial assets in nonfinancial firms negatively comoves with aggregate output. When the economy is slowing, firms tend to substitute financial assets for production capital in the real sector.\(^3\) A major part of a firm’s financial assets is property investment. The upper-right panel in Figure 2 shows that an upward trend in the share of financial assets of the firm’s total assets (the solid line) is associated with a housing boom (the dashed line). The figure

\(^1\) According to an official report by China Central Depository & Clearing Corporation (CCDC), a large portion of WMPs (approximately 40%) are attributed to the shadow banking sector. The major portion of these liquidities are believed to be allocated to the housing market. Source: www.people.com.cn.

\(^2\) In 2016, approximately 100 public firms relied mainly on the sale of real estate assets to earn profits. The market value of housing sales for these firms exceeds two billion RMB. As of 2018, more than one-half of public firms choose to invest in housing assets. The market value of housing assets held by these firms is almost one trillion RMB. Source: www.finance.china.com.cn.

\(^3\) The real sector and production sector will be used interchangeably in our paper.
Figure 1: Transaction cost for financial investments in China

Notes: The fee rate of a wealth management product is defined as the ratio of commission income to total sales. {“Beijing”, “Citic”, “CEB”, “ICBC”, and “Ping An”} represent the Bank of Beijing, China Citic Bank, China Everbright Bank, Industrial and Commercial Bank of China, and Ping An Bank, respectively. Due to the lack of available data, the series for some banks are not continuous, and the sample periods vary among banks. Data source: WIND database.
Figure 2: Share of financial assets in the firm side and aggregate economy

Notes: For the left panel, the share of a financial asset (the left y-axis) is defined as the average ratio of the financial asset to the sum of the financial assets and fixed assets across nonfinancial listed firms. The right y-axis indicates real GDP growth. For the right panel, the house price (the right y-axis) is the country-level real house price. Both series are presented in quarterly frequency. Due to data availability, the share of financial assets is from 2011Q1 to 2016Q3. All the data series are seasonally adjusted. For the HP-filtered series, the smooth parameter takes a value of 1600.
reveals that during periods of economic slowdown, the strong demand for investing in real estate markets from the firm side may largely contribute to the surge in house prices. The data also imply that a higher return on real estate investment may boost the firm’s financial investment and lead to an expansion in the housing sector. Therefore, a firm’s portfolio choice may provide a new channel to understand China’s recent housing boom. The above pattern is robust to the HP-filtered time series; see the two panels on the bottom row. In particular, the correlation between the share of financial assets and real GDP is -0.43, and the correlation between the share of financial assets and real house prices is 0.41.

The standard macroeconomic theory of housing dynamics, e.g., Iacoviello (2005), Liu, Wang, and Zha (2013), predicts a pro-cyclical housing market because of the use of housing as collateral. However, the Chinese housing market presents a strong counter-cyclical pattern in the past decade, which suggests that an alternative mechanism is required to explain the fluctuations in the housing market. We propose a dynamic general equilibrium with heterogeneous firms’ portfolio decision between housing and production assets. Individual firms are assumed to receive idiosyncratic investment efficiency shocks to production capital. Intuitively, the investment decision follows a trigger strategy, where the threshold is the ratio between the return on housing and the return on production capital. If the investment efficiency is low, the firm opts to invest in housing assets; otherwise, the firm invests in physical capital. Moreover, firms can finance their investment from the banking sector. We introduce financial frictions by following Boissay, Collard, and Smets (2016) to assume that firms can divert bank loans to storage technology. Since a firm’s investment efficiency cannot be observed and verified by a bank, the bank imposes an incentive compatibility (IC) constraint on leverage to prevent firms from diverting bank loans. The endogenous upper limit of a firm’s leverage is positively related to the return of housing investment. We can further decompose firms’ aggregate housing into two components: the extensive margin, i.e., the measure of firms investing in housing, and the intensive margin, i.e., the amount of housing assets that firms can purchase. When the economy is hit by a negative shock, for instance, a negative TFP shock, the relative return on housing increases. Then, more firms invest in housing, i.e., the extensive margin increases. This
process would further boost house prices and the return on housing assets. The relatively high return on housing implies that firms have less incentive to divert bank loans to storage technology. Therefore, the financial constraint (IC constraint) is relaxed, and the intensive margin increases. Consequently, a negative shock can raise housing prices.

Similarly, we can decompose the aggregate demand for production capital into the extensive and intensive margin. With a negative shock, the extensive margin declines because fewer firms invest in physical capital. Meanwhile, the intensive margin increases because higher house prices boost the amount of bank loans that firms can obtain due to a loosened borrowing constraint. The intensive margin captures the impact of housing prices via the collateral effect on real investment. After calibrating the model for the Chinese macroeconomy, we show that, for physical investment, the collateral effect is dominated by the crowding-out effect (the extensive margin). In turn, a negative TFP shock may dampen the real sector while stimulating the housing sector. Therefore, our model can account for the counter-cyclical housing market in China. Moreover, the model-implied firm-level portfolio choice between housing and physical capital is consistent with the empirical pattern from disaggregate data.

Since a housing boom may crowd out investment in the real sector, a natural question is what type of policies can we use to mitigate the adverse impact. To address this question, we quantitatively evaluate miscellaneous policies, including a monetary policy that targets house prices, a macroprudential policy that regulates firms’ leverage, and a capital subsidization policy that directly raises the return on physical capital. Our quantitative exercise suggests that the capital subsidization policy outperforms the other policies because the capital subsidization policy more effectively stabilizes the housing markets and mitigates the crowding-out effect caused by the housing boom.

**Literature Review** Both the collapse of the housing market in Japan in the early 1990s and the recent Great Recession have shed light on the impact of the fluctuation of housing markets on firms and households. In this sense, our paper falls into the strand of literature on housing markets in developed and developing economies. There is a large volume of literature on housing; we do not
attempt to provide a comprehensive survey. Instead, we focus on the papers that are most closely related to ours.

To begin, using the burst of housing bubbles of Japan in the early 1990s as a natural experiment, Gan (2010) illustrates the collateral channel (i.e., crowding-in channel) of housing by showing that landholding firms reduced investment more than non-landholding firms. Moreover, the structural analyses by Iacoviello (2005), Chaney, Sraer, and Thesmar (2012), and Liu, Wang, and Zha (2013) demonstrate that the collateral channel induced by housing can stimulate private investment in the US. Additionally, Miao, Wang, and Zha (2014) introduce housing assets into a heterogeneous-firm model with financial constraint. They show that the liquidity premium generated by housing assets provides an important channel to amplify US business cycles.

Moreover, the literature on the crowding-out channel of the housing boom has grown rapidly. Miao and Wang (2014) develop a two-sector growth model to show that one sector’s bubble-type boom can inefficiently crowd out investment in other sectors. Similarly, Bleck and Liu (2017) show that banks may allocate too much credit to firms in the bubble sector. Hirano, Inaba, and Yanagawa (2015) theoretically illustrate that whether the boom of asset bubbles crowd out or crowd in real investment crucially depends on the financial development of the model economy. Dong et al. (2018) introduce housing into an Aiyagari-type incomplete financial market model. They show that housing can be purely used as a store of value that may crowd out real sector investment.

Finally, our paper is strongly connected to the burgeoning literature on Chinese housing markets; see Glaeser (2017) for a survey on Chinese housing markets. Chen and Wen (2017) build an OLG model with rational housing bubbles in China to model and quantify resource misallocation due to inefficient boom in the housing sector. More relevantly, the recent empirical work of Chen et al. (2017) finds that both the collateral channel and speculation channel are relevant in China, but in general, the housing boom in China is characterized by the crowding out of physical investment in real sectors. Moreover, Han, Han, and Zhu (2018) link house values to fundamental economic variables such as income growth, demographics, migration, and land supply. Additionally, Fang et al. (2016) empirically find that housing prices have experienced enormous appreciation in the
decade preceding 2012, accompanied by equally impressive growth in household income, except in a few first-tier cities. Dong et al. (2018) empirically and quantitatively document the driving force of the recent housing boom in China through the lens of a household’s demand for safe assets. In their model, due to the underdeveloped financial market, housing (especially in Tier 1 cities) becomes a desirable saving instrument when the economy becomes more uncertain.

Complementary to the literature, we build a dynamic general equilibrium model to investigate the Chinese housing dynamics through the lens of portfolio decision by heterogeneous firms. Because of the tractability of the model, we can clearly decompose the collateral and crowding-out effect of a housing boom on the real sector. We also contribute to the literature with quantitative evaluations of miscellaneous policies related to the Chinese housing market, which have not been sufficiently studied in the literature.

The rest of this paper proceeds as follows. Section 2 uses a toy model to account for the stylized facts in the introduction. Section 3 translates the toy model into a fully fledged DSGE model. Section 4 and Section 5 present the quantitative analysis and policy evaluation, respectively. Section 6 concludes. Proofs and the summary of the dynamic system are provided in the appendices.

2 Toy Model

Our empirical analysis suggests that firms’ investment portfolios provide a promising channel to explain housing market dynamics. We use this section to develop a stylized and static model that captures firms’ heterogeneous portfolio decisions between housing assets and real capital. We then analytically derive the aggregate demand for housing and for physical capital. The model’s tractability helps to decompose the impact of housing on physical capital into two components: the crowding-in effect (i.e., the collateral effect) and the crowding-out effect.

The model economy is populated by a continuum of entrepreneurs with unit measure. Each entrepreneur is endowed with one unit of capital. An entrepreneur can obtain $\phi$ amount of bank loans with an exogenous loan rate $R^b$, thus, $\phi$ is also the loan-to-capital ratio. With $1 + \phi$ capital
in hand, an entrepreneur has two investment options. The entrepreneur can invest \( k \) amount of capital in production assets with rate of return \( R^k \) and invest \( h \) amount of capital in financial assets with rate of return \( R^h \). The associated flow of funds constraint is given by \( 1 + \phi = k + h \). In this section, we assume the rates of return of all assets are exogenous such that \( R^k > R^h > R^b \). We introduce an idiosyncratic investment efficiency shock \( \varepsilon \) for investment in production assets. The effective rate of return that the entrepreneur obtains is \( \varepsilon R^k \), where \( \varepsilon \) is i.i.d. across entrepreneurs and has CDF \( F(\varepsilon) \) on support \([0, 1]\). We interpret \((1 - \varepsilon) R^k\) as the operating cost for managing a production project. Furthermore, the realization of \( \varepsilon \) occurs before the entrepreneur’s investment and borrowing decision.

We follow Boissay, Collard, and Smets (2016) in introducing financial friction between entrepreneurs (borrowers) and banks (lenders). Specifically, the investment efficiency shock \( \varepsilon \) is an entrepreneur’s private information that the bank cannot observe. This information asymmetry is necessary for loan market friction. Following Boissay, Collard, and Smets (2016), we assume that the entrepreneur has a linear storage technology with unit marginal return. The entrepreneur can store \( \omega \) fraction of the bank loan and obtain the payoff \( \omega \phi \), where \( \omega > R^h - R^b \). Once the entrepreneur diverts the bank loans to the storage technology, the bank cannot seize the entrepreneur’s assets; thus, there is a moral hazard problem between the bank and the entrepreneur. To prevent the entrepreneur from diverting the bank loans to the storage technology, the bank has to impose an incentive compatibility (IC) constraint on all entrepreneurs:

\[
1 + \omega \phi \leq (1 + \phi) \left(1 + R^h\right) - \phi \left(1 + R^b\right) . \tag{1}
\]

The left-hand side of the IC constraint is the gross rate of return for diverting bank loans to storage technology, and the right-hand side is the minimum gross rate of return for an entrepreneur who pays back the bank loan. Since investing in a financial asset is always an option for the firm, the minimum marginal rate of return for the firm is \( R^h \). Note that as the right-hand side of the IC constraint is the minimum return that an entrepreneur can obtain, the above condition prevents all
entrepreneurs from diverting bank loans.

We now discuss the entrepreneur’s optimal investing decision. Let $\rho \equiv \frac{k}{k + h}$ denote the share of production assets of the entrepreneur’s total assets. Given investment efficiency shock $\varepsilon$, the entrepreneur’s profit is defined as $\varphi(\varepsilon) = (1 + \phi) \left[ \rho \varepsilon R^k + (1 - \rho) R^h \right] - \phi R^b$. The entrepreneur’s optimization problem is to choose $\phi$ and $\rho \in [0, 1]$ to maximize profit $\varphi(\varepsilon)$ subject to the IC constraint (1). It is straightforward to show that under the optimal decisions, the IC constraint holds with equality. Therefore, the optimal loan-to-capital ratio can be solved as

$$\phi \left( R^h; \omega \right) = \frac{R^h}{\omega - (R^h - R^b)}.$$  

The optimal loan-to-capital ratio increases with the return on financial asset $R^h$ and decreases with the fraction of the loan the entrepreneur can divert, $\omega$. The intuition is straightforward. A higher $R^h$ or lower $\omega$ represents less incentive for the entrepreneur to divert bank loans, indicating a less severe moral hazard. As a result, the bank sets a relatively high limit for the loan-to-capital ratio.

Because of the linear structure, the policy function on investment, $\rho$, follows a trigger strategy. Specifically, for $\varepsilon^* = \frac{R^h}{R^k}$, the entrepreneur allocates all the capital in hand $(1 + \phi)$ to the financial asset if $\varepsilon < \varepsilon^*$; otherwise, the entrepreneur invests all the capital in production assets. Therefore, the optimal decision rule of $\rho$ follows

$$\rho(\varepsilon) = \begin{cases} 1 & \text{if } \varepsilon \geq \varepsilon^* \\ 0 & \text{if } \varepsilon < \varepsilon^* \end{cases}.$$  

The entrepreneur’s portfolio decision implies $k(\varepsilon) = (1 + \phi) \rho(\varepsilon)$ and $h(\varepsilon) = (1 + \phi) [1 - \rho(\varepsilon)]$. Note that the cutoff of investment efficiency $\varepsilon^*$ is the ratio of marginal return between financial assets and production assets. When the return on financial assets increases (e.g., a boom in house prices) or the return on production assets declines (e.g., a negative technology shock), more entrepreneurs will invest in financial assets. Therefore, $\varepsilon^*$ describes the intensive margin of the entrepreneur’s
investment.

The aggregate demand for housing assets, $H$, is then given by

$$H \equiv \int h(\varepsilon) dF(\varepsilon) = \left[ 1 + \phi (R^h; \omega) \right] F(\varepsilon^*). \tag{4}$$

The first term on the right-hand side of this equation, i.e., $1 + \phi (R^h; \omega)$, increases with the return on financial assets, $R^h$, which reflects the positive collateral effect of a housing boom ($R^h$ increases) on housing demand. The second term $F(\varepsilon^*)$ increases with $R^h$ (see the definition of $\varepsilon^*$), which reflects the crowding-in effect of a housing boom on housing demand.

The aggregate production capital is defined as $K = \int k(\varepsilon) dF(\varepsilon)$. According to the optimal decision rules, the aggregate production capital can further be expressed as:

$$K = \left[ 1 + \phi (R^h; \omega) \right] \left[ 1 - F(\varepsilon^*) \right]. \tag{5}$$

Similar to the housing demand function, the first term on the right-hand side of the last equation represents the collateral effect of a housing boom on the real investment, while the second term, i.e., $1 - F(\varepsilon^*)$, which decreases with $R^h$, captures the crowding-out effect of a housing boom. Therefore, a housing boom that increases $R^h$ generates competing effects on the real economy.

### 3 Fully Fledged Dynamic Model

We now introduce the setup of the static model into a standard New Keynesian framework. Based on the fully fledged model, we aim to study the macroeconomic impact of the housing market through the channel of the entrepreneur’s asset portfolio decisions. We also evaluate various macroprudential policies.

The economy consists of six sectors: entrepreneurs, households, banks, retailers, capital goods producers and house producers. Entrepreneurs can invest in production projects (physical capital) that produce intermediate goods and in financial projects that purchase housing. House-
holds consume, provide labors to the production side, and rent housing from housing owners (the entrepreneurs that invest in financial projects). The banks channel household deposits to the entrepreneur sector through bank loans. Following Bernanke, Gertler, and Gilchrist (1999), we introduce monopolistic competitive retailers to model the nominal price rigidity. Capital goods producers use final goods to produce new physical capital. Finally, house producers create new houses by using final goods and sell them to investors (entrepreneurs).

3.1 Entrepreneurs

The economy is populated by a continuum of risk-neutral entrepreneurs with unit measure. Each entrepreneur lives only two periods. We assume that each entrepreneur born in period $t$ is endowed with a same amount of equity or net worth $N_t$ (in nominal terms) The entrepreneur’s behavior is the same as that in the static model. In the first period, the entrepreneur can choose to invest in production projects (physical capital) and financial projects (housing). In the second period, the returns on projects are realized. Then, the entrepreneur sells physical capital and housing assets and exits the market. In the following analysis, we first characterize the two types of projects, taking the portfolio decision as given. Then, we solve the entrepreneur’s optimal portfolio decision.

Production Projects A production project is operated as follows. In period $t$, when the entrepreneur is born, she decides to purchase $k_t$ amount of physical capital at nominal price $Q_t^k$. In period $t+1$, the entrepreneur hires labor $l_{t+1}$ at the nominal wage rate $W_{t+1}$ to produce intermediate goods and earn profit. At the end of period $t+1$, the entrepreneur sells physical capital and exits the market. The intermediate good $y_{t+1}$ is produced according to a Cobb-Douglas production function $y_{t+1} = A_{t+1} k_t^\alpha l_{t+1}^{1-\alpha}$, where $A_{t+1}$ is an aggregate TFP shock and the capital share $\alpha$ satisfies $\alpha \in (0, 1)$. The nominal profit is defined as $D_{t+1}^k = P_{m_{t+1}} y_{t+1} - W_{t+1} l_{t+1}$, where $P_{m_{t+1}}$ is the price of intermediate goods. The optimal labor demand is obtained by solving the profit maximization problem, which is linear in physical capital: $l_{t+1} = \left( \frac{P_{m_{t+1}} A_{t+1}}{W_{t+1}} \right)^{\frac{1}{\alpha}} k_t$. It is straightforward to show that the nominal profit $D_{t+1}^k$ is a linear function of capital, i.e., $D_{t+1}^k = Z_{t+1}^k k_t$, where $Z_{t+1}^k = \alpha \left( \frac{P_{m_{t+1}} A_{t+1}}{W_{t+1}} \right)^{\frac{1}{\alpha}} W_{t+1}^{-\frac{1-\alpha}{\alpha}}$. 
is the nominal marginal product of capital. At the end of period $t + 1$, the entrepreneur sells all the physical capital (after depreciation) at price $Q_{k,t+1}^k$. Therefore, the overall expected revenue that an entrepreneur can obtain from investing in a production project is $[Z_{t+1}^k + Q_{t+1}^k (1 - \delta^k)] k_t$. Let $\mathcal{R}_{t,t+1}^k$ denote the expected rate of return on production projects, which is given by

$$
\mathcal{R}_{t,t+1}^k = E_t \left[ \frac{Z_{t+1}^k + Q_{t+1}^k (1 - \delta^k)}{Q_t^k} \right] - 1.
$$

(6)

Similar to the static model, we introduce an idiosyncratic investment efficiency $\varepsilon_t$ to the production project. Following Banerjee and Moll (2010), we assume that $\varepsilon_t$ is realized in period $t$, so there is no idiosyncratic risk involved in the portfolio decision. We interpret $1 - \varepsilon_t$ as the operating cost (as a fraction of the rate of return) paid for managing the production project.\footnote{We assume that the operating cost is transferred to the households, so there is no deadweight loss.} Therefore, the effective rate of return that an entrepreneur can obtain by investing in production projects is $\varepsilon_t \mathcal{R}_{t,t+1}^k$. Again, we assume that $\varepsilon_t$ is an entrepreneur’s private information, which is i.i.d. across entrepreneurs and follows CDF $F(\varepsilon)$ on support $[0, 1]$. The individual heterogeneity in the investment efficiency represents the extensive-margin effect of a housing boom on the real economy.

**Financial Projects** Financial projects channel the entrepreneur’s equity to the housing market. In reality, in China, a financial project corresponds to a trust or wealth management product. A financial project is operated as follows. Upon birth in period $t$, the entrepreneur decides to purchase $h_t$ amount of housing assets at the nominal house price $Q_t^h$. In period $t + 1$, houses are rented to households at the nominal rental rate $Z_{t+1}^h$. At the end of period $t + 1$, the housing assets (after depreciation) are sold at the nominal price $Q_{t+1}^h$. Similar to production projects, the expected rate of return on financial projects $\mathcal{R}_{t,t+1}^h$ is given by

$$
\mathcal{R}_{t,t+1}^h = E_t \left[ \frac{Z_{t+1}^h + Q_{t+1}^h (1 - \delta^h)}{Q_t^h} \right] - 1.
$$

(7)

As in the toy model, we assume that investment in financial projects incurs a cost (such as
management fees), which is a share of the rate of return, $1 - \sigma$, where $\sigma \in [0, 1]$. As a result, the effective rate of return that the entrepreneur can obtain from a financial project is $\sigma R_{t,t+1}^h$.

**Portfolio Decision** Denote $\rho_t(\varepsilon)$ as the share of equity that the entrepreneur allocates to the production project. Given the rates of return $\{R_{k,t,t+1}, R_{h,t,t+1}\}$ and the efficiency shock $\varepsilon_t$, the entrepreneur’s optimal portfolio decision is to choose $\rho_t(\varepsilon)$ to maximize the expected rate of return

$$\bar{R}_{t,t+1}(\varepsilon_t) = \max_{\rho_t} \rho_t \varepsilon_t R_{k,t,t+1} + (1 - \rho_t) \sigma R_{h,t,t+1}.$$  

Clearly, the optimal decision follows a trigger strategy, as in the static model. Specifically, there exists a cutoff investment efficiency shock $\varepsilon_t^* = \frac{\sigma R_{h,t,t+1}}{\bar{R}_{k,t,t+1}}$ such that $\rho_t(\varepsilon_t) = 1$ if $\varepsilon_t \geq \varepsilon_t^*$ and $\rho_t(\varepsilon_t) = 0$ if $\varepsilon_t < \varepsilon_t^*$. Note that under the optimal portfolio decision, the minimum of $\bar{R}_{t,t+1}(\varepsilon_t)$ is $\sigma R_{h,t,t+1}$.

**Moral Hazard** An entrepreneur born in period $t$ can borrow $\phi_t N_t$ from the bank at loan rate $R_b^t$. We assume $\sigma R_{h,t,t+1} > R_b^t$, so the entrepreneur always has incentive to seek external funds from the bank. Similar to the static model, due to the moral hazard problem, the bank imposes an IC constraint on bank loans to prevent the entrepreneur from diverting bank loans to a storage technology. In particular, the IC constraint is

$$1 + \omega \phi_t \leq (1 + \phi_t) \left(1 + \sigma R_{h,t,t+1}^h\right) - \phi_t \left(1 + R_b^h\right).$$

At equilibrium, the IC constraint holds with equality, implying an optimal loan-to-equity ratio

$$\phi_t = \frac{\sigma R_{h,t,t+1}^h}{\omega - (\sigma R_{h,t,t+1}^h - R_b^h)},$$

which indicates that the optimal leverage is increasing in the expected return to the financial project, $\sigma R_{h,t,t+1}^h$. Therefore, either a reduction in transaction cost ($\sigma$ increases) or an increase in expected return on housing assets will increase the entrepreneur’s leverage. Given the loan-to-equity ratio, the entrepreneur’s demands for physical capital and housing (financial assets) are given by
\( k_t(\varepsilon_t) = (1 + \phi_t) [1 - \rho_t(\varepsilon_t)] \frac{N_t}{Q_t^k} \) and \( h_t(\varepsilon_t) = (1 + \phi_t) \rho_t(\varepsilon_t) \frac{N_t}{Q_t^h} \), respectively.

At equilibrium, \( 1 - F(\varepsilon_t^*) \) fraction of entrepreneurs choose to invest in production projects due to their relatively high investment efficiency. The remaining \( F(\varepsilon_t^*) \) fraction of entrepreneurs with relatively low efficiency invest in financial assets. Therefore, the aggregate physical capital \( K_t = \int k_t(\varepsilon_t) \, dF(\varepsilon_t) \) can be written as

\[
K_t = [1 - F(\varepsilon_t^*)] \frac{(1 + \phi_t) N_t}{Q_t^k}.
\]  (11)

The last equation decomposes the impact of a housing boom on the aggregate physical capital \( K_t \) into two effects: the first term \( 1 - F(\varepsilon_t^*) \) reflects the extensive margin, i.e., the number of entrepreneurs willing to invest in physical capital; the second term \( \frac{(1 + \phi_t) N_t}{Q_t^k} \) reflects the intensive margin, i.e., the quantity of physical capital the entrepreneur wants to allocate. Since both the cutoff \( \varepsilon_t^* \) and the leverage \( \phi_t \) increase with expected return on housing assets \( R_{t,t+1}^h \), a housing boom \( (R_{t,t+1}^h \text{ increases}) \) may lead to a crowding-out effect through the extensive margin: fewer entrepreneurs are willing to invest in the real sector. A housing boom may also cause a positive collateral effect through the intensive margin: the entrepreneur’s borrowing constraint is relaxed.

Similarly, the aggregate demand for housing assets, \( H_t \), is obtained by aggregating the individual demands and is given by

\[
H_t = \int h_t(\varepsilon_t) \, dF(\varepsilon_t) = F(\varepsilon_t^*) \frac{(1 + \phi_t) N_t}{Q_t^h}. \]  (12)

**Entrepreneur Net Worth** The net worth in the entrepreneur sector in period \( t \) is endogenously determined. We follow Bernanke, Gertler, and Gilchrist (1999) to assume that when old entrepreneurs exit the market, they consume a fixed fraction \( \theta \in (0, 1) \) of their wealth (net worth plus profits).\(^5\) The remaining portion of entrepreneur wealth is evenly transferred to the next

\(^5\)Indeed, the parameter \( \theta \) resembles the exit rate (or death rate) of entrepreneurs in a standard financial friction model with infinite horizons, e.g., Bernanke, Gertler, and Gilchrist (1999).
generation. Thus, the law of motion of net worth in the entrepreneur sector is

\[ N_t = (1 - \theta) N_{t-1} + (1 - \theta) N_{t-1} \left[ (1 + \phi_{t-1}) \int \mathcal{R}_t (\varepsilon) dF (\varepsilon) - \phi_{t-1} \mathcal{R}_t R_{t-1} \right]. \tag{13} \]

Here, we define \( \mathcal{R}_t (\varepsilon) = \rho_{t-1} \varepsilon \mathcal{R}_t^k + (1 - \rho_{t-1}) \sigma \mathcal{R}_t^h \), where

\[ \mathcal{R}_t^h = \frac{Z_t^h + Q_t^h (1 - \delta_t^h)}{Q_{t-1}^h} - 1 \quad \text{and} \quad \mathcal{R}_t^k = \frac{Z_t^k + Q_t^k (1 - \delta_t^k)}{Q_{t-1}^k} - 1 \tag{14} \]

are the realized rate of return on capital at period \( t \).\(^6\)

### 3.2 Household

The household is representative. Each period, a household consumes \( C_t \), provides labor \( L_t \), rents house \( H_t \), and saves \( S_{t+1} \) in the bank at deposit rate \( R_t \). The household’s optimization problem is given by

\[ \max_{\{C_t, H_t, L_t, S_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u (C_t, L_t, H_t), \tag{15} \]

subject to the budget constraint

\[ P_t C_t + Z_t^h H_t + S_t = W_t L_t + (1 + R_{t-1}) S_{t-1} + \Pi_t, \tag{16} \]

where the utility function takes the form of \( u (C_t, L_t, H_t) = \frac{c_t^{1-\eta_c}}{1-\eta_c} + \chi_h \frac{h_t^{1-\eta_h}}{1-\eta_h} - \chi_l L_t^{1+\eta_l} \) and \( \chi_h > 0, \chi_l > 0; \Pi_t \) is the nominal profit and transfers distributed from the production side and from the banks; and \( P_t \) is the aggregate price level. Let \( \Lambda_t \) denote the Lagrangian multiplier for the

\(^6\)Specifically, the realized rates of return on capital are simply the expected ones in (6) and (7) without the expectation operator.
household budget constraint. The first-order conditions for \(\{C_t, H_t, L_t, S_t\}\) are given by

\[
C_t^{-\eta_c} = P_t \Lambda_t, \quad (17)
\]

\[
\chi_h H_t^{-\eta_h} = Z_t^h \Lambda_t, \quad (18)
\]

\[
\chi_l L_t^\eta = W_t \Lambda_t, \quad (19)
\]

\[
1 = \beta \mathbf{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} (1 + R_t). \quad (20)
\]

### 3.3 Other Sectors

Other sectors, including capital goods producers, housing producers, retailers, banks and the monetary authority, are either standard or degenerated. We model these sectors according to the literature.

**Capital Goods Producers** Physical capital is supplied by physical capital producers, which are representative and stylized. A capital goods producer combines final goods as input, \(I_t^k\), to produce new production capital, \(I_t\), and sells new capital to entrepreneurs. The total capital evolves as

\[
K_t = (1 - \delta^k) K_{t-1} + I_t. \quad (21)
\]

We assume the production function for capital goods is linear, i.e., \(I_t = I_t^k\), so at equilibrium, the real price of physical capital \(Q_t^k\) is simply \(P_t\).

**Housing Producers** We assume housing is produced from final goods through a Cobb-Douglas production function \(X_t = (I_t^h)^v\), where \(v \in (0, 1)\). Solving the profit optimization problem \(\max_{I_t^h} Q_t^h X_t - P_t I_t^h\) yields the optimal decision \(I_t^h = \left(v \frac{Q_t^h}{P_t}\right)^{\frac{1}{1-v}}\). The supply of new housing is then given by \(X_t = \left(v \frac{Q_t^h}{P_t}\right)^{\frac{1}{1-v}}\). The law of motion of aggregate housing assets \(H_t\) is

\[
H_t = (1 - \delta^h) H_{t-1} + X_t. \quad (22)
\]
where $\delta_h$ is the depreciation rate of housing.

**Retailers** The retail sector is populated by a continuum of retailers indexed by $i \in [0, 1]$. The retailers are monopolistic competitive. Final goods are produced by combining retail goods according to a CES production function, $Y_t = \left[ \int_0^1 Y_t(i)^{(\epsilon-1)/\epsilon} \right]^{\epsilon/(\epsilon-1)}$, where $\epsilon > 1$. Let $P_t(i)$ denote the price of retail goods in period $t$, and let $P_t$ denote the price of final goods. The demand function faced by retailer $i$ is $Y_t(i) = \left[ P_t(i)^{-\epsilon} Y_t \right]^{-1}$. Retailers purchase intermediate goods at the nominal price $P_t^m$ to produce retail goods. To introduce price inertia, we assume that the retailer can change its price in a given period with probability $1 - \gamma$, following Calvo (1983). Let $P_t^*$ denote the optimal price set by the retailers who are able to adjust their prices. Retailer $i$ chooses her price to maximize the expected discounted profits, given by

$$
\sum_{\tau=0}^{\infty} (\beta^\gamma)^\tau E_t \left[ \frac{\Lambda_{t+\tau} + P_{t+\tau} - P_{t+\tau}^*}{\Lambda_t P_t} \left( \frac{P_t^*}{P_t} \right)^{-\epsilon} Y_{t+\tau} \right].
$$

The aggregate price indexation equation implies $P_t = \left[ \gamma P_{t-1} + (1 - \gamma) (P_t^*)^{1-\epsilon} \right]^{1/(1-\epsilon)}$.

**Banks** We follow Chang et al. (2016) to model the banks’ behavior. In period $t$, the bank receives deposits $S_t$ from households. The bank chooses the amount of loans $B_t$ supplied to entrepreneurs at loan rate $R_t^b$. The bank is subject to a convex cost of loan processing, $\Psi(B_t/P_t)$, which increases in the total amount of loans. Specifically, $\Psi(B_t/P_t) = \frac{\xi_1}{1+\xi_2} \left( \frac{B_t}{P_t} \right)^{1+\xi_2} b$, for $\xi_1 > 0$ and $\xi_2 > 0$, and $b$ is the steady-state value of $B_t/P_t$. The bank’s problem is therefore

$$
V_t = \max_{B_{t+1}} \left[ \Pi_t^B + \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1} \right],
$$

where $\Pi_t^B$ is the dividend defined as

$$
\Pi_t^B = (1 + R_{t-1}^b) B_{t-1} - P_t \Psi(B_t/P_t) - B_t + S_t - (1 + R_{t-1}) S_{t-1}.
$$

The optimal condition for $B_{t+1}$ is given by

$$
\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} (1 + R_t^b) = 1 + \xi_1 \left( \frac{B_t}{P_t} \right)^{\xi_2}.
$$
Monetary Authority  The central authority implements the monetary policy. In our benchmark analysis, we specify a standard interest rule

$$1 + \frac{R_t}{\bar{R}} = \left(\frac{\pi_t}{\bar{\pi}}\right)^{\varphi_{\pi}} \left(\frac{Y_t}{\bar{Y}}\right)^{\varphi_{y}}, \text{ for } \varphi_{\pi}, \varphi_{y} > 0,$$

(26)

where $\bar{\pi}$ and $\bar{Y}$ denote, respectively, the steady-state inflation and aggregate output.

3.4 Aggregation and General Equilibrium

Due to the constant return to scale production technology in the intermediate goods sector, the aggregate production function for the whole economy is

$$Y_t = \Delta_t A_t K_t^\alpha L_t^{1-\alpha},$$

(27)

where $\Delta_t \equiv 1/ \int \frac{P_i(0)}{P_t} - \epsilon \, di$ denotes the price dispersion. In the general equilibrium, under the market prices, all the agents achieve their own optimum and all markets clear. For the loan market, we have $B_t = N_t \phi_t$. The goods market clearing condition derives the resource constraint

$$C_t + I_t^k + I_t^h + \Psi(B_t/P_t) = Y_t.$$

(28)

We summarize the full dynamic system in Appendix B.

4 Quantitative Analysis

4.1 Calibration

We now calibrate the model according to the aggregate moments in the Chinese economy. One period in our model corresponds to one quarter. We set the discount rate $\beta$ to 0.993, implying an annual risk-free rate of 3%. For the capital share in the production function, we follow the literature (e.g., Song, Storesletten, and Zilibotti, 2011) and set it to 0.5. The parameter in the
production function of new housing, $v$, represents the elasticity between housing investment and new housing produced, i.e., $v = \partial \log X_t / \partial \log I_t^h$. We use the data counterpart to define its value as 0.57. According to the standard business cycle literature, we set the parameters of a household’s utility to satisfy $\eta_c = 2$, $\eta_h = 2$ and $\eta_n = 1$. We set the fixed capital $\delta^k$ to 0.025. The parameter for the depreciation rate of housing $\delta^h$ is set to 0.01 according to Iacoviello (2005). Following the standard New Keynesian business cycle literature, we set the elasticity parameter in the CES production function of final goods $\epsilon$ to 11, implying a markup of 10%. We follow Zhang (2009) and set the probability of not setting prices, $\gamma$, to 0.84. The parameter in the adjustment cost function for bank loans, $\xi_2$, is set to 20, according to Chang et al. (2016). Through the optimal condition of bank loan, we can determine the value of parameter $\xi_1$. Parameter $\sigma$ is the share of returns that the firm eventually obtains from investing in financial projects. We compute its value as the ratio of the net return and net cost of the trust funds in the data. Specifically, the net annual return of the trust funds is 7%, and the net annual cost of the trust funds is 2.8%; thus, $\sigma = 0.72$.\textsuperscript{7} We jointly calibrate the parameter $\omega$, i.e., the fraction of bank loans that the firm can divert to storage technology, and the coefficient in front of the utility of housing service, $\chi_h$, to match the average marginal product of capital (0.05) and the share of financial assets to total assets (0.20) in the data. We obtain $\chi_h = 0.15$ and $\omega = 0.013$. Parameter $\theta$ is equivalent to the exit rate of the firm, and we calibrate it to be 0.03, implying an annual exit rate of 12% (Brandt, Biesebroeck, and Zhang, 2012). For the Taylor rule, we set the coefficient in front of the inflation target, $\varphi_\pi$, to 1.5 (see Chang, Liu, and Spiegel, 2015) and the coefficient in front of the output gap, $\varphi_y$, to 0.05.\textsuperscript{8} Table 1 summarizes the calibration values of the parameters. Appendix A provides more details about the Chinese data utilized in our calibration exercise.

\textsuperscript{7}In our model, the net return of trust funds corresponds to $\sigma R^h$, and the cost of trust funds is $(1 - \sigma)R^h$. In the data, the ratio between these two variables is $\frac{\sigma}{1 - \sigma} = \frac{7\%}{2.8\%} \simeq 2.5$; therefore, $\sigma = 0.72$.

\textsuperscript{8}Under the calibration values of the other parameters, the unique solution of the dynamic system (i.e., BK condition is satisfied) requires a small $\varphi_y$. 
Table 1: Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.993</td>
<td>discounting factor</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.5</td>
<td>capital share in production</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.57</td>
<td>elasticity in housing production</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.72</td>
<td>share of return obtained from financial investment</td>
</tr>
<tr>
<td>$\eta_c$</td>
<td>2</td>
<td>elasticity of utility of consumption</td>
</tr>
<tr>
<td>$\eta_h$</td>
<td>2</td>
<td>elasticity of utility of housing</td>
</tr>
<tr>
<td>$\eta_l$</td>
<td>1</td>
<td>elasticity of labor supply</td>
</tr>
<tr>
<td>$\chi_h$</td>
<td>0.15</td>
<td>coefficient of utility of housing</td>
</tr>
<tr>
<td>$\delta^k$</td>
<td>0.025</td>
<td>depreciation rate of fixed capital</td>
</tr>
<tr>
<td>$\delta^h$</td>
<td>0.01</td>
<td>depreciation rate of housing asset</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>11</td>
<td>elasticity in CES production</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.84</td>
<td>probability of not setting price</td>
</tr>
<tr>
<td>$\xi_1$</td>
<td>0.01</td>
<td>coefficient in adjustment cost of bank loan</td>
</tr>
<tr>
<td>$\xi_2$</td>
<td>20</td>
<td>coefficient in adjustment cost of bank loan</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.013</td>
<td>fraction of bank loan be diverted to storage technology</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.03</td>
<td>exit rate</td>
</tr>
<tr>
<td>$\varphi_\pi$</td>
<td>1.5</td>
<td>parameter of inflation target in Taylor rule</td>
</tr>
<tr>
<td>$\varphi_y$</td>
<td>0.05</td>
<td>parameter of output gap in Taylor rule</td>
</tr>
</tbody>
</table>
4.2 Understanding Housing Dynamics

In this section, we conduct a quantitative analysis to understand the underlying mechanism of housing dynamics observed in the data. The previous empirical facts show that during China’s recent economic slowdown, the housing market presents strong counter-cyclicality. In our quantitative exercise, to generate an economic slowdown, we introduce a negative technology shock to the model economy. Figure 3 reports the impulse responses for key aggregate variables. From the figure, it can be seen that a negative technology shock leads to a decline in aggregate output and a boom in house prices. It also increases real investment in the housing sector and dampens that in the real sector. Therefore, our benchmark model is able to replicate the Chinese empirical pattern.

To understand the counter-cyclical housing dynamics, note that the aggregate housing demand is characterized by equation (12), i.e.,

\[ H_t = F(\varepsilon_t^*) \left(1 + \phi_t\right) n_t \]

where \( n_t \) is the real net worth and \( q_h^t \) is the real house price.\(^9\) There are two components in the demand function: extensive margin, \( F(\varepsilon_t^*) \), and intensive margin, \( (1 + \phi_t) n_t \). A negative technology shock reduces the rate of return on investment in the real sector and, thus, raises the relative return on financial investment. As a result, more firms will invest in the housing sector (\( \varepsilon_t^* \) increases), resulting in a strong positive impact on the extensive margin. By contrast, the intensive margin relies on changes in leverage \( \phi_t \) and net worth \( n_t \). When a negative technology shock hits the economy, net worth \( n_t \) experiences a persistent drop, while the leverage ratio \( \phi_t \) declines slightly initially and then starts to increase.\(^10\) The offsetting between \( \phi_t \) and \( n_t \) leads to a weaker response of the intensive margin than that of the extensive margin. Figure 4 presents the responses of the extensive margin and intensive margin under a negative technology shock. In our benchmark model, the extensive margin has a larger response than the intensive margin. Therefore, a negative technology shock shifts the housing demand curve

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\(^9\)Here, we express net worth \( n_t \) and house price \( q_h^t \) in real terms by removing the nominal price.

\(^10\)According to the definition of leverage ratio, \( \phi_t = \frac{\sigma R_{h,t+1}}{\omega - (\sigma R_{h,t+1} - R_{t}^f)} \), a negative technology shock raises expected return \( R_{h,t+1} \) but also increases the loan rate due to a lower credit supply and a higher inflation rate. The dynamics of the leverage ratio depend on which force dominates. In our benchmark model, in the initial period, the change in loan rate dominates the change in financial return, leading to a weakly negative response in the leverage ratio. As house prices continue to increase, the positive change in financial returns dominates; therefore, the leverage increases.
upward, creating a housing boom.\textsuperscript{11} This explains the counter-cyclical housing dynamics observed in China’s recent economic slowdown. Note that the negative comovement between real investment in two sectors reflects a compounding effect of the negative shock and the crowding out from the housing boom. To sum up, our benchmark model is able to replicate the empirical pattern observed in the real data.

The existence of the option to invest in the housing market provides a key mechanism to generate counter-cyclical housing dynamics. To further evaluate the importance of this mechanism, we specify a very low rate of return to financial investment, $\sigma R_{t,t+1}$ (i.e., the parameter $\sigma$ approaches zero). In this case, investing in the housing sector is no longer attractive, so fewer firms choose to invest in the housing sector. As a result, the crowding-out effect caused by the housing market is greatly mitigated. The dashed lines in Figure 3 present the corresponding impulse responses. In the absence of an attractive investment opportunity in the housing market, a negative technology shock leads to recession in both the real sector and the housing sector. Since the decrease in aggregate output reduces the rental rate of housing due to the weak demand from the household side, the firm’s demand for housing investment is largely depressed, resulting in strongly pro-cyclical housing dynamics. Regarding the decomposition of housing demand, Figure 4 shows that in the control model, both the extensive margin and intensive margin present negative responses. This result explains the pro-cyclical housing dynamics in the control model. The negative extensive margin on housing demand also implies that without strong housing investment on the firm side, a negative productivity shock does not produce a crowding-out effect on the real economy. This result is also reflected by the dynamics of investment in the real sector ($I_t^k$). Figure 4 shows that the decline in $I_t^k$ is much smaller than that in the benchmark case.

\textsuperscript{11}Note that the supply curve for new housings is given by $X_t = \left(vq_t^h\right)^{1\over \alpha}$, which does not depend on the firm side condition.
**Figure 3: Impulse responses to a negative technology shock**

Notes: Impulse responses to a 1-standard-deviation negative productivity shock. The productivity $A_t$ is assumed to follow an AR(1) process with persistence parameter $\rho = 0.9$. All vertical axes are in percentage. For the control model, we set $\sigma = 0.01$. The solid lines represent responses in the benchmark model. The dashed lines represent responses in the control model.
Figure 4: Decomposing housing demand

Notes: Impulse responses to a 1-standard-deviation negative productivity shock. The productivity $A_t$ is assumed to follow an AR(1) process with persistence parameter $\rho = 0.9$. All vertical axes are in percentage. For the control model, we set $\sigma = 0.01$. The solid lines represent responses in the benchmark model. The dashed lines represent responses in the control model. The extensive margin of housing demand is $F(\varepsilon_t^*)$. The intensive margin is $(1 + \phi_t)n_t$. 
4.3 Further Discussions

Housing Demand from Households In our baseline model, we consider disturbances only from the supply side. The existing literature, e.g., Liu, Wang, and Zha (2013); Dong et al. (2018), suggests that housing demand shocks are also important for housing market dynamics. In an extended version, we introduce housing demand shocks to the household side into our baseline model. Appendix D provides more details about the model setup. Figure 5 presents the impulse responses of key aggregate indicators when the economy is hit by a positive housing demand shock. A rise in housing demand boosts the housing market but crowds out the real sector. This result confirms our previous findings. Moreover, if the channel of a firm’s portfolio decision is muted (σ is very small), then a positive housing demand will lead to a boom in both the housing and real sectors, as predicted by the standard housing model. The above analysis indicates that the key propagation mechanism proposed in this paper works for various sources of exogenous disturbance.

Alternative Monetary Policy Rule Some literature argues that the Chinese central bank (People’s Bank of China) frequently uses the growth of the money supply as an instrument for monetary policy; therefore, we replace the nominal interest rate by the growth rate of the money supply in the monetary policy. As our baseline model does not explicitly model money, we introduce money demand, $M_t$, through the Money-in-Utility (MIU).

The quantity rule of the money supply takes the following form

$$g_{Mt} = \left( \frac{\pi_t}{\pi} \right)^{\bar{\varphi}_x} \left( \frac{Y_t}{Y} \right)^{\bar{\varphi}_y}, \quad (29)$$

where $g_{Mt}$ is the growth rate of the money supply. In our quantitative analysis, we follow the literature, e.g., Zhang (2009), and set $\bar{\varphi}_x = 1$ and $\bar{\varphi}_y = 0.25$. Figure 6 shows that the impulse responses appear to be very similar to those in the baseline analysis; thus, the main findings are robust to various types of monetary policy rules.

---

12In particular, the utility function is $u(C_t, L_t, H_t, M_t) = \frac{C_t^{1-\eta_c-1}}{1-\eta_c} + \chi_h \frac{H_t^{1-\eta_h-1}}{1-\eta_h} - \chi_l \frac{L_t^{1+\eta_l}}{1+\eta_l} + \chi_m \frac{M_t^{1-\eta_m-1}}{1-\eta_m}$, where $\chi_m > 0$. 

25
Figure 5: Impulse responses to a positive housing demand shock

Notes: Impulse responses to a 1-standard-deviation positive housing demand shock. The demand shock $\chi_{ht}$ is assumed to follow an AR(1) process with persistence parameter $\rho = 0.9$. All vertical axes are in percentage. For the control model, we set $\sigma = 0.01$. The solid lines represent responses in the benchmark model. The dashed lines represent responses in the control model.
Figure 6: Impulse responses to a negative technology shock under the quantity rule

Notes: Impulse responses to a 1-standard-deviation negative productivity shock. The productivity $A_t$ is assumed to follow an AR(1) process with persistence parameter $\rho = 0.9$. All vertical axes are in percentage. For the control model, we set $\sigma = 0.01$. The solid lines represent responses in the benchmark model. The dashed lines represent responses in the control model. The monetary policy takes the form of the quantity rule.
**Other Sensitivities** Finally, we conduct a sensitivity analysis by varying the values of the key parameters \{\omega, \chi_h, \sigma\} in our baseline model. As \omega and \chi_h are jointly calibrated, in the sensitivity analysis, we consider the range for \omega and \chi_h to be 0.005 \sim 0.03 and 0.05 \sim 0.4, respectively. The model implied marginal product of capital varies from 18\% to 22\%, and the model implied share of financial assets to total assets varies from 12\% to 28\%. For \sigma, the share of returns obtained from financial investment, we consider a range of 0.5 \sim 0.95.\textsuperscript{13} The impulse responses indicate that the main findings in the previous analysis remain valid.\textsuperscript{14} That is, an adverse TFP shock depresses the real sector but boosts the housing sector, resulting in counter-cyclical housing market dynamics.

**5 Policy Evaluations**

A housing boom hinders the real economy by stimulating the firm’s investment in the housing market and crowding out investment in the real sector. One important question is what types of policy can mitigate the adverse impact of a housing boom. We use this section to quantitatively evaluate several macroeconomic policies.

**Monetary Policy** We first consider the lean-against-the-wind monetary policy that targets house prices. In particular, we specify the extended Taylor rule as

\[
1 + R_t \left(\frac{\pi_t}{\pi}\right) \left(\frac{Y_t}{Y}\right) \left(\frac{q_{ht}}{q_h}\right) = \frac{1 + \bar{R}}{1 + \bar{R}} \left(\frac{\varphi_y}{\varphi} \left(\frac{Y_t}{Y}\right) \left(\frac{q_{ht}}{q_h}\right) \varphi_y \right), \quad \text{for } \varphi, \varphi_y, \varphi_h > 0.
\] (30)

The above policy rule indicates that the government conducts tighter monetary policy against the housing boom. To observe the aggregate consequence of this policy, we compare the impulse responses in the model with the extended Taylor rule and those in the benchmark model. Figure 7 shows that the extended Taylor rule effectively curbs the expansion of the housing market by dampening the boom in house prices. However, although the policy mitigates the crowding-out

\textsuperscript{13}To compute the value of \sigma from real data, we consider a fairly wide range of net return of trust funds (5\% \sim 20\%) and cost of trust funds (1\% \sim 5\%). Under these two ranges, the implied \sigma varies from 0.5 to 0.95.

\textsuperscript{14}To save space, we do not report the corresponding impulse responses. The detailed results are available upon request.
effect of the housing boom, the impact on aggregate output and investment in the real sector is very limited because a housing boom that induces a tightened policy also has an adverse impact on aggregate demand. As a result, the overall effect of the extended Taylor rule on the aggregate economy is negligible, which indicates that the lean-against-the-wind monetary policy plays a limited role in stabilizing the aggregate economy.

Macroprudential Policy We now consider a macroprudential policy that regulates the firm’s leverage ratio. Specifically, we assume that with the regulation, the leverage ratio becomes

\[ \phi_t = \frac{\sigma R_{h,t+1}}{\omega_t - (\sigma R_{h,t+1} - R_b t)} \]

where \( \Gamma_t \) is the policy instrument that targets the house price \( q_h^{15} \)

\[ \hat{\Gamma}_t = \psi_h \hat{q}_h, \text{ for } \psi_h > 0, \]

(31)

where the variables with hats indicate the percentage deviation from the steady state. The economic meaning of the revised leverage ratio is that the bank becomes more conservative in providing loans to the firm. The above policy indicates that in response to a housing boom, the government will implement a more aggressive deleveraging policy. Figure 8 reports the impulse responses under the macroprudential policy. The dashed lines in the figure show that the deleveraging policy can reduce the response of the leverage ratio under a negative technology shock. However, the policy amplifies the housing boom and the aggregate volatility because the reduction in the leverage ratio also impedes investment in the real sector and thus the aggregate demand. This further depresses the relative return between production capital and financial capital (housing). As a result, under the macroprudential policy, the housing boom is amplified and the recession in the real economy is prolonged. The above analysis indicates that a deleveraging policy that targets the housing market fails to curb the housing boom and stabilize the aggregate volatility.

\[ ^{15} \text{An alternative regulation rule is one in which the government controls the firm’s leverage directly, i.e., } \phi_t = \frac{1}{\Gamma_t} \frac{\sigma R_{h,t+1}}{\omega - (\sigma R_{h,t+1} - R_b t)}. \] Our quantitative results show that the change in the main results is minimal.
Figure 7: Impulse responses: extended Taylor rule

Notes: Impulse responses to a 1-standard-deviation negative productivity shock. The productivity $A_t$ is assumed to follow an AR(1) process with persistence parameter $\rho = 0.9$. All vertical axes are in percentage. For the case of the extended Taylor rule, we set $\varphi_h = 0.1$. The solid lines represent responses in the benchmark model. The dashed lines represent responses in the model with the extended Taylor rule.
Figure 8: Impulse responses: macroprudential policy

Notes: Impulse responses to a 1-standard-deviation negative productivity shock. The productivity $A_t$ is assumed to follow an AR(1) process with persistence parameter $\rho = 0.9$. All vertical axes are in percentage. For the case of macroprudential policy, we set $\psi_h = 3$. The solid lines represent responses in the benchmark model. The dashed lines represent responses in the model with the macroprudential policy.
**Capital Subsidization Policy**  A recession creates a housing boom in our model because investing in the housing sector is relatively profitable. One plausible policy is to directly subsidize the fixed capital in production, i.e., the expected rate of return to fixed capital under the subsidization policy $R_{t,t+1}^k$ where satisfies

$$R_{t,t+1}^k = E_t (1 + \tau_{t+1}) \pi_{t+1} \left[ \frac{z_{t+1}^k + q_{t+1}^k (1 - \delta^k)}{q_t^k} - 1 \right], \quad (32)$$

where $\tau_t$ is the rate of subsidy that targets house prices

$$\hat{\tau}_t = \psi \hat{q}_t^h, \text{ for } \psi > 0. \quad (33)$$

Figure 9 reports the impulse responses under the subsidization policy. The figure shows that the policy largely mitigates the crowding-out effect caused by the housing boom because of an increased subsidy rate. As a result, the declines in aggregate output and investment in the real sector are substantially alleviated comparing to those in the benchmark case. Meanwhile, the dampened crowding-out effect stimulates the demand in the housing market through the intensive margin, $(1 + \phi_t)n_t$, because of a relative increase in the net worth compared to the benchmark case. Consequently, a capital subsidization policy can effectively stabilize the aggregate economy and also stimulate the housing market.

**Welfare Implication of Policies**  We further compare the welfare implications of the three types of policies. To compute the welfare, we follow Galí (2015) to approximate the utility function by the second-order Taylor expansion. Figure 10 plots social welfare as a function of the coefficient in the corresponding government policy. For instance, the first panel reports the welfare as a function of $\varphi_h$ in the case of the extended Taylor rule.

The first panel of Figure 10 suggests that the tightness of the house prices targeted ($\varphi_h$) in the extended Taylor rule has a negative impact on welfare; that is, welfare decreases with $\varphi_h$. Note that

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16 The capital subsidization policy is essentially equivalent to the tax reduction policy.
Figure 9: Impulse responses: capital subsidization policy

Notes: Impulse responses to a 1-standard-deviation negative productivity shock. The productivity $A_t$ is assumed to follow an AR(1) process with persistence parameter $\rho = 0.9$. All vertical axes are in percentage. For the case of the capital subsidization policy, we set $\psi_r = 3$. The solid lines represent responses in the benchmark model. The dashed lines represent responses in the model with the capital subsidization policy.
Figure 10: Welfare implication of policies

Notes: The welfare reported here is computed as a function of the policy coefficient. For the case of the extended Taylor rule, coefficients $\varphi_\pi$ and $\varphi_y$ in the Taylor rule are fixed at 1.5 and 0.05, respectively. The welfare at $\varphi_h = \psi_h = \psi_\tau = 0$ corresponds to the welfare in the benchmark case. In the quantitative exercise, only technology shock $A_t$ is considered. The persistence parameter of $A_t$ is set to 0.9.

the targeting of house prices in the Taylor rule may have two competing forces: on the one hand, the term $\varphi_h \hat{q}_h$ in the Taylor rule reduces the crowding-out effect of a housing boom, which improves welfare; on the other hand, the term also leads to a tighter monetary policy on the aggregate economy, which negatively affects welfare. The latter effect dominates. The second panel of Figure 10 shows that the macroprudential policy harms welfare, which is monotonically decreasing in the policy parameter $\psi_h$, because the macroprudential policy unambiguously amplifies the crowding-out effect and the aggregate volatilities. The final panel in Figure 10 shows that welfare is improved by implementing a capital subsidization policy. Furthermore, in this case, welfare is strictly increasing in the policy parameter $\psi_\tau$. The above results suggest that the capital subsidization policy that targets house prices is more desirable than other policies in terms of welfare improvement.
6 Conclusion

The surge in the shadow banking system in China after the global financial crisis is associated with the recent housing boom and economic slowdown. The recent literature documents that a large portion of firm-level financial investment has been channeled to the Chinese housing market via the shadow banking system. Therefore, a nonfinancial firm’s portfolio decision may provide an important mechanism to understand China’s recent housing boom.

We develop a macroeconomic theory with firm-level portfolio decision to study the investment-driven housing boom in China. An individual firm that is facing an idiosyncratic investment efficiency shock aims to optimally allocate its investment between housing and production capitals. The firm opts to invest in housing if the efficiency of investing in physical capital is relatively low. The firm finances its investment through bank loans. Due to financial friction, the firm’s leverage is positively related to the expected house price. The aggregate housing demand on the firm side can be decomposed into two components: the number of firms that are willing to hold housing assets and the amount of housing the firm is able to purchase. When the economy is hit by an adverse shock to the real sector, the reduction in return on physical capital causes more firms to allocate housing assets, leading to a boom in house prices. The housing boom produces two competing effects on real investment. On the one hand, a relatively high return on housing reduces the number of firms that invest in production capital (crowding-out effect). On the other hand, a rise in house prices relaxes the firm’s financial constraints, leading to larger leverage (crowding-in effect). The overall effect of a housing boom on real investment depends on which force dominates. After calibrating our model for the Chinese economy, the quantitative results suggest that the crowding-out effect dominates. Thus, our model can explain the counter-cyclical housing market in China in terms of firm-side investment decision.

One important question is what type of macroeconomic policies can mitigate the adverse impact of an investment-driven housing boom. Our quantitative exercises consider various policies, including a monetary policy that targets house prices, a macroprudential policy that regulates the firm-level leverage ratio, and a capital subsidization policy that directly increases the relative re-
turn on physical capital. We find that the capital subsidization policy that targets house prices outperforms the monetary and the deleveraging policies since it can effectively stabilize the housing market and the real economy.
References


A Data

This appendix describes the data used in our calibration.

1. The parameter in the production function of new housing, $v$, represents the elasticity between housing investment and newly built houses. The time series of housing investment and newly built houses are downloaded from the WIND database. The sample period is from 2005 to 2017. We regress the logarithm of the gross floor area of newly built houses on the logarithm of the PPI-deflated amount of housing investment. The value of the coefficient is used to calibrate $v$.

2. Parameter $\sigma$ is the share of return that the firm can obtain from investing in financial projects. We compute its value through the ratio between the net return and the net cost of the trust funds in the data. The net annual cost of the trust funds is calculated as the total income from the trust business of the entire trust company divided by the total amount of corresponding trust funds. All these data are downloaded from the WIND database. The sample period covers 2011Q1-2016Q4.

3. We jointly calibrate the parameter $\omega$, the fraction of bank loans that the firm can divert to storage technology, and the coefficient in front of the utility of housing service, $\chi^h$, to match the average marginal product of capital (0.05) and the share of financial assets to total assets (0.20) in the data. The average marginal product of capital is taken from Bai, Hsieh, and Qian (2006). The share of financial assets to total assets of listed firms is from the CSMAR database. The sample period is from 2011Q1 to 2016Q4. Financial assets include the following eight accounting items according to the Chinese Accounting Standard: financial assets held for trading, available-for-sale financial assets, held-to-maturity investment, long-term equity investment, interest receivable, dividend receivable, investment property, and financial assets held under resale agreements.
B Full Dynamic System of the Real Economy

We now present the full dynamic system. We use lowercase variables to represent real variables.

1. Definition of the expected rate of return on production projects ($R^k_{t,t+1}$)

$$R^k_{t,t+1} = E_t \pi_{t+1} \left[ \frac{z^k_{t+1} + q^k_{t+1} (1 - \delta^k)}{q^k_t} \right] - 1.$$  \hfill (B.1)

where the marginal product of capital is given by $z^k_t = \alpha (p^m_t A_t)^{\frac{1}{2}} w_t^{-\frac{1-\alpha}{\alpha}}$.

2. Definition of the expected rate of return on financial projects ($R^h_{t,t+1}$)

$$R^h_{t,t+1} = E_t \pi_{t+1} \left[ \frac{z^h_{t+1} + q^h_{t+1} (1 - \delta^h)}{q^h_t} \right] - 1.$$  \hfill (B.2)

3. Cutoff for investment efficiency ($\varepsilon^*_t$)

$$\varepsilon^*_t = \frac{\sigma R^h_{t,t+1}}{R^k_{t,t+1}}.$$  \hfill (B.3)

4. Aggregate labor demand ($L_t$)

$$L_t = \left( \frac{p^m_t A_t}{w_t} \right)^{\frac{1}{\alpha}} K_{t-1}.$$  \hfill (B.4)

5. Aggregate output ($Y_t$)

$$Y_t = \Delta_t A_t K_{t-1}^{\alpha} L_t^{1-\alpha},$$  \hfill (B.5)

where $\Delta_t \equiv 1/ \int P(i) P_k \left[ \frac{P(i)}{P_k} \right] - e d i$.

6. Loan-to-equity ratio ($\phi_t$)

$$\phi_t = \frac{\sigma R^h_{t,t+1}}{\omega - (\sigma R^h_{t,t+1} - R^k_t)}.$$  \hfill (B.6)
7. Aggregate demand for physical capital ($q^k_t$)

\[ K_t = [1 - F(\varepsilon^*_t)] \frac{(1 + \phi_t) n_t}{q^k_t}. \]  

(B.7)

8. Aggregate demand for housing assets ($q^h_t$)

\[ H_t = F(\varepsilon^*_t) \frac{(1 + \phi_t) n_t}{q^h_t}. \]  

(B.8)

9. Net worth ($N_t$)

\[ n_t = (1 - \theta) \frac{n_{t-1}}{\pi_t} + (1 - \theta) n_{t-1} \left[ (1 + \phi_{t-1}) \int R_t(\varepsilon) dF(\varepsilon) - \phi_{t-1} R^h_{t-1} \right], \]  

(B.9)

where $R_t(\varepsilon) = \rho_{t-1} \varepsilon R^k_t + (1 - \rho_{t-1}) \sigma R^h_t$, $R^h_t = \pi_t \frac{z^h + q^h(1 - \delta^h)}{q^h_{t-1}} - 1$ and $R^k_t = \pi_t \frac{z^k + q^k(1 - \delta^k)}{q^k_{t-1}} - 1$.

10. Consumption ($C_t$)

\[ C_t^{-\eta_c} = \lambda_t. \]  

(B.10)

11. Demand for housing service ($z^h_t$)

\[ \chi_h H_t^{-\eta_h} = z^h_t \lambda_t. \]  

(B.11)

12. Supply for labor ($w_t$)

\[ \chi_l L_t^{-\eta_l} = w_t \lambda_t. \]  

(B.12)

13. Euler equation for saving ($R_t$)

\[ 1 = \beta E_t \frac{\lambda_{t+1} + R_t}{\pi_{t+1}}. \]  

(B.13)

14. Law of motion for aggregate capital ($K_t$)

\[ K_t = (1 - \delta^k) K_{t-1} + I_t^k. \]  

(B.14)
15. Law of motion for aggregate housing \((H_t)\)

\[
H_t = (1 - \delta^h) H_{t-1} + X_t. \tag{B.15}
\]

16. Supply of new housing \((X_t)\)

\[
X_t = (I_t^h)^\nu. \tag{B.16}
\]

17. Investment in housing production \((I_t^h)\)

\[
I_t^h = (v q_t^h)^{\frac{1}{1-\nu}}. \tag{B.17}
\]

18. Supply for loans \((R_t^b)\)

\[
\beta E_t \lambda_{t+1} \frac{1 + R_t^h}{\lambda_t} \frac{\lambda_t}{\pi_{t+1}} = 1 + \xi_1 \left( \frac{b_t}{b} \right)^{\xi_2}. \tag{B.18}
\]

19. Loan market clearing condition \((b_t)\)

\[b_t = \phi_t n_t. \tag{B.19}\]

20. Resource constraint \((\Lambda_t)\)

\[C_t + I_t^k + I_t^h + \Psi(b_t) = Y_t. \tag{B.20}\]

21. Monetary policy \((\pi_t)\)

\[
\frac{R_t}{\bar{R}} = \left( \frac{\pi_t}{\bar{\pi}} \right)^{\varphi_x} \left( \frac{Y_t}{\bar{Y}} \right)^{\varphi_y}, \text{ for } \varphi_x > 0 \text{ and } \varphi_y > 0, \tag{B.21}\]

22. New Keynesian Phillips curve determines \(P_{t+\tau}^m\)

\[
\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{(1 - \gamma)(1 - \beta \gamma)}{\gamma} \hat{p}_t^m. \tag{B.22}\]
C Solving the Steady State

In the steady state, we specify \( \pi = 1 \). The Euler equations for deposits and bank loans imply

\[
R = \frac{1}{\beta} - 1, \tag{C.1}
\]

\[
R^b = \frac{1 + \xi_1}{\beta} - 1. \tag{C.2}
\]

Given \( R^h, R^k \) and \( b \), the cutoff is given by

\[
\epsilon^*_t = \frac{\sigma R^h}{R^k}. \tag{C.3}
\]

The loan-to-equity ratio is given by

\[
\phi = \frac{\sigma R^h}{\omega - (\sigma R^h - R^b)}. \tag{C.4}
\]

Then, \( z^k \) can be solved directly as

\[
z^k = R^k + \delta^k. \tag{C.5}
\]

Since \( z^k = \alpha p^m Y \frac{K}{K^L} \), from the definition of \( R^k \), we have

\[
1 = \alpha p^m z^k \left( \frac{K}{L} \right)^{\alpha-1}, \tag{C.6}
\]

where \( p^m = \frac{\epsilon-1}{\epsilon} \). Given steady-state labor \( L \), the last equation can express \( K \) as a function of \( R^h \) and \( R^k \)

\[
K = \left( \alpha p^m z^k \right)^{\frac{1}{1-\alpha}} L. \tag{C.7}
\]

The housing demand \( H = F(\epsilon^*) \frac{(1+\phi)n}{q^h} \) and \( K = [1 - F(\epsilon^*)] \frac{(1+\phi)n}{q^h} \) implies

\[
H = \frac{F(\epsilon^*)}{1 - F(\epsilon^*)} \frac{q^k}{q^h} K. \tag{C.8}
\]
From the housing supply, we have

\[ \delta^h H = (vq^h)^{\frac{v}{1-v}}. \]

Thus, we can solve \( q^h \) as

\[ q^h = \left[ \delta^h \frac{F(\varepsilon^*)}{1 - F(\varepsilon^*)} q^k K \right]^{1-v} \left[ \frac{1}{v} \right]. \quad (C.9) \]

Given \( q^h \), we can directly obtain \( H \) and \( z^h \) as

\[ z^h = (\mathcal{R}^h + \delta^h) q^h, \quad (C.10) \]
\[ n = \frac{Hq^h}{F(\varepsilon^*) (1 + \phi)}. \quad (C.11) \]

Additionally, we can solve \( b \) as

\[ b = \phi n. \quad (C.12) \]

The law of motion of net worth implies

\[ 1 = (1 - \theta) \left[ 1 + (1 + \phi) \int \mathcal{R}(\varepsilon) dF(\varepsilon) - \phi \mathcal{R}^h \right], \quad (C.13) \]

where \( \int \mathcal{R}(\varepsilon) dF(\varepsilon) = [1 - F(\varepsilon^*)] \mathcal{R}^h \mathcal{E}_t(\varepsilon | \varepsilon > \varepsilon^*) + F(\varepsilon^*) \sigma \mathcal{R}^h \). Note that \( \mathcal{E}_t(\varepsilon | \varepsilon > \varepsilon^*) = \frac{\int_{\varepsilon^*}^{\infty} dF(\varepsilon)}{1 - F(\varepsilon^*)} \).

The last equation gives the second implicit function for \( \mathcal{R}^h \) and \( \mathcal{R}^k \).

From the housing service demand, we can obtain consumption as

\[ C^{-\eta_c} = \lambda = \chi_h \frac{H^{-\eta_h}}{z^h}. \quad (C.14) \]

From the labor demand, we can solve the wage rate as

\[ w = (1 - \alpha) \frac{P^m Y}{L}. \quad (C.15) \]
The optimal labor supply determines the parameter value

$$\chi_l = \frac{w \lambda}{L^n}.$$  \hspace{1cm} (C.16)

Since we already know $$Y = AK^\alpha L^{1-\alpha}$$, $$C$$, $$I^k = \delta K$$ and $$I^h = (vq^h)^{\frac{1}{1-\nu}}$$, from the resource constraint, the steady-state loan $$b$$ is determined by

$$\frac{\xi_1}{1 + \xi_2} b = Y - C - I^h - I^k.$$  \hspace{1cm} (C.17)

Finally, equations (C.13) and (C.17) jointly determine $$\{\mathcal{R}^h, \mathcal{R}^k\}$$.

### D Model with Housing Demand Shocks

We extend our baseline model to introduce housing demand shocks. In addition to the housing demand shocks, we assume that the household can purchase financial assets, which are eventually used for purchasing housing assets.\(^{17}\) In particular, the household consumes $$C_t$$, provides labor $$L_t$$, rents house $$h_t$$, saves $$S_{t+1}$$ in the bank at deposit rate $$R_t$$, and purchases financial assets $$H_{t+1}^H$$ (similar to those on the firm side). The optimization problem is given by

$$\max \left\{ C_t, h_t, H_{t+1}^H, L_t, S_t \right\} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t, L_t, h_t),$$ \hspace{1cm} (D.1)

subject to the budget constraint

$$P_t C_t + Z_t^h h_t + S_t + H_{t+1}^H = W_t L_t + (1 + R_{t-1}) S_{t-1} + (1 + \mathcal{R}_t^h) H_t^H - v^h \left( \frac{H_t^H}{1 + \kappa^h} \right)^{1+\kappa^h} + \Pi_t.$$ \hspace{1cm} (D.2)

\(^{17}\)In principle, we do not need to introduce the households’ demand for financial (or housing) assets as an investment instrument. However, in the absence of this setup, the total supply of housing for the rental market would be very small in the case of low $$\sigma$$ due to the low supply from the firm side. As a result, a positive demand shock is greatly amplified in the case of low $$\sigma$$.  

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where the utility function takes the form of

\[ u(C_t, L_t, h_t) = \frac{c_t^{1-\eta_c} - 1}{1-\eta_c} + \chi_{ht}^{h_{ht}^{1-\eta_h} - 1} - \chi_{lt}^{l_{lt}^{1-\eta_l}} \]

and \( \chi_t > 0 \); \( \Pi_t \) is the nominal profit and transfers distributed from the production side and from the banks; \( P_t \) is the aggregate price level; the term \( v^h \left( \frac{H_t^{H_t}}{1+\kappa_h} \right)^{1+\kappa_h} \) captures the transaction cost for purchasing financial assets for the households; and \( R_h \) is the realized rate of return on housing assets, which is defined in (14). The transaction cost is necessary to generate an upward-sloping demand for financial assets (as a function of \( R_h \)) from the household side. The housing demand shock \( \chi_{ht} \) is assumed to follow an AR(1) process

\[ \log \chi_{ht} - \log \chi_h = \rho \chi_h (\log \chi_{ht-1} - \log \chi_h) + \varepsilon_{\chi_{h,t}}. \] (D.3)

### E Welfare Function

This appendix derives a second-order approximation of the utility of the representative consumer when the economy remains in the neighborhood of an efficient steady state. A second-order approximation of utility is derived around a given steady-state allocation. Frequent use is made of the following second-order approximation of relative deviations in terms of log deviations

\[ \frac{x_t - \bar{x}}{\bar{x}} \approx \tilde{x}_t + \frac{1}{2} \tilde{x}_t^2, \] (E.1)

where \( \tilde{x}_t = \ln x_t - \ln \bar{x} \) is the log deviation from the steady state for a generic variable \( x_t \). The second-order Taylor expansion of \( U_t \) around a steady state yields

\[ U_t - \bar{U} \approx U_{c\bar{c}} \left( \frac{c_t - \bar{c}}{\bar{c}} \right)^2 + \frac{1}{2} U_{cc} \left( \frac{c_t - \bar{c}}{\bar{c}} \right)^2 + U_{h\bar{h}} \left( \frac{h_t - \bar{h}}{\bar{h}} \right)^2 + \frac{1}{2} U_{hh} \left( \frac{h_t - \bar{h}}{\bar{h}} \right)^2 + U_{l\bar{l}} \left( \frac{l_t - \bar{l}}{\bar{l}} \right) + \frac{1}{2} U_{ll} \left( \frac{l_t - \bar{l}}{\bar{l}} \right)^2. \] (E.3)
In terms of log deviations,

\[ U_t - \bar{U} \approx U_c \left( \hat{c}_t + \frac{1 - \eta_c}{2} \hat{c}_t^2 \right) + U_h \left( \hat{h}_t + \frac{1 - \eta_h}{2} \hat{h}_t^2 \right) + U_l \left( \hat{l}_t + \frac{1 + \eta_l}{2} \hat{l}_t^2 \right), \]  

(E.4)

where \( \eta_c = -\frac{U_c}{U_{cc}} \), \( \eta_h = -\frac{U_h}{U_{ch}} \), and \( \eta_l = \frac{U_l}{U_{cl}} \).

Taking the sum for the whole life and then taking expectation, we have

\[ \mathbb{E}_0 \sum_{t=1}^{\infty} \beta^t (U_t - \bar{U}) \approx \frac{\beta}{1 - \beta} U_c \left[ \frac{1 - \eta_c}{2} \text{Var} (\hat{c}) + \frac{1 - \eta_h}{2} \frac{U_h}{U_{cc}} \text{Var} (\hat{h}) + \frac{1 + \eta_l}{2} \frac{U_l}{U_{cl}} \text{Var} (\hat{l}) \right] \].

(E.5)

Under the utility function \( U(c_t, h_t, l_t) = c_t^{1 - \eta_c} + h_t^{1 - \eta_h} - l_t^{1 + \eta_l} \), we eventually obtain

\[ \mathbb{E}_0 \sum_{t=1}^{\infty} \beta^t (U_t - \bar{U}) \approx \frac{\beta}{1 - \beta} \left[ (1 - \eta_c) \text{Var} (\hat{c}) + (1 - \eta_h) \frac{U_h}{U_{cc}} \text{Var} (\hat{h}) - (1 + \eta_l) \frac{U_l}{U_{cl}} \text{Var} (\hat{l}) \right] \].

(E.6)