Cycles of Credit Expansion and Misallocation: 
The Good, The Bad and The Ugly*

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Abstract

The Austrian School of business cycle theory views boom-bust cycles in economic activities as an endogenous consequence of over-investment driven by excessive credit creation by commercial banks under fractional reserve banking. We formalize the Austrian theory in a general equilibrium model with banks and financially constrained heterogeneous firms. In our model, a moderate credit expansion has a non-monotonic positive impact on aggregate output, but an excessive credit expansion can trigger an interbank-market crisis and result in a discontinuous sharp fall in aggregate output. In a dynamic setting, this mechanism can generate endogenous boom-bust business cycles despite the absence of adverse shocks.

Keywords: Credit Expansion, Volume-Composition Tradeoff, Financial Risk Capacity, Financial Crisis, Credit Cycles.

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“The wavelike movement affecting the economic system, the recurrence of periods of boom which are followed by periods of depression, is the unavoidable outcome of the attempts, repeated again and again, to lower the gross market rate of interest by means of credit expansion.”


1 Introduction

The idea that a credit boom may sow the seed for a crisis can be dated back at least to the Austrian School. The school views the business cycle as the consequence of an excessive growth in bank credit under fractional reserve banking. In particular, economists of the Austrian School argue that excessive issuance of bank credit may be exacerbated if central bank monetary policy sets interest rates too low, and the resulting expansion of the money supply causes a “boom”, in which resources are misallocated or “malinvested” because of artificially low interest rates. Eventually, the boom cannot be sustained and is followed by a “bust”, in which the malinvestments are liquidated (sold for less than their original cost), and the money supply contracts.

Recent empirical findings seem consistent with the spirit of the Austrian School’s conjecture that the reason for an excessive credit expansion to eventually lead to an economic bust is because it ultimately worsens resource misallocations among firms through malinvestment. For example, Gopinath et al. (2017) show that capital inflow into Italy and Portugal lowers the interest rate, which in turn delivers a significant decline in sectoral TFP by misallocating credit toward firms that are not necessarily more productive. See also Reis (2013) for similar findings. Moreover, Boissay, Collard, and Smets (2016) document that one country’s recent path of credit growth helps predict the country’s financial recession. Based on the information of credit spread, Krishnamurthy and Muir (2017) find that credit expansions are a precursor to crises.1 However, the empirical literature also finds that not all credit booms lead to busts. For example, based on a cross-country dataset, Gorton and Ordoñez (2016) reveal that 34 out of 87 credit booms end up with a recession, and those recessions led by credit booms account for 70% of all financial crises.

Motivated by these empirical facts, this paper aims to formalize the Austrian School of thought or the Austrian Business Cycle Theory (ABCT) in a more rigorous analytical framework. We highlight the interaction between a credit expansion and the “malinvestment” caused by firm-level credit resource misallocation.2 We also show why not all credit booms necessarily lead to

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1Similar patterns are also documented in Schularick and Taylor (2012), Mendoza and Terrones (2008), etc.
2The credit boom-led misallocation is puzzling through the lens of the financial accelerator theory. In this type of model, agency costs are negatively related to a firm’s net worth, which tends to be procyclical. Therefore, the financial-accelerator models generally imply that credit booms are “good” times that relax borrowing constraint and alleviate misallocations; see Bernanke and Gertler (1989), Bernanke, Gertler, and Gilchrist (1999), and Kiyotaki and Moore (1997) for the pioneering work and Jermann and Quadrini (2012), among others, for recent developments.
economic busts. In particular, we ask the following questions. (i) Why do some credit expansions generate crises, whereas others do not? (ii) Why can excessive credit expansions generate credit misallocations, and why do they matter to the aggregate economy? (iii) How can financial accelerator theory be reconciled with the empirical facts?

Our analytical framework is closely related to that developed by Boissay, Collard, and Smets (2016), but there are critical differences. In our baseline static model, a typical bank meets two types of firms, one from a high-productivity sector (H sector) and another from a low-productivity sector (L sector). Firms in the H sector are heterogeneous in terms of productivity, and those in the L sector are homogeneous. However, not all firms in the H sector are more productive than L-sector firms. In particular, we assume that H-sector firms are more productive than L-sector firms only on average. The introduction of productivity heterogeneity allows us to analyze the within-sector credit misallocation, which turns out to be critical for the credit boom led financial crisis.

Households deposit their savings into the banking system, which undertakes investment projects by offering loans to firms. If a bank decides to undertake an investment project (or offer corporate loans to a firm) in the H sector (with firm productivity drawn from a publicly observable distribution), it can either utilize its internal funds (household savings), with a required reserve ratio controlled by the credit policy, or obtain external funds from other banks through the interbank market.

We assume that projects in the L sector (akin to the storage technology in Boissay, Collard, and Smets (2016)) are not traceable and cannot be seized by lenders. Thus, there is a moral hazard problem within the banking sector, which implies that a borrowing bank always has an incentive to divert its interbank loans and invest in the L sector. As a consequence, the lending banks opt to limit the quantity of debts that the borrowers can obtain such that the incentive compatibility condition is satisfied. This financial friction prevents the economy from achieving the first-best equilibrium.

When the credit policy is tight, i.e., the required reserve ratio is large or the quantity of banks’ available internal funds is small, the economy reaches a second-best equilibrium in which the interbank market can be supported. In this equilibrium, those banks that meet firms in the H sector with relatively high productivities would undertake investment and become net borrowers in the interbank market. The remaining banks opt to provide interbank loans to the borrowing banks and earn interests on the interbank loans. We label this scenario as the “normal regime”. We analytically show that in this regime, the property of the equilibrium critically depends on the magnitude of the credit policy. When the credit policy is very tight (below an endogenous threshold), the economy has a unique equilibrium, which is labeled as the good equilibrium or Good.

When the credit policy becomes moderate, multiple equilibria emerge. In this scenario, in
addition to the Good, a less efficient equilibrium emerges, which is labeled as the bad equilibrium or the Bad. In the good equilibrium, a credit expansion has non-monotonic effects on the level of aggregate output. On the one hand, the credit expansion raises credit supply to productive firms, resulting in a positive effect on aggregate output along the intensive margin, but on the other hand, it lowers the credit standard and induces a larger number of inefficient projects to be financed, leading to a negative effect on aggregate output along the extensive margin. This extensive margin captures within-sector misallocations across firms is a novel feature of our model not shared by Boissay, Collard, and Smets (2016).

When the credit policy becomes excessively expansionary, the goods market cannot effectively absorb the over-supplied credit because too much credit is flooded into unproductive firms in the H sector, leading to an excessively lowered rate of returns to investment projects and breaking the incentive compatibility constraint. Consequently, the interbank loan market may eventually collapse, leading to a financial crisis, as documented in Boissay, Collard, and Smets (2016). We show that the crisis equilibrium is unique and label it as the ugly equilibrium or the Ugly. We want to highlight that the within-sector misallocations of credit resources across firms make the financial crisis more prolonged. Our analysis suggests that when the economy is trapped in the Ugly, the quantity of inefficient projects that are financed becomes enormously large, implying that resource misallocations among firms in the H sector become extremely severe. This channel of misallocation amplifies the adverse impact of credit expansions on the aggregate economy during a financial crisis.

To generate endogenous boom-bust cycles, we extend the baseline model to a dynamic environment, in which the different equilibrium scenarios in the static model are captured by the different steady states in the dynamic model. The characteristics of each steady-state equilibrium thus crucially rely on the tightness of the credit policy. When the credit supply is tight, the steady state is Good. When the credit supply is loose, the steady state becomes Bad. Most interestingly, if the credit supply is relaxed further, the economy starts to periodically switches between the Good and the Ugly. If the credit supply becomes excessively expansionary, the interbank loan market may freeze, and the economy settles down into the Ugly steady state, in which the social welfare is the lowest. The rich dynamics in our model is due to the aforementioned resource misallocation channel within the H sector. This channel also causes the impulse response of the aggregate output to be non-monotonic and hump-shaped.

**Literature Review** The current paper is generally related to an extensive volume of literature, which we do not attempt to go through here. Instead, we only highlight papers that are most closely related. First, one of the purpose of this paper is to develop a model in which agency frictions in the interbank markets influence investment in both economic downturns and economic booms. Our paper is definitely not the first paper to use agency problems in financial markets to

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3Since the Bad is strictly Pareto dominated by the Good, our analysis mainly focuses on the Good one.
generate endogenous instability and fluctuations. The idea that the separation of creditors and investors may generate macroeconomic (in)stability dates back at least to Keynes (1936) and Harrod (1939). Theoretical progress made on endogenous credit cycles due to agency frictions includes Aghion, Banerjee, and Piketty (1999), Suarez and Sussman (1997), Gu et al. (2013), Matsuyama (2013) and, more recently, Boissay, Collard, and Smets (2016) and Gorton and Ordoñez (2016).

Our paper is highly related to the literature that studies credit boom led financial crises. The most relevant research is Boissay, Collard, and Smets (2016). Their empirical analysis indicates that a banking crisis is more likely to burst after intensive credit expansion and in turn tends to generate deep and long-last recessions.\(^4\) Similarly, Gorton and Ordoñez (2016) shows that some credit booms end in a crisis, whereas others do not. Moreover, these authors find that credit booms start with an increase in productivity growth, which subsequently falls faster during bad booms. Additionally, Bleck and Liu (2017) develop a model of credit expansion and credit misallocation across sectors.\(^5\) The main difference between our paper and the above three papers is that we emphasize the importance of the interaction between within-sector misallocation and cross-sector misallocation. As illustrated by Proposition 3 in this paper, the within-sector misallocation in our model not only implies an inverted U-shape for the relationship between credit expansion and output but also generates a quantitatively larger effect at the regime switch point. We show that the within-sector misallocation channel largely amplifies the adverse impacts of credit expansions, making the economy more vulnerable to exogenous disturbances (e.g., negative TFP shocks) and the recession more severe. Boissay, Collard, and Smets (2016) also generates a non-monotonic demand curve because of the moral hazard problem. In their paper, a large credit boom may trigger an interbank market freeze equilibrium. Gorton and Ordoñez (2016) provides an alternative mechanism, in which a recession after a credit boom is due to the regime switch between the information insensitive and sensitive equilibria.\(^6\) However, in these papers, the credit booms are consequences of productivity growth; thus, they do not particularly study the credit expansion policy. Moreover, the financial risk capacity due to within-sector misallocation can be considerably lower than that in the literature, including Boissay, Collard, and Smets (2016).\(^7\)

Moreover, our model predicts the asymmetric effects of shocks to the economy, which is in line with empirical findings. In particular, Ordoñez (2013) empirically shows the asymmetric effects of financial frictions: quickly during crises but slowly during recoveries. This asymmetry is stronger.

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\(^4\)See section 3.4 for more comparisons between our paper and Boissay, Collard, and Smets (2016).

\(^5\)In Bleck and Liu (2017), the asymmetric financial structure of two sectors leads to a non-monotonic credit demand scheme. As a result, a credit expansion that shifts the credit supply curve may have asymmetric effects on the two sectors. The inefficient one may absorb more capital from the market, preventing the resources from being efficiently allocated to the productive sector. Credit booms then go wrong. In our paper, in addition to the cross-sector distortionary effect of the credit expansion, we particularly study within-sector misallocations.

\(^6\)See also Dong, Miao, and Wang (2016), who develop a dynamic model with adverse selection in the financial market to study the interaction between funding liquidity and market liquidity and the implications of credit shocks.

\(^7\)See Bigio (2015), among others, for a discussion of the financial risk capacity of banking sector under adverse selection in financial markets, which is used to explain the slow recovery of bank capital and economic activity.
in countries with less-developed financial systems and greater financial frictions. Early works that study the asymmetry of financial and real variables include Veldkamp (2005), Jovanovic (2006) and Van Nieuwerburgh and Veldkamp (2006).

Finally, our paper contributes to the literature on the misallocation consequences of credit expansion (liquidity injection) policy, especially after the recent global financial crisis. Using Portugal as a case study, Reis (2013) shows that the capital inflow can encourage the expansion of relatively unproductive firms in the nonchargeable sector at the expense of more productive tradable-sector firms. Gopinath et al. (2017) show that capital inflow into Italy and Portugal lower the interest rate, which in turn implies a significant decline in sectoral TFP by misallocating credit toward firms that are not necessarily more productive. Our theoretical predictions are consistent with the aforementioned empirical findings.

The rest of the paper proceeds as follows. Section 2 introduces a stylized static model in which the bank’s own capital stock is fixed. Section 3 characterizes the equilibria under different scenarios and illustrates the key mechanism of the credit expansion. Section 4 embeds the static analysis of the baseline model to an infinite-horizon dynamic general framework to study under what conditions endogenous cycles of credit misallocation emerge. We conclude in section 5. All the proofs are put in the appendix.

2 A Static Model

We start with a stylized static model to convey the basic idea. The economy has unit measure of banks. Each bank is endowed with \( K \) units of capital. We assume that the quantity of capitals that the bank can employ is \( \xi K \). The bank can convert its own capital to loans potentially available to production sectors or to other banks in the interbank market. The parameter \( \xi \) reflects the tightness of the central bank’s credit policy. For instance, \( \xi \) could be treated as reserve requirement ratio; thus, it is less than 1.\(^8\) Additionally, it could be the loan-to-deposit ratio. In the latter case, \( \xi \) is the central bank’s macro-prudential policy instrument. More broadly speaking, we treat \( \xi \) as a measure of credit expansion.

2.1 Production Sectors

There are two production sectors in the economy. We label the sector with high productivity as \( H \) and the one with low productivity as \( L \). Later, we will present a formal assumption regarding the sectoral productivities. Each individual bank meets one firm in \( H \) sector and one in \( L \) sector. We assume that the firms in \( H \) sector are heterogeneous in the sense that each firm receives an idiosyncratic productivity shock \( z \) with independent and identical CDF \( F(z) \), with \( E(z) = 1 \).

\(^8\)For simplicity, we normalize the interest rate paid to the reserves \( (1 - \xi) K \) to zero.
firm’s physical capital is fully financed from the bank if the bank decides to invest (i.e., provide loans). Following Gertler and Karadi (2011), we assume that there is no asymmetric information or agency problem between the individual bank and the firm. Thus, the bank takes all the capital income from the firm as the payment to the loans. Note that the banks in our model are indeed heterogeneous because of the productivity heterogeneity of the firms. This setup generates the interbank market (Boissay, Collard, and Smets, 2016).

A typical firm with idiosyncratic productivity $z$ in H sector uses capital $k_h$ and labor $n_h$ to produce goods according to the Cobb-Douglas production technology $y_h = A_h (zk_h)^\alpha n_h^{1-\alpha}$, $\alpha \in (0, 1)$, where $A_h$ denotes the sectoral productivity of the H sector. The physical capital $k_h$ is fully financed from the bank loan. Note that $k_h$ is not necessarily equal to the individual bank’s endowed capital, $\xi K$, because of the presence of interbank markets. The optimal labor decision is the solution of the static problem $\Pi_h(z) = \max_{n_h} A_h (zk_h)^\alpha n_h^{1-\alpha} - Wn_h$, where $W$ is the wage rate. The first-order condition implies that the labor demand satisfies

$$n_h(z) = \left[\frac{(1-\alpha)A_h}{W}\right]^{\frac{1}{\alpha}} zk_h.$$  

(1)

Due to the constant return to scale technology, the capital income $\Pi_h(z)$ can be expressed as $\Pi_h(z) = \pi_h zk_h$, where the sectoral average marginal rate of return $\pi_h$ satisfies

$$\pi_h = \alpha A_h^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{W}\right)^{\frac{1-\alpha}{\alpha}}.$$  

(2)

Thus, the bank’s marginal return of capital by investing in the firm with productivity $z$ in H sector is simply $\pi_h z$.

For the firms in the inefficient L sector, we assume that they are homogeneous and produce the final goods by only using capitals. The production technology follows a linear form: $y_l = A_l k_l$, where $A_l$ is the sectoral productivity. Thus, the bank’s marginal return of capital by investing the firm in L sector is simply $A_l$. We can interpret the L sector as a storage technology with a constant return. For instance, the L sector can be the international financial market with an exogenously given interest rate in a small open economy, or agents can put their money under the mattress. In the latter case, the marginal return $A_l$ is just 1.

For simplicity, we assume that the households inelastically provide one unit of labor. To characterize the first-best scenario, we make the following assumption.

**Assumption 1** $\alpha A_h z_{\max}^{\alpha} K^{\alpha-1} > A_l$.

This assumption says that given the capital stock $K$ the marginal product of capital for the firm with highest productivity is greater than that in the inefficient sector. Therefore, a socially
optimal allocation implies that all the capitals should be allocated to the most productive firms in H sector, and with complete credit expansion ($\xi = 1$). We will go back to this assumption later after the characterization of the equilibrium.

2.2 Banks and Interbank Market

There is an interbank market from which the individual bank can supply or obtain loans. We denote $R^f$ as the competitive interest rate prevalent in the interbank market. Given $\xi K$ units of capital available, an individual bank that meets a firm in H sector with idiosyncratic productivity $z$ can choose to (i) lend to other banks in the interbank market with the interest rate $R^f$ or (ii) borrow from the interbank market with $R^f$ and invest (or provide loans to) the firm in the H sector with the rate of return $\pi_hz$.

Let $\lambda$ denote the ratio of interbank loan to bank’s endowed capital. Then, $\lambda\xi K$ is the quantity of loans the bank borrows from the interbank market. If the bank decides to invest in the H sector, the overall capital available is $(1 + \lambda)\xi K$. The net rate of return is $\pi_hz(1 + \lambda) - R^f\lambda$. Instead, if the bank chooses to lend all the deposits to other banks, the rate of return is simply $R^f$.

As we assume each bank only meets one firm in the H sector with productivity $z$, the banks are essentially heterogeneous in terms of their potential investment projects. In the first-best scenario, as suggested by the Assumption 1, all banks with $z < z_{\text{max}}$ should lend their capital ($\xi K$) to the banks with $z = z_{\text{max}}$. However, the presence of financial frictions, such as moral hazard problems, may discourage credit trade in the interbank market. In particular, following Boissay, Collard, and Smets (2016), we assume that the borrowers may divert $\theta \in (0, 1)$ proportion of the interbank loans, combining all their resources (endowed capitals plus interbank loans) together and resort to the inefficient L sector. Thus, the total amount of capital that the bank can invest in the L sector is $\xi K + \theta\lambda\xi K$. Due to the linear production function, the marginal rate of return per unit of bank’s own capital for producing in the L sector is $A_l(1 + \theta\lambda)$.

In sum, the rate of return of the bank with $z$ under the aforementioned options is given by

$$R(z) = \max \{ R^f, \quad \pi_hz(1 + \lambda) - R^f\lambda, \quad A_l(1 + \theta\lambda) \} .$$

Due to the moral hazard problem, the lending banks want to deter the borrowing banks from diverting the interbank loans. To do so, they can limit the quantity of loans that the marginal borrowers (those are indifferent with the first and the second options) can borrow such that they

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9In the current setup, the severity of moral hazard problem is captured by the parameter $\theta$. We could also endogenize $\theta$ by considering risk-taking behaviors when the bank chooses to undertake an inefficient project in the L sector. We show that in this case, $\theta$ is decreasing in the interbank market rate $R^f$. The endogenous severity of moral hazard problem serves as an additional amplifier for the credit expansion policy. The detailed proof is available upon request.
have no interest in diverting:

\[ A_l (1 + \theta \lambda) \leq R^f. \]  \hspace{1cm} (4)

It can be shown that the above incentive-compatibility (IC) condition holds with equality at the optimum.\(^{10}\) Thus, the market funding ratio in the interbank market can be expressed as

\[ \lambda = \frac{R^f - A_l}{\theta A_l}. \]  \hspace{1cm} (5)

The market funding ratio \( \lambda \) increases with the market interest rate \( R^f \) and decreases with the rate of return in low productivity sector, \( A_l \), and the severity of moral hazard problem, \( \theta \). As discussed in Boissay, Collard, and Smets (2016), the positive relationship between \( \lambda \) and \( R^f \) reflects the positive selection effect of interbank market rate on the borrowers. That is, when \( R^f \) rises, only those banks with efficient projects (\( z \) is high) intend to borrow, which in turn mitigates the moral hazard problem and therefore induces a higher market funding ratio \( \lambda \).

Given that the IC condition (4) is satisfied, we now discuss the bank’s optimal borrowing/lending strategies. It is straightforward to show that a bank tends to borrow from the interbank market if and only if the productivity \( z \) is above a threshold \( z^* \) that satisfies

\[ z^* \equiv \frac{R^f}{\pi_h}, \]  \hspace{1cm} (6)

where the sectoral average marginal return to capital in the H sector, \( \pi_h \), is given by (2). If \( z < z^* \), investing in the H sector is less profitable than lending to the interbank market. As a result, the bank would strictly prefer the latter option. Otherwise, for the case of \( z > z^* \), the bank would choose the former option.

### 3 Characterizing Equilibrium

Before the discussion of capital market equilibrium, we first specify the aggregate labor \( N \) and aggregate output in H sector, \( Y_h \). The aggregate output in L sector, \( Y_l \), will be defined later. Let \( K_h \) and \( K_l \) denote the aggregate capital in the H sector and L sector, respectively. From the individual labor demand (1), the aggregate labor \( N \) is given by

\[ N = \int_{z \geq z^*} n_h (z) dF (z) = \left[ \frac{(1 - \alpha) A_h}{W} \right]^\frac{1}{\alpha} K_h. \]  \hspace{1cm} (7)

\(^{10}\)To be rigorous, the optimization problem regarding \( \lambda \) is given by \( \max_{\lambda} \pi_h z (1 + \lambda) - R^f \lambda \), subject to the IC constraint. The first-order condition implies that the IC condition always binds at the optimum, i.e., the borrowers would always achieve the borrowing limit.
Here, \( \bar{K}_h \equiv K_h (1 + \lambda) \int_{z \geq z^*} zdF (z) \) is the effective capital used in the H sector, which depends on the quantity \( K_h (1 + \lambda) \) and the average quality \( \int_{z \geq z^*} zdF (z) \). For the aggregate output in the H sector, we have \( Y_h = \int_{z \geq z^*} y_h (z) dF (z) = \frac{W}{1 - \alpha} N \), where the second equality is due to the optimal condition of labor demand. Combining last two equations leads to the aggregate production function in the H sector

\[
Y_h = A_h \bar{K}_h \alpha N^{1-\alpha}.
\] (8)

With the inelastic labor supply (i.e., \( N = 1 \)), the marginal rate of return to capital in the H sector is obtained by substituting (7) into (2):

\[
\pi_h = \alpha A_h \bar{K}_h^{\alpha-1}.
\] (9)

To sharpen the analysis, we assume for the rest of the paper that individual productivity \( z \) conforms to a Pareto distribution with CDF \( F (z) = 1 - \left( \frac{z}{z_{\min}} \right)^{-\eta} \) and \( \eta > 2 \).\(^{11}\) We set \( z_{\min} = \frac{\eta - 1}{\eta} \) so that \( E (z) = 1 \).

3.1 Interbank Market Equilibrium: The Good and The Bad

We start with the case in which the interbank market is not collapsed. In this case, we must have \( R_f > A_l \), and all capital in the banking sector, \( \xi K \), will be allocated to H sector. Then, we have \( K_h = \xi K \) and \( K_l = 0 \).

The interbank capital market clearing condition implies the demand of loans, \( [1 - F (z^*)] \lambda \xi K \), equals the supply of loans, \( F (z^*) \xi K \). This equilibrium condition can be further expressed as

\[
[1 - F (z^*)] \lambda = F (z^*),
\] (10)

where the market funding ratio or leverage \( \lambda \) is given by (5). The RHS of the above equation indicates that the supply of loans only depends on the extensive margin \( F (z^*) \), which monotonically increases with the cutoff value \( z^* \), whereas the LHS of the equation shows that the aggregate demand of loans consists of the extensive margin \( 1 - F (z^*) \) and the intensive margin \( \lambda \). It is straightforward to show that the extensive margin declines with the cutoff \( z^* \). The intensive margin \( \lambda \) is increasing in \( z^* \). To see this, under the market clearing condition, the aggregate effective capital can be expressed as \( \bar{K}_h = \xi K \mathbb{E} (z | z \geq z^*) \), which is the product of the quantity of capital available to the bank and the average production efficiency.\(^{12}\) From (6) and (9), the equilibrium

\(^{11}\)Since \( z_{\max} = \infty \) under a Pareto distribution, Assumption 1 is automatically satisfied. Moreover, we have performed numerical analysis for other widely used distributions, e.g., lognormal, uniform, etc., and all the results are qualitatively preserved. The numerical analysis is available upon request.

\(^{12}\)From the definition of \( \bar{K}_h \), we have \( \bar{K}_h = \xi K (1 + \lambda) \int_{z \geq z^*} zdF (z) = \xi K \mathbb{E} (z | z \geq z^*) (1 + \lambda) [1 - F (z^*)] \). From the market clearing condition (10), we immediately have \( (1 + \lambda) [1 - F (z^*)] = 1 \).
interest rate satisfies

\[ R^f = \pi_hz^* = \alpha z^* E^{\alpha-1} (z | z \geq z^*) A_h (\xi K)^{\alpha-1}. \]  

(11)

Notice that with the Pareto distribution, we have \( E \left( z | z \geq z^* \right) = \frac{z^*}{z_{\min}} \). Thus, the equilibrium interest rate \( R^f \) strictly increases with \( z^* \). The positive relationship between \( R^f \) and \( z^* \) reflects the positive selection of the market rate on the production efficiency. From the incentive compatibility constraint (5), \( \lambda = \frac{R^f - A_l}{\theta A_l} \) increases with \( R^f \), implying that the leverage increases with \( z^* \) as well. Therefore, the relationship between the aggregate demand of loans \( [1 - F(z^*)] \lambda \) and the cutoff value \( z^* \) could be non-monotonic. A rise in the cutoff \( z^* \) would raise the borrowing capacity of banks—intensive margin; meanwhile, it reduces the number of firms that choose to borrow and produce—extensive margin.

Under the first-best scenario, in which market frictions are absent, the socially optimal allocation implies that all the capital (with a complete credit expansion) should be allocated to the firm in H sector with \( z = z_{\max} \). The equilibrium condition (11) indicates that the market rate equates the marginal product of capital, i.e., \( R^f = \alpha z_{\max} (K)^{\alpha-1} > A_l \), where the second inequality comes from Assumption 1.

To give an intuitive illustration of interbank market equilibrium, Figure 1 plots the demand and supply schemes of loans against the cutoff value \( z^* \). The demand curve (the blue lines) presents an inverted-U shape.\(^{13}\) The intersection point of the demand and supply curves corresponds to the equilibrium cutoff \( z^* \). Note that the equilibrium condition (10) indicates that the credit policy \( \xi \) only affects the loan demand. It is straightforward to show that a rise in \( \xi \) shifts the demand curve downward.\(^ {14}\) As the supply side \( F(z^*) \) does not depend on \( \xi \), the magnitude of the credit policy essentially determines the properties of the equilibria. As shown in Figure 1, when the credit policy is tight (\( \xi \) is small), the demand and the supply only cross once; thus, the interbank market equilibrium is unique, whereas if the credit is further expanded, the demand and the supply may cross twice, implying a situation of multiple equilibria; when the credit policy is sufficiently loose, the demand may not intersect with the supply, such that an interbank market equilibrium does not exist. We will show later that in the last scenario, there exists a unique interbank market collapse equilibrium.

To provide a rigorous analysis, we express the equilibrium condition (10) more explicitly as a function of the cutoff \( z^* \) by employing the market funding ratio equation (5), the cutoff value

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\(^{13}\)This inverted-U shape is intuitive. Consider an extreme case of \( z^* \rightarrow z_{\min} \): the extensive margin becomes 1, so the demand curve fully relies on the intensive margin \( \lambda \), which increases with \( z^* \). Meanwhile, for \( z^* \rightarrow z_{\max} \), no firm would produce and borrow, so the demand shrinks to zero, which means the demand curve must decline for a sufficiently large \( z^* \).

\(^{14}\)Given the cutoff \( z^* \) fixed, (11) implies a larger \( \xi \) leads to a lower \( R^f \). Since the market funding ratio \( \lambda \) decreases with \( R^f \), an expansionary policy may shift the demand curve downward.
Notes: The blue lines indicate the credit demand under different values of credit policy $\xi$: $[1 - F(z^*)] \lambda$, and the black line indicates the credit supply: $F(z^*)$.

One advantage of the above expression of market clearing condition is that we can easily

\[ \frac{\alpha (\xi K)^{\alpha - 1} A_h}{A_l} = \Gamma (z^*) \]

where $\Gamma (z^*) \equiv \frac{\theta \frac{F(z^*)}{F(z_{max})} + 1}{[E(z|z \geq z^*)]^{\alpha - 1} z^*}$ with $\lim_{z^* \to z_{min}} \Gamma (z^*) = \frac{1}{z_{min}}$ and $\lim_{z^* \to z_{max}} \Gamma (z^*) = \infty$. \(^{15}\)

One advantage of the above expression of market clearing condition is that we can easily

\(^{15}\)Note that the LHS of (12) after multiplying $z_{max}^\alpha$ is the wedge between the first-best marginal product of capital (MPK) and that in the L sector. Thus, $\Gamma (z^*)$ is proportional to this wedge. It can be further decomposed into two components: the gap between the first-best MPK and the second-best MPK, and the gap between the second-best MPK and that in the L sector. Here, the second best MPK is defined as $\frac{\partial Y}{\partial K}$. The first component decreases with the cutoff $z^*$, and the second one increases with the cutoff $z^*$. The intuition is simple. A higher cutoff $z^*$ means less severe misallocation among firms, which narrows the gap of MPK between the first best and the second best but enlarges the gap of MPK between the second best and that in the L sector.
analyze the impact of credit policy $\xi$ on the equilibrium cutoff $z^*$, since the LHS of (12) does not rely on $z^*$. Thus, the property of equilibrium (unique or multiple) fully depends on the shape of $\Gamma(z^*)$. We now characterize the property of $\Gamma(z^*)$.

**Assumption 2** The capital share $\alpha$, the shape parameter for the Pareto distribution $\eta$ and the severity of moral hazard problem $\theta$ satisfy the condition $1 + \alpha < \eta < \frac{\alpha}{\theta}$.

Lemma A.1 in the appendix provides a full characterization of the property of $\Gamma(z^*)$. In particular, $\Gamma(z^*)$ is strictly convex in $z^*$ because of the condition $\eta > 1 + \alpha$. Moreover, $\Gamma(z^*)$ achieves its minimum at $\hat{z} = \left(1 + \frac{\alpha/\theta - \eta}{\eta - \alpha}\right)^{\frac{1}{\eta}} z_{\min}$. The condition $\eta < \frac{\alpha}{\theta}$ guarantees that the minimum is interior, i.e., $\hat{z} > z_{\min}$.

As discussed previously, the properties of equilibria, given $\Gamma(z^*)$, mainly rely on the value of credit policy, $\xi$. The following proposition gives a full characterization.

**Proposition 1** There exist two thresholds of credit policy $\xi^{**} = \left[\frac{\alpha K^{n-1} A_{z^*}}{\Gamma(z_{\min})}\right]^{-\frac{1}{1-\alpha}}$ and $\xi^* = \left[\frac{\alpha K^{n-1} A_{\hat{z}}}{\Gamma(\hat{z})}\right]^{-\frac{1}{1-\alpha}}$ with $\xi^* > \xi^{**} > 0$ such that (i) if $\xi < \xi^{**}$, the interbank market equilibrium is unique; (ii) if $\xi^{**} < \xi < \xi^*$, there may exist multiple interbank market equilibria with $z^* < \hat{z}$ and $z^* > \hat{z}$; and (iii) if $\xi > \xi^*$, the interbank equilibrium cannot be supported.

Part (iii) of Proposition 1 shows that once the total supply of credit in the whole economy exceeds $\xi^* K$, the interbank market equilibrium cannot be supported. In other words, $\xi^* K$ reflects the limit of capacity that the market can absorb and efficiently allocate. This finding implies that a large credit expansion may trigger a banking crisis due to the discontinuity between different equilibria.\(^\text{16}\) We label the regime for the existence of interbank market equilibrium (i.e., $\xi < \xi^*$) as Regime 1 and the regime for the interbank market collapse equilibrium (i.e., $\xi > \xi^*$) as Regime 2.

The parts (i) and (ii) in Proposition 1 state the conditions for the credit policy under which the economy may have unique or multiple equilibria. If the credit policy is sufficiently tight ($\xi$ is below $\xi^{**}$), the interbank market equilibrium is unique, whereas if the credit policy is moderate, i.e., $\xi \in (\xi^{**}, \xi^*)$, multiple equilibria emerge. In this scenario, the equilibrium cutoff of productivity $z^*$ is high in one equilibrium and low in the other. Since the value of $z^*$ reflects the efficiency of the credit allocation, we label the equilibrium with high $z^*$ as the **Good** and the equilibrium with low $z^*$ as the **Bad**. Figure 2 gives a graphic illustration for the Proposition 1.

The non-monotonicity of the credit demand regarding $z^*$ (the LHS in (10)) is crucial for the existence of multiple equilibria. As the credit supply (the RHS in (10)) strictly increases with $z^*$,

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\(^\text{16}\)That a credit boom may induce a banking crisis has been comprehensively documented in Boissay, Collard, and Smets (2016). The major differences between our paper and theirs are (i) we focus on the credit policy instead of technology growth led credit boom; (ii) we highlight the interaction between the within-sector misallocations and cross-sector misallocations.
Good
Bad
Ugly
Γ(𝑧∗) Φ(𝑧∗)
𝑧∗
𝛼(ξ𝐾) 𝛼−1 𝐴ℎ/𝐴𝑙
ξ < ξ∗∗
ξ∗ < ξ < ξ∗∗
ξ > ξ∗

Figure 2: Equilibria under Different Regimes
to support an equilibrium with high cutoff \( z^* \) (the **Good**), the demand side must be large as well. This can always be supported by a large market funding ratio \( \lambda \). To sustain an equilibrium with low cutoff \( z^* \) (the **Bad**), the demand side must be low in order to match a low credit supply, which can happen only if the market funding ratio \( \lambda \) is small. However, if the credit policy is sufficiently tight (less than \( \xi^{**} \)) such that the market funding ratio cannot be small,\(^{17}\) the **Bad** equilibrium may not be supported. In this scenario, the **Good** is the unique equilibrium, which gives part (i) of Proposition 1.

**Risk Capacity** The thresholds \( \xi^* \) and \( \xi^{**} \) reflect the different types of risk capacity for the financial market. In particular, a small \( \xi^* \) indicates that the interbank market is vulnerable to crisis, whereas a small \( \xi^{**} \) implies that the interbank market is vulnerable for generating multiple equilibria. Corollary A.1 in the appendix shows that two risk capacities \( \xi^* \) and \( \xi^{**} \) increase with the dispersion of sectoral TFP, \( \frac{A_h}{A_l} \), and decrease with the volatility of idiosyncratic productivity shock \( z \). Moreover, the threshold \( \xi^* \) decreases with the severity of the moral hazard problem \( \theta \).

The properties of risk capacity \( \xi^* \) have several important implications. First, they suggest that a small TFP shock to the H sector that changes the cross-sector dispersion \( \frac{A_h}{A_l} \) may generate asymmetric effects on the aggregate economy. For instance, when the H sector suffers a negative TFP shock (i.e., \( \frac{A_h}{A_l} \) decreases), the threshold \( \xi^* \) declines. Given the value of \( \xi \), this decline may lead the economy to the collapsed equilibrium. The above channel is consistent with the analysis in Boissay, Collard, and Smets (2016). One novel aspect of our model is that the firm-level misallocation largely amplify the adverse impact caused by the bank crisis. We will go back to this point in the later analysis. Second, these properties imply that a rise in microeconomic uncertainty (variance of \( z \)) may trigger an interbank market crisis by reducing the threshold value of regime switch \( \xi^* \).\(^{18}\) The above two arguments can also be applied to the risk capacity \( \xi^{**} \) for the multiple equilibria. That is, a lower sectoral TFP ratio \( \frac{A_h}{A_l} \) or a higher volatility of \( z \) may make the economy more likely to enter the multiple-equilibria regime, in which the **Bad** equilibrium emerges. Finally, the property of \( \xi^* \) implies that a more severe moral hazard problem (\( \theta \) is larger) may make the economy more vulnerable to the financial crisis. Intuitively, if a bank can divert a larger fraction of loans from the interbank market, it has a stronger incentive to invest in the L sector. Thus, it is more likely the interbank market cannot be sustained.

**Allocation Efficiency** For Regime 1 (\( \xi < \xi^* \)), in which the interbank market equilibrium exists, all the credit is allocated to the H sector; thus, there are no cross-sector misallocations. In this regime, the cutoff \( z^* \) is a sufficient statistic for the allocation efficiency. A larger \( z^* \)

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\(^{17}\) According to the equilibrium condition for the market rate \( R^f \), (11), given \( z^* \), a small \( \xi \) implies a large market rate \( R^f \), which further induces a large market funding ratio \( \lambda \).

\(^{18}\) The fact that \( \xi^* \) strictly decreases with microeconomic uncertainty holds not only for Pareto distribution but is also true for other frequently used distributions, including uniform distribution and lognormal distribution. We use Pareto distribution mainly for analytical considerations. The numerical results for the robust analysis are available upon request.
indicates more efficient allocation of the market credit. It is straightforward to show that for the Good equilibrium, a credit expansion ($\xi$ increases) reduces the cutoff value (see Figure 2) and therefore exacerbates the within-sector misallocations. The model-implied adverse impact of credit expansion on the firm-level allocation efficiency conforms to the empirical findings in the recent literature, e.g., Gopinath et al. (2017). This within-sector productivity misallocation is the novel channel in our model that departs from recent banking crisis theories.

**Equilibrium Output** Since all the capital is allocated to the H sector, we have $K_h = \xi K$, $K_l = 0$, and $Y_l = 0$. The aggregate output $Y$ is

$$Y = Y_h = A_h \left[ E(z|z \geq z^*) \xi K \right]^{\alpha}.$$

(13)

We first discuss aggregate output in the Good equilibrium (the blue points in Figure 2). In this equilibrium, as the equation (13) shows, the credit expansion has two offsetting effects on the output. On the one hand, it directly raises the market credit that is used for the production, i.e., $\xi K$ increases. On the other hand, it induces the banks to finance more low-efficiency projects and thus reduces the average productivity, i.e., $E(z|z \geq z^*)$ declines. As a result, given the aggregate TFP $A_h$ and the capital stock $K$, the output non-monotonically responds to the change in credit policy.

**Proposition 2** Under Regime 1, where the interbank market equilibrium is supported (i.e., $\xi < \xi^*$), the aggregate output in the Good equilibrium is strictly concave in the credit policy $\xi$ and achieves its optimal level at $\xi = \tilde{\xi}$, where $\tilde{\xi} = \left[ \frac{\alpha K^{\alpha-1} A_h}{A_h} \right]^{\frac{1}{\alpha}} < \xi^*$ and $\tilde{z} = \left[ \frac{1 - \theta}{(\eta - 1)\theta} \right]^{\frac{1}{\eta}} z_{\min}$.

Proposition 2 indicates that in the Good equilibrium, there may exist an optimal credit policy so that the output achieves the optimum. When the credit supply below this optimal level (i.e., $\tilde{\xi} K$), the credit expansion would stimulate the output. When the credit supply exceeds the optimum, a further expansion may impede the aggregate economy. If the credit expands further and pass the absorption capacity (i.e., $\xi^* K$), the interbank market will freeze, inducing a banking crisis. The former negative impact of credit expansion is due to the within-sector misallocation, and the latter one is mainly due to the cross-sector misallocation. We will discuss this scenario shortly.

We now briefly discuss the Bad equilibrium (the green point in Figure 2). The output in this case also takes the form of (13). The only difference is the equilibrium cutoff $z^*$. It is straightforward to show that a credit expansion ($\xi$ increases) in this case raises the cutoff and thus the production efficiency (see Figure 2). Meanwhile, a larger $\xi$ also expands the market credit available for the banks. Therefore, for the Bad equilibrium, the credit expansion unambiguously raises the aggregate output.
Note that given the credit policy, the overall efficiency in the Bad equilibrium is below that in the Good equilibrium. Corollary A.2 in the appendix compares the aggregate output in both equilibria. This comparison shows that in the multiple equilibria regime where $\xi \in (\xi^{**}, \xi^{*})$, the output in the Good is strictly higher than that in the Bad. Since the cutoff $z^{*}$ in two equilibria converges to $\hat{z} \equiv \arg \min \Gamma(z^{*})$ when $\xi = \xi^{*}$, the output in two equilibria also converges to the risk capacity for the banking crisis $\xi^{*}$, i.e., $Y_{\text{Good}}(\xi^{*}) = Y_{\text{Bad}}(\xi^{*})$. Note that for the Bad equilibrium, if the credit policy is at the risk capacity for the multiple equilibria, $\xi = \xi^{**}$, the equilibrium output is simply $Y = A_{h} \left( \alpha \frac{A_{h}}{A_{l}} \right)^{1-\alpha} \frac{\alpha}{\min}$. In this case, the moral hazard parameter $\theta$ no longer matters for the aggregate economy.\(^{19}\)

### 3.2 Financial Crisis Equilibrium: The Ugly

We now consider Regime 2, where the interbank market equilibrium cannot be supported, i.e., $\xi > \xi^{*}$. We label this financial autarky (or crisis) equilibrium as the Ugly. Because of the collapse of interbank market, the banks cannot finance from outside; thus, $\lambda = 0$. The bank’s investment options are reduced to two: investing either in the H sector or L sector. Remember that the marginal rates of return for these two options are $\pi_{h}z$ and $A_{l}$, respectively. The bank’s investment decision rule follows trigger strategy: invest in the H sector if $z > z^{*}$, and invest in the L sector otherwise. The cutoff $z^{*}$ equates two marginal rates of return, i.e.,

$$\pi_{h}z^{*} = A_{l}. \quad (14)$$

Thus, the aggregate capitals allocated to H sector and L sector are given by $K_{h} = \xi K [1 - F(z^{*})]$ and $K_{l} = \xi K F(z^{*})$, respectively. Comparing to the interbank market equilibrium where all the capital is allocated to the efficient H sector, in the Ugly equilibrium, the cross-sector misallocation emerges.

To determine the equilibrium cutoff $z^{*}$, from (14) and the definition of $\pi_{h}$, we have $\alpha A_{h} \tilde{K}_{h}^{\alpha-1} z^{*} = A_{l}$, where the effective capital satisfies $\tilde{K}_{h} = \xi K [1 - F(z^{*})] E(z | z \geq z^{*})$. Analogously to the previous analysis for Regime 1, we rearrange the terms and obtain

$$\alpha \frac{A_{h}}{A_{l}} \left( \xi K \right)^{\alpha-1} = \Phi(z^{*}), \quad (15)$$

where $\Phi(z^{*}) \equiv \frac{[1 - F(z^{*})]^{1-\alpha}}{[E(z | z \geq z^{*})]^{\alpha-1} z^{*}}$, $\lim_{z^{*} \to z^{\min}} \Phi(z^{*}) = \frac{1}{z^{\min}}$ and $\lim_{z^{*} \to z^{\max}} \Phi(z^{*}) = 0$. Again, the LHS in the last condition is a constant, and the RHS only depends on the cutoff $z^{*}$. Therefore, the property of the Ugly equilibrium relies on the shape of the function $\Phi(z^{*})$. Lemma A.2 in the appendix shows

\(^{19}\)This is because in this type of Bad equilibrium, even the bank with lowest productivity $z_{\min}$ has an incentive to borrow from the interbank market, in other words, the supply of credit, $F(z^{*})$, is zero. Therefore, the moral hazard problem is not essential because of the market freeze.
that under Assumption 2, the function $\Phi (z^*)$ strictly decreases with the cutoff $z^*$. Therefore, there may exist a unique Ugly equilibrium when the credit policy is sufficiently loose, i.e., $\xi > \xi^*$.

A graphical illustration for the above analysis is also presented in Figure 2. Since $\xi < \xi^*$, the line $\alpha \frac{A_h}{A_i} (\xi K)^{\alpha - 1}$ is below the curve $\Phi (z^*)$ and only crosses it once. See the yellow point in Figure 2. The intersection point is the unique Ugly equilibrium. Under the Pareto distribution, the equilibrium condition (15) admits an analytical solution to $z^*$ such that $z^* = \left[ \frac{A_i \xi^{1-\alpha}}{\alpha A_h K^{\alpha-1} z_{min}} \right]^{\frac{1}{\alpha + \eta - \alpha \eta}} z_{min}$.

Obviously, the cutoff $z^*$ in the Ugly equilibrium strictly increases with the credit policy $\xi$. In Figure 2, the positive relationship between $\xi$ and $z^*$ can be easily verified by shifting the line $\alpha \frac{A_h}{A_i} (\xi K)^{\alpha - 1}$ downward. Thus, a credit expansion improves the firm-level efficiency in the H sector.

Regarding the aggregate output, the impact of $\xi$ on the aggregate output now consists of three effects. First, a higher $\xi$ reduces the number of banks that invest in the H sector, i.e., $1 - F(z^*)$ declines, so it exacerbates the cross-sector capital misallocation. Second, a higher $\xi$ improves the within-sector misallocation because of a larger cutoff $z^*$. Finally, a higher $\xi$ raises the endowed capital stock that the bank can use. It turns out that the latter two effects dominate the first one. Therefore, in the Ugly equilibrium, the aggregate output strictly increases with credit policy $\xi$. It is worth noting that as $\Phi (z_{min}) = \Gamma (z_{min}) = \frac{1}{z_{min}}$, the Ugly equilibrium is coincident with the Bad equilibrium at $\xi = \xi^{**}$. As a result, we must have $Y_{Bad}(\xi^{**}) = Y_{Ugly}(\xi^{**})$.

Discontinuity Thus far, we have analyzed the impact of credit policy $\xi$ on the aggregate output for different equilibria. To give a complete and rigorous characterization of the relationship between the output and the credit policy, we need to discuss the discontinuity of the aggregate output around the risk capacity $\xi^*$.

**Proposition 3** The equilibrium cutoff $z^*$ and the aggregate output are discontinuous for Regime 1 and Regime 2 at $\xi = \xi^*$. In particular, $z_{Good}^*(\xi = \xi^*) = z_{Bad}^*(\xi = \xi^*) > z_{Ugly}^*(\xi = \xi^*)$ and $Y_{Good} (\xi = \xi^*) = Y_{Bad} (\xi = \xi^*) > Y_{Ugly} (\xi = \xi^*)$.

The proposition indicates that a credit expansion policy that makes $\xi$ exceed the risk capacity $\xi^*$ will induce a discontinuous drop in the allocation efficiency $z^*$ and in the aggregate output. Now, we can fully characterize the relationship between the aggregate output and the credit policy. Figure 3 presents a graphic description.

The two thresholds $\xi^{**}$ and $\xi^*$ divide the space into three areas. The left blue area corresponds to the unique Good equilibrium. The right green area corresponds to the unique Ugly equilibrium. For the purple area in the middle, there coexist three equilibria. According to the previous discussions, in Regime 1, the output is concave in the credit policy $\xi$. It increases when $\xi$ is less than the threshold $\bar{\xi}$ and decreases when $\xi \in (\bar{\xi}, \xi^*)$. For the Bad and the Ugly, the output is monotonically increasing in the credit policy $\xi$. In the area of multiple equilibria, where
Figure 3: Aggregate Output in Different Equilibria
\( \xi \in (\xi^{**}, \xi^*) \), the output in different equilibria ranks as follows: \( Y_{\text{Good}} > Y_{\text{Bad}} > Y_{\text{Ugly}} \). In addition, Proposition 3 suggests that the output presents a drop at \( \xi = \xi^* \) between Regime 1 and Regime 2.

For the within-sector allocation efficiency, the relationship between the equilibrium cutoff \( z^* \) and the credit policy \( \xi \) generally presents a similar pattern to that for the output. Figure 4 presents a graphic description. It shows that after a credit expansion (\( \xi \) increases), the efficiency for the \textbf{Good} equilibrium is deteriorated (instead of the non-monotonic relationship for the case of output), whereas for the \textbf{Bad} and the \textbf{Ugly}, the efficiency is improved after the credit expansion. In the area of multiple equilibria, the cutoff \( z^* \) ranks \( z^*_{\text{Good}} > z^*_{\text{Bad}} > z^*_{\text{Ugly}} \). Moreover, Proposition 3 implies that when \( \xi \) exceeds the risk capacity \( \xi^* \), the efficiency experiences a sharp decline.

### 3.3 From The Good to The Ugly

In the area of multiple equilibria where \( \xi \in (\xi^{**}, \xi^*) \), the ranking for the output and the allocation efficiency shows that the \textbf{Good} Pareto dominates the \textbf{Bad} and the \textbf{Ugly}. Thus, following Boissay, Collard, and Smets (2016), throughout the rest of the paper, we exclude the \textbf{Bad} and the \textbf{Ugly} and only focus on the \textbf{Good} equilibrium whenever multiple equilibria emerge.

Proposition 3 also conveys an important message that when the economy switches from Regime 1 (normal time) to Regime 2 (crisis time), the output would undertake a sharp reduction. In this sense, our model provides a novel channel through which a small credit expansion policy may lead to excessive aggregate volatility. For instance, given risk capacity \( \xi^* \), if a small credit expansion makes the \( \xi \) that is originally less than but close to \( \xi^* \) exceed \( \xi^* \), the aggregate output will experience a large drop instead of a continuous reduction due to the regime switch between the \textbf{Good} and the \textbf{Ugly}.

In addition, as we discussed earlier, the risk capacity \( \xi^* \) depends on various fundamental factors; thus, a small change in economic fundamental may trigger a large fluctuation in real economy. For instance, Corollary A.1 in the appendix suggests that \( \xi^* \) strictly decreases with the sectoral TFP dispersion \( \frac{A_h}{A_l} \). Thus, a small negative TFP shock in H sector \( (A_h) \) may cause a large recession due to the collapse of interbank market. This prediction is in line with the technology-led credit boom-bust cycles comprehensively documented in Boissay, Collard, and Smets (2016). One crucial difference between our model and theirs is that the magnitude of the recession in our model is greatly amplified by the within-sector misallocation. Similarly, as \( \xi^* \) strictly decreases with the dispersion of idiosyncratic productivity \( z \), a small uncertainty shock that raises the variance of \( z \) would cause a large recession in aggregate output. Therefore, our model also provides a new channel to transmit the uncertainty shocks that have been identified to be an important source of

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\(^{20}\) The part of \( Y_{\text{Good}} > Y_{\text{Bad}} \) comes directly from Corollary A.2 in the appendix. The part of \( Y_{\text{Bad}} > Y_{\text{Ugly}} \) is due to the facts that (i) \( Y \) strictly increases with \( \xi \) for both the \textbf{Bad} and the \textbf{Ugly} and (ii) \( Y_{\text{Bad}}(\xi^{**}) = Y_{\text{Ugly}}(\xi^{**}) \) and \( Y_{\text{Bad}}(\xi^*) > Y_{\text{Ugly}}(\xi^*) \).
Figure 4: Efficiency in Different Equilibria
aggregate fluctuations, e.g., Bloom et al. (2016).

3.4 The Role of Within-Sector Misallocation

It is worth noting that the crisis due to the interbank market freeze would be exacerbated by the within-sector productivity misallocation. To highlight this channel, we compare the relationship between the output and the credit policy $\xi$ in the baseline model and that in the model without TFP misallocation. In particular, for the latter setup, we follow Boissay, Collard, and Smets (2016) by assuming that the firms are identical, i.e., the production function for the firms in $h$ sector is $y_h = A_h k_h^n n_h^{1-\alpha}$ and the banks are heterogeneous in terms of their rate of return. Thus, the bank’s problem remains the same as that in the baseline model. However, the aggregate output is different since the aggregate production becomes

$$Y = \begin{cases} A_h (\xi K)^\alpha & \text{Good} \\ A_h \{[1 - F(z^*)] \xi K\}^\alpha + A_f \xi K F(z^*) & \text{Ugly} \end{cases}$$

In the baseline model, when credit expansion triggers the interbank market freeze, the cutoff of productivity $z^*$ falls sharply due to the discontinuity of $z^*$ around $\xi^*$, which deteriorates the allocation efficiency. This adverse impact on the output is reflected by the term $E(z|z \geq z^*)$ in production function (13). In contrast, in the model with homogeneous productivity, the within-sector misallocation channel is absent.

Figure 5 compares the relationship between the aggregate output and the credit policy in these two models. To make the different models comparable, in our numerical exercise, we set the same values for the common parameters in both models. The figure shows that the recession caused by the interbank market freeze is much more severe in the baseline model. Moreover, the within-sector misallocation also leads to a non-monotonic impact of credit expansion on the output in the normal time (the Good equilibrium), whereas in the model without within-sector misallocation, the credit expansion monotonically affects the aggregate output in the Good equilibrium. Moreover, the within-sector misallocation reduces the risk capacity ($\xi^{**}$), making the financial system more vulnerable.

4 Dynamics

The previous analysis is based on a static model in which the aggregate capital stock $K$ is exogenously given. We now extend the static model to a dynamic one and document the dynamics under the credit policy. The banking and production sectors in the dynamic model are essentially the same as those in the static model. To introduce the dynamics, we assume that the banker (corresponding to the bank in the static model) accumulates capital. The workers again inelasti-
Figure 5: Credit Expansion and Aggregate Output: Model Comparison
cally provide labor. To simplify the analysis, we assume that the workers do not have access to the capital market and are hand-to-mouth. They consume all of their labor income each period. The idiosyncratic productivity $z_t$ is assumed to follow Pareto distribution and i.i.d. across individuals and over time.

The banker that meets a firm in H sector with idiosyncratic productivity $z_t$ chooses consumption $c_t$ and capital stock in the next period $k_{t+1}$ to maximize the flow of utility $\sum_{t=0}^{\infty} \beta^t \log c_t$, where $\beta \in (0, 1)$ is the discount rate. The banker’s budget constraint is

$$c_t + k_{t+1} - (1 - \delta) k_t = R_t (z_t) \xi_t k_t,$$

where $R_t (z_t)$ is the rate of return to the capitals that is defined in (3); $\xi_t \in [0, 1]$ is a time-varying credit policy instrument that is exogenous to the banker. Because of the log utility, the optimal capital decision satisfies

$$k_{t+1} (z_t, k_t) = \beta [R_t (z_t) \xi_t + (1 - \delta)] k_t.$$  \hspace{1cm} (18)

Let $\mu_t (k)$ denote the distribution of the capital $k$ in the economy. The aggregate capital satisfies

$$K_{t+1} = \int \int k_{t+1} (z_t, k_t) dF (z_t) d\mu_t (k_t).$$

Since $z_t$ is i.i.d., the aggregate capital $K_{t+1}$ can be expressed as

$$K_{t+1} = \beta \left[ \xi_t \int R_t (z_t) dF (z_t) + (1 - \delta) \right] K_t = \beta \left[ \alpha Y_{h,t} + Y_{l,t} + (1 - \delta) K_t \right].$$  \hspace{1cm} (19)

The second equality holds because of the fact that the aggregate capital wealth consists of the income from the H and L sectors and the value after depreciation. According to the analysis in the static model, the sectoral outputs are given by

$$Y_{h,t} = A_h [\mathbb{E} (z | z \geq z^*_t) K_{h,t}]^\alpha,$$

$$Y_{l,t} = A_l K_{l,t}.$$  \hspace{1cm} (20)

Recall that the cutoff values of $z_t^*$ for the different regimes are determined by

$$\frac{A_h}{A_l} (\xi_t K_t)^{\alpha - 1} = \begin{cases} \Gamma (z_t^*), & \text{if Regime 1} \\ \Phi (z_t^*), & \text{if Regime 2} \end{cases}. \hspace{1cm} (22)$$

Unlike the static model, given the credit policy $\xi_t$, the risk capacity $\xi^*$ now is also determined by the aggregate capital stocks. In particular, let $K_t^*$ denote the critical value of capital stock for the switch between Regime 1 and Regime 2, which satisfies

$$\alpha (\xi_t K_t^*)^{\alpha - 1} \frac{A_h}{A_l} = \Gamma (\tilde{z}_t),$$  \hspace{1cm} (23)
where \( \hat{z}_t = \arg \min_{z \in (z_{\text{min}}, z_{\text{max}})} \Gamma (z) \). Analogously to the static model, we must have

\[
K_{h,t} = \begin{cases} 
\xi_t K_t, & K_t < K_t^*, \text{ (Regime 1)} \\
\xi_t K_t [1 - F(z_t^*)], & K_t > K_t^*, \text{ (Regime 2)} 
\end{cases}
\]  

(24)

The capital allocated to the L sector is given by

\[ K_{l,t} = \xi_t K_t - K_{h,t}. \]  

(25)

To completely describe the dynamics, we need to define the threshold for the capital stock for the existence of multiple equilibria. In particular, let \( K_t^{**} \) satisfy

\[
\alpha A_h \left( \xi_t K_t^{**} \right)^{\alpha - 1} = \Gamma (z_{\text{min}}).
\]  

(26)

It is straightforward to see that if \( K_t \in [K_t^*, K_t^{**}] \), the Good, the Bad and the Ugly equilibria coexist. In the end, the full dynamic system is described by (19) to (26).

Equations (22) to (25) imply that the RHS of (19) is a function of \( K_t \) under either Regime 1 or 2, which can be denoted as \( g(K_t) \). Similar to proofs for the static model, we can prove that \( g^{\text{Good}} (K_t) \) is strictly concave in \( K_t \), and both \( g^{\text{Bad}} (K_t) \) and \( g^{\text{Ugly}} (K_t) \) strictly increase with \( K_t \) with \( g^{\text{Good}} (K_t^*) = g^{\text{Bad}} (K_t^*) \) and \( g^{\text{Bad}} (K_t^{**}) = g^{\text{Bad}} (K_t^{**}) \), where \( (K_t^*, K_t^{**}) \) are defined in (23) and (26), respectively. Figure 6 provides a numerical example of the phase diagram of the capital stock \( K_t \).

4.1 The Steady States

We now discuss the steady state of the dynamic system. From the law of motion of aggregate capital, in the steady state, we must have \( r K = \alpha Y_h + Y_l \), where \( r = \frac{1}{\beta} - 1 + \delta \). Notice that the steady-state capital is the intersection between the policy function \( g(K_t) \) and the 45 degree line. It turns out that the steady-state capital depends on the credit policy \( \xi \). The following proposition provides a summary.

**Proposition 4** Under Assumption 2 and \( \xi < \xi_X \equiv \frac{r}{A_l} \), the dynamic economy has a unique steady state. If the credit policy satisfies \( \xi < \xi_L \equiv \frac{r - \alpha}{\eta (1 - \theta) A_l} z_{\text{min}} \), the unique steady state is the Good equilibrium; if \( \xi_L < \xi < \xi_H \equiv \frac{r_{\text{min}}}{A_l} \), the steady state is the Bad equilibrium; and if \( \xi_H < \xi < \xi_X \), the steady state is the Ugly equilibrium.

Figure 7 presents graphical illustrations of the above proposition. A credit expansion (\( \xi \) increases) reduces the two threshold values of capital stock, \( K^* \) and \( K^{**} \), and shifts the policy...
Figure 6: Phase Diagram for Aggregate Capital
function of aggregate capital \( g(K_t) \) towards the left. If the credit policy \( \xi \) equals \( \xi_L \), the steady-state capital is exactly \( K^* \) (the upper-left panel in Figure 7). Therefore, for any \( \xi < \xi_L \), the steady state capital would be the **Good** equilibrium. If the credit policy equals \( \xi_L \), the unique intersection between the policy function \( g(K_t) \) and the 45 degree line is at \( K^{**} \) (the upper-right panel in Figure 7). Thus, for any \( \xi \in (\xi_L, \xi_H) \), the unique steady state is the **Bad** equilibrium. For \( \xi = \xi_X \), the policy function \( g(K_t) \) does not intersect the 45 degree line (the bottom-left panel in the Figure); therefore, for any \( \xi > \xi_X \), a steady state does not exist.

### 4.2 Dynamics under Credit Expansions

In this section, we aim to discuss the dynamic effect under the expansionary credit policy. The policy function of capital \( g(K_t) \) shows that when \( K_t \in [K^{**}, K^*] \), the **Good**, the **Bad** and the **Ugly** equilibria coexist. Since these three equilibria can be Pareto-ranked, following Boissay, Collard, and Smets (2016), we exclude the coordination failure problem and assume that the most efficient equilibrium (the **Good**) is always selected for the regime of multiple equilibria.

Assume that the economy initially stays in the steady state with the **Good** equilibrium, i.e., the initial level of credit policy satisfies \( \xi_0 < \xi_L \). In the first period, the credit policy \( \xi_t \) then undertakes a permanent increase. According to the previous analysis, the dynamic impact of this credit policy may depend on the magnitude of the expansion as well as the initial level of \( \xi_t \). We now discuss the different cases based on the quantitative analysis.

First, we conduct the following parameterization. The capital share in the production function \( \alpha \) is set to 0.4. The shape parameter in the Pareto distribution \( \eta \) is set to 2.5. To make Assumption 2 hold, the moral hazard parameter \( \theta \) is set to 0.01, implying that an individual borrower can divert 1% of the loan. We normalize the sectoral TFP in L sector \( A_l \) to 1 and set the sectoral TFP in H sector \( A_h \) to 1.2. The discount rate \( \beta \) and the depreciation rate \( \delta \) are set to 0.9 and 0.1, respectively.

We start with the case in which the initial credit policy is tight (i.e., \( \xi_0 \) is far below \( \xi_L \)). In particular, we set \( \xi_0 \) to 0.25 \( \times \xi_L \). We assume that in the first period, \( \xi_t \) permanently increases by 1%, 2% or 3%. Figure 8 presents the transition dynamics of the economy. It shows that when the initial level of \( \xi_t \) is low, a credit expansion would unambiguously raise the output in the H sector. Since, in this case, \( \xi_t \) is sufficiently below the threshold \( \xi_L \), the economy would not hit the Regime 2 along the transition. As a result, the L sector is degenerated (i.e., \( Y_{lt} = 0 \)), and all the capital is allocated to the H sector. As a result, the cross-sector misallocation is absent. In addition, the magnitudes of the responses for the output and capital are proportional to the level of credit expansion.

Now, consider a case in which the initial credit policy \( \xi_0 \) is close to but below the critical value \( \xi_L \). In particular, we specify \( \xi_0 = 0.92 \times \xi_L \). We then conduct the same exercise as for the previous
Figure 7: Steady States under Different $\xi$
Figure 8: Transition Dynamics under Credit Expansions: $\xi_0 = 0.25 \times \xi_L$
Figure 9 presents the transition dynamics. From the figure, it can be seen that unlike the previous case, a credit expansion has an adverse effect on the aggregate economy. Moreover, for a relatively small credit expansion with the increment of 1% or 2%, the reduction of aggregate variables are proportional to the change of credit policy, whereas for a further large expansion ($\xi_t$ is still less than $\xi_L$), namely, increasing $\xi_t$ by 3%, the economy may hit Regime 2 during the transition, in which the interbank market is temporally collapsed. As a result, the output and the capital stock experience sharp declines in the short run (see the blue lines in Figure 9). Since the steady state remains in the Good equilibrium, the transition path eventually converges.

Figure 10 provides a graphical illustration of the dynamics of capital stock through the phase diagram. The initial steady state is at point A. A credit expansion in this case reduces the threshold $K^*_t$ and shifts the second half part of the policy function $g(K_t)$ in Regime 1 (the thick line) downward and the Regime 2 part (the dot-dashed line) upward. The intuition is that for Regime 1, credit expansion has a non-monotonic impact on the capital. On the one hand, it raises
the capital that the bank can use (intensive margin); on the other hand, it induces more banks to invest in less-efficient firms (extensive margin), exacerbating the resource misallocation. When the capital is close to \( K_t^* \), the adverse impact dominates, which explains the downward shift of \( g(K_t) \) in the Regime 1. For Regime 2, in which the interbank market is collapsed, a credit expansion unambiguously increases the capital (see the discussion of the output in the \textbf{Ugly} equilibrium), which leads to a upward shift of \( g(K_t) \). As a result, during the transition, the economy may hit Regime 2, leading to a temporally collapse of the interbank market. This explains the large drop of capital and output during the transition after a credit expansion.

In the previous two cases, the credit expansion does not alter the property of the steady state. We now consider a case in which the credit expansion results in regime switch between different steady states (from the \textbf{Good} to the \textbf{Ugly}) and generates endogenous business cycles. In particular, we specify that the initial credit policy is close to \( \xi_L \), namely, \( \xi_0 = 0.96 \times \xi_L \). The steady state in the initial period is the \textbf{Good} equilibrium. We again consider various small credit expansions, i.e., \( \xi_t \) permanently increases by 1%, 2% and 3%, respectively. It turns out that under the credit expansion of 3%, \( \xi_t \) exceeds the threshold value \( \xi_L \) (but is still less than \( \xi_H \)). In light
of the previous steady-state analysis, when $\xi_t \in (\xi_L, \xi_H)$ the economy has multiple equilibria, and the steady state is the Bad equilibrium (see Figure 7). Since for the case of multiple equilibria, only the Good equilibrium is selected, the steady state in this case does not exist. The credit expansion eventually leads to endogenous fluctuations. To see this, Figure 11 plots the transition dynamics for the expansionary credit policies with increments of 1%, 2% and 3% respectively. The figure shows that when a credit expansion does not make $\xi_t$ exceed $\xi_L$, the economy first experiences a sharp drop due to the temporal collapse of the interbank market and then converges to the steady state—the Good (see the red and the black lines). However, if the credit expansion induces $\xi_t$ to pass the threshold $\xi_L$, the transition dynamics will present oscillations, as shown by the blue lines.

The intuition can be observed from the phase diagram Figure 12. A sufficiently large credit expansion shifts the Regime 1 part of the policy function $g(K_t)$ (the thick red line) downward and the Regime 2 part upward. Because of the discontinuity, the new policy function does not intersect the 45 degree line. In this case, the steady state is not well defined. As a result, after the
credit expansion, the capital stock periodically switches between the Good and Ugly equilibria, resulting in oscillation dynamics.

In sum, the credit expansion has an asymmetric impact on the aggregate economy and leads to rich dynamics for the transitions. More specifically, a rise in $\xi_t$ may increase or reduce the aggregate output and capital depending on the initial level of credit policy. This rise may also cause endogenous fluctuations because of the regime switches between different equilibria.

Although we focus on the impact of credit policy, our model can also generate financial crises caused by unfavorable technology shocks or the slowdown of TFP growth. This prediction conforms to those empirical evidences documented in Boissay, Collard, and Smets (2016) and Gorton and Ordoñez (2016). In particular, we show that when the technology progress in the efficient sector relative to inefficient sector declines, the economy may experience severe crisis and endogenous boom-bust credit cycles. Appendix B provides more detailed analysis.
5 Conclusion

An increasing number of empirical works reveal that a large portion of financial crises follow credit expansions, a fact that has been explained by recent theoretical work through the sector-level misallocation channel. Although firm-level misallocations have been proven to be important for the real economy (Hsieh and Klenow, 2009), their consequences for credit expansion policy remain unclear.

To fill the gap, we introduce firm-level misallocations into a banking crisis model (Boissay, Collard, and Smets, 2016) to evaluate both the aggregate and disaggregate consequences of credit expansion policy. In the model economy, banks can offer loans to firms with idiosyncratic productivities. The quantity of credits that the banks can provide is controlled by the government through the credit policy. The heterogeneity of bank’s investment return gives rise to an interbank market. The moral hazard problem and asymmetric information leads to different regimes of the market equilibrium. A credit expansion, on the one hand, raises the credit supply, which may stimulate the production side; on the other hand, it causes more inefficient projects to be financed. In a stylized static model, we analytically show that the credit condition determines the properties of the equilibrium. A unique Good equilibrium in which the interbank market is well-functioning can be achieved when the credit policy is sufficiently tight. Multiple equilibria (the Good and the Bad) may emerge when the credit policy becomes looser. In the Bad equilibrium, the within-sector misallocation is relatively acute even though the interbank market still works. If the credit expansion policy is sufficiently large, the unique Ugly equilibrium emerges, in which the interbank market collapses. Thus, an intensive credit expansion may cause regime switch between the Good equilibrium to the banking crisis equilibrium (the Ugly). The contraction effect of crisis is exacerbated by the within-sector misallocations due to the credit expansion at the firm level. In a dynamic setup, the firm level misallocations provide a novel channel through which a large credit expansion may cause interbank market collapse and endogenous boom-bust cycles for the real economy.

To make the transmission mechanism as transparent as possible, the setup of our work is intentionally stylized, and therefore, we cannot do full justice to reality. To make the model setup more realistic, we could introduce the labor input into the inefficient sector, which may make this sector coexist with the efficient sector when the equilibrium is the Good. We believe that this extension will make the model dynamics richer and fit the data better. Our model can also be flexibly extended to a small open economy by taking the inefficient sector as an international capital market. The extended model can be used to discuss impacts of the financial integration or foreign interest rate shocks on the domestic capital market. This extended model may offer new insights into capital control for the international capital flows. We can also embed our model into a growth model to address the connections between cycles and trends, as discussed in Gorton and
Ordoñez (2016). We leave all these potential topics for future research.
References


Appendix

A  Proofs

Lemma A.1 Under Assumption 2, \( \Gamma (z^*) \) is strictly convex in \( z^* \) over \((z_{\min}, z_{\max})\) and achieves its minimum at \( \hat{z} = \left( 1 + \frac{\alpha/\theta - \eta}{\eta - \alpha} \right)^{-\frac{1}{\eta}} z_{\min} > z_{\min} \).

Proof of Lemma A.1: As \( z \) follows a Pareto distribution such that \( F(z) = 1 - (z/z_{\min})^{-\eta} \) with \( \eta > 1 \) and \( E(z) = \frac{\eta}{\eta - 1} z_{\min} = 1 \), we can easily prove that \( E(z|z \geq z^*) = \frac{\eta}{\eta - 1} z^* = \frac{z^*}{z_{\min}} \). Then, \( \Gamma (z^*) \) can be simplified as

\[
\Gamma (z^*) = z_{\min}^{\alpha - 1} \left[ \frac{\theta}{z_{\min}^{\eta}} (z^*)^{\eta - \alpha} + (1 - \theta) (z^*)^{-\alpha} \right].
\]

(A.1)

Since \( \eta > 1 > \alpha \), we must have \( \eta - \alpha > 0 \). Furthermore, we can derive

\[
\Gamma' (z) = z_{\min}^{\alpha - 1} \left[ \frac{\theta}{z_{\min}^{\eta}} (\eta - \alpha) z^{\eta - \alpha - 1} - \alpha (1 - \theta) z^{-\alpha - 1} \right],
\]

\[
\Gamma'' (z) = z_{\min}^{\alpha - 1} \left[ \frac{\theta}{z_{\min}^{\eta}} (\eta - \alpha) (\eta - \alpha - 1) z^{\eta - \alpha - 2} + \alpha (\alpha + 1) (1 - \theta) z^{-\alpha - 2} \right].
\]

(A.2)

Under the first part of Assumption 2, i.e., \( \eta > 1 + \alpha \), it is easy to show \( \Gamma'' (z) > 0 \), implying that \( \Gamma (z^*) \) is strictly convex in \( z^* \). Thus, the minimum of \( \Gamma (z^*) \) is achieved under the first-order condition \( \Gamma' (z^*) = 0 \), from which we can solve \( z^* \)

\[
z^* = \hat{z} \equiv \left( 1 + \frac{\alpha/\theta - \eta}{\eta - \alpha} \right)^{-\frac{1}{\eta}} z_{\min}.
\]

(A.3)

We can obtain interior solution of \( \Gamma' (z^*) = 0 \), i.e., \( \hat{z} \in (z_{\min}, z_{\max}) \), if and only if \( \eta < \frac{\alpha}{\theta} \), which gives the second half of Assumption 2.

Notice that the solution of the equilibrium equation (12) guarantees the condition \( R^f > A_l \). To see this, from (6), we have \( R^f = \pi_h z^* \). Thus, the condition \( R^f > A_l \) is equivalent to

\[
\alpha (\xi K)^{\alpha - 1} \frac{A_h}{A_l} > \frac{1}{[E(z|z \geq z^*)]^{\alpha - 1} z^*}.
\]

(A.4)

the non-collapse condition \( R^f > A_l \) is satisfied. Q.E.D.

Proof of Proposition 1: When \( \xi > \xi^* \), the condition
\[
\alpha (\xi K)^{\alpha - 1} \frac{A_h}{A_l} \geq \Gamma (\hat{z}) \quad (A.5)
\]

is violated. Thus, the equation (12) does not have a solution, i.e., the interbank market equilibrium cannot be supported. Therefore, the interbank market equilibrium exists if and only if \( \xi \leq \xi^* \).

From the definition of two thresholds,

\[
\xi^{**} = \left[ \frac{\alpha K^{\alpha - 1} A_h}{\Gamma (\hat{z})} \right]^{\frac{1}{1 - \alpha}}, \\
\xi^* = \left[ \frac{\alpha K^{\alpha - 1} A_h}{\Gamma (\hat{z})} \right]^{\frac{1}{1 - \alpha}},
\]

we immediately know that \( \xi^{**} < \xi^* \). The property of \( \Gamma (z) \) further implies that under \( \xi < \xi^{**} \), the equilibrium condition (12) has unique solution. Instead, if \( \xi^{**} < \xi < \xi^* \), there exist two equilibria: one is less efficient, and the other is Pareto-improving. We label the equilibrium for \( z^* > \hat{z} \) as the Good equilibrium and the one for \( z^* < \hat{z} \) as the Bad equilibrium. Q.E.D.

**Corollary A.1** Under Assumption 2, the risk capacity \( \xi^* \) and \( \xi^{**} \) increase with the sectoral TFP dispersion \( A_h/A_l \) and decrease with the volatility of the idiosyncratic productivities. Moreover, \( \xi^* \) decreases with the severity of moral hazard problem \( \theta \).

**Proof of Corollary A.1:** The definition of \( \xi^{**} \) in (A.6) immediately suggests that \( \xi^{**} \) increases with \( A_h/A_l \), and is independent of \( \theta \). Furthermore, with the Pareto distribution, \( \xi^{**} \) can be further expressed as

\[
\xi^{**} = \left[ \frac{\alpha K^{\alpha - 1} A_h \eta - 1}{A_l \eta} \right]^{\frac{1}{1 - \alpha}}, \quad (A.8)
\]

which implies \( \xi^{**} \) increases with \( \eta \). Since \( \text{Var} (z) \) decreases with \( \eta \), we know that \( \xi^{**} \) decreases with \( \text{Var} (z) \).

The comparative statistics of \( \xi^* \) with respect to \( A_h/A_l \) is obvious. As shown in the proof of Lemma A.1, we have

\[
\hat{z} = \left( 1 + \frac{\alpha/\theta - \eta}{\eta - \alpha} \right)^{\frac{1}{\eta}} \hat{z}_{\min}, \quad (A.9)
\]

and therefore, we have

\[
\Gamma (\hat{z}) = \left( \frac{\eta}{\eta - 1} \right) \left( \frac{\eta}{\eta - \alpha} \right) \left( \frac{\eta - \alpha}{\alpha} \right)^{\frac{\alpha}{\eta}} \theta^{\frac{\alpha}{\eta}} (1 - \theta)^{1 - \frac{\alpha}{\eta}}. \quad (A.10)
\]

We can easily verify that \( \Gamma (\hat{z}) \) strictly increases with the severity of moral hazard \( \theta \) under the condition \( \theta < \frac{\alpha}{\eta} \), which is satisfied under Assumption 2. Note that the definition of \( \xi^* \) implies that
\(\xi^*\) strictly decreases with \(\theta\).

Finally, the variance of \(z\) under the Pareto distribution is given by

\[
\text{Var}(z) = \frac{[\mathbb{E}(z)]^2}{\eta(\eta - 2)} = \frac{1}{\eta(\eta - 2)},
\]

which is well defined and strictly decreases with \(\eta\) provided \(\eta > 2\). Moreover, we can verify that \(\Gamma(\hat{z})\) strictly increases with \(\eta\). Therefore, \(\xi^*\) decreases with \(\text{Var}(z)\). Q.E.D.

**Proof of Proposition 2:** Under the Pareto distribution, (12) and (A.1) imply

\[
\xi = \left(\alpha \frac{A_h}{A_l}\right)^{\frac{1}{\alpha}} \frac{1}{K} \Gamma(z^*)^{-\frac{1}{1-\alpha}}.
\]

Inserting the last equation into (13) and recalling that \(\mathbb{E}(z|z \geq z^*) = \frac{z^*}{z_{\min}}\), we can express the output as

\[
Y = A_h \left(\alpha \frac{A_h}{A_l}\right)^{\frac{1}{\alpha}} \Omega(z^*)^{-\frac{1}{1-\alpha}},
\]

where

\[
\Omega(z^*) = \frac{\theta}{z_{\min}^\eta} (z^*)^{\eta-1} + (1 - \theta)(z^*)^{-1}.
\]

We can easily show that \(\Omega''(z^*) > 0\), i.e., \(\Omega(z^*)\) is convex in \(z^*\) and reaches its minimum at \(\hat{z} = \left[\frac{1-\theta}{(\eta-1)\theta}\right]^\frac{1}{\eta} z_{\min}\). According to Assumption 2, we have \(\theta < \frac{\alpha}{\eta} < \frac{1}{\eta}\), and therefore, \(\frac{1-\theta}{(\eta-1)\theta} > 1\). Then, we immediately know that \(\hat{z} > z_{\min}\). Since \(\xi\) is strictly decreasing in \(z^*\), we must have that the aggregate output achieves its optimal level when \(\xi = \tilde{\xi} = \left[\frac{\alpha K^{\alpha-1} A_h}{\Gamma(z)}\right]^{-\frac{1}{1-\alpha}}\). Recall that the threshold value of the regime switch \(\xi^*\) satisfies \(\xi^* = \left[\frac{\alpha K^{\alpha-1} A_h}{\Gamma(z)}\right]^{-\frac{1}{1-\alpha}}\), where \(\tilde{z} = \left(1 + \frac{\alpha/\theta - \eta}{\eta - \alpha}\right)^\frac{1}{\eta} z_{\min}\). Since \(\hat{z} < \tilde{z}\), we must have \(\tilde{\xi} < \xi^*\).

It remains for us to prove that \(Y\) is concave in \(\xi\). First, since \(\Omega(z^*)\) is convex in \(z^*\), equation (A.13) implies that \(Y\) is concave in \(z^*\). Second, equation (12) implies that

\[
\xi = \left[\frac{\Gamma(z^*)}{\alpha K A_h/A_l}\right]^{-\frac{1}{1-\alpha}}.
\]

We can easily prove that \(\Gamma(z^*)\) is convex with \(z^*\). Meanwhile, the function \(x^{-\frac{1}{1-\alpha}}\) can be easily verified to be a convex function. Therefore, \(\xi\) is convex in \(z^*\), or equivalently, \(z^*\) is concave in \(\xi\). Consequently, the concavity of both \(Y(z^*)\) and \(z^*(\xi)\) implies that \(Y = Y(\xi) = Y(z^*(\xi))\) is concave with \(\xi\).
Since
\[
\Gamma(\tilde{z}) = \frac{\theta \frac{F(\tilde{z})}{1-F(\tilde{z})} + 1}{|E(z|z \geq \tilde{z})|^{\alpha-1} \tilde{z}} = \left( \frac{\eta}{\eta - 1} \right)^2 (\eta - 1)^{\frac{\alpha}{\eta}} \theta^{\frac{\alpha}{\eta}} (1 - \theta)^{1 - \frac{\alpha}{\eta}},
\] (A.16)
we can easily prove that \( \tilde{\xi} \) increases with cross-sector TFP dispersion \( \frac{A_h}{A_l} \), decreases with aggregate capital stock \( K \) and decreases with the severity of moral hazard problem \( \theta \), similar to the comparative statistics of \( \xi^* \) in Proposition A.1. Q.E.D.

Corollary A.2 Under Regime 1 for the Bad equilibrium, i.e., \( \xi \in [\xi^{**}, \xi^*] \), the aggregate output \( Y \) is strictly increasing in \( \xi \), and \( Y^{Bad}(\xi^*) = Y^{Good}(\xi^*) \).

Proof of Corollary A.2: From the definition of \( \tilde{\xi} \), we know that \( \tilde{\xi} \) satisfies \( \alpha \left( \tilde{\xi} K \right)^{\alpha-1} \frac{A_h}{A_l} = \Gamma(\tilde{z}) \), where \( \tilde{z} = \left[ \frac{1 - \theta}{(\eta - 1)\theta} \right]^{\frac{1}{\eta}} z_{\min} \). Under the Assumption 2, we have \( \tilde{z} > \tilde{z} > z_{\min} \). As the function \( \Omega(z^*) \) is convex, i.e., \( \Omega''(z^*) > 0 \), we must have \( \frac{\partial Y}{\partial z^*} > 0 \) for any \( z^* \in [z_{\min}, \tilde{z}] \). From the previous analysis, we already have \( \frac{\partial x}{\partial z^*} > 0 \), therefore we eventually obtain \( \frac{\partial Y}{\partial z^*} = \frac{\partial Y}{\partial z^*} \frac{\partial z^*}{\partial \xi} > 0 \). Moreover, when \( \xi = \xi^* \), the Bad equilibrium converges to the Good one since \( \Gamma(z) \) now reaches minimum at \( \tilde{z} \). Therefore, we must have \( Y^{Bad}(\xi^*) = Y^{Good}(\xi^*) \). Q.E.D.

Lemma A.2 Under the Assumption 2, there exists a unique equilibrium (the Ugly) in the Regime 2 where \( \xi > \xi^* \).

Proof of Lemma A.2: Under the Pareto distribution, we have
\[
\Phi(z^*) = z_{\min}^{-(1-\alpha)(1-\eta)} (z^*)^{(1-\alpha)(1-\eta)-1},
\] (A.17)
with the properties \( \lim_{z^* \to z_{\min}} \Phi(z^*) = \lim_{z^* \to z_{\min}} \Gamma(z^*) = \frac{1}{z_{\min}} \) and \( \lim_{z^* \to +\infty} \Phi(z^*) = 0 \). Since \( \Phi(z^*) \) is monotonically decreasing, to show the existence of unique solution of (15) is equivalent to show \( \alpha \frac{A_h}{A_l} (\xi K)^{\alpha-1} < \Phi(z_{\min}) \). This is indeed the case. Remember that under the Assumption 2, i.e., \( \eta < \frac{\alpha}{\theta} \), we have \( \Gamma(z_{\min}) > \Gamma(\tilde{z}) \) because \( \tilde{z} \) solve the problem \( \min_{\tilde{z}} \Gamma(z) \). Moreover, as \( \Phi(z_{\min}) = \Gamma(z_{\min}) \), we immediately have \( \Phi(z_{\min}) > \Gamma(\tilde{z}) \). Since in the Regime 2, the credit policy \( \xi \) must satisfy \( \xi > \xi^* \) or equivalently \( \alpha \frac{A_h}{A_l} (\xi K)^{\alpha-1} < \Gamma(\tilde{z}) \). Then, we must have
\[
\alpha \frac{A_h}{A_l} (\xi K)^{\alpha-1} < \Gamma(\tilde{z}) < \Phi(z_{\min}).
\] (A.18)
Consequently, equation (15) has a unique solution. Q.E.D.

Lemma A.3 The sectoral outputs \( Y_h \) and \( Y_l \) and the aggregate output \( Y \) increase with the credit policy \( \xi \).
Proof of Lemma A.3: Once the equilibrium $z^*$ is solved from (15), it is straightforward to obtain the sectoral outputs. In particular, we have

$$Y_h = A_h \{\xi K [1 - F (z^*)] \mathbb{E} (z \mid z \geq z^*) \}^\alpha = \frac{A_h \xi K}{\alpha} \int_{z \geq z^*} \frac{z}{z^*} dF (z) , \tag{A.19}$$

where the second equality is obtained by using (15), and $Y_l = A_l \xi K F (z^*)$.

Meanwhile, substituting (15) into (A.19) implies that $Y_h$ strictly increases with $\xi$. On the other hand, (15) implies that $z^*$ strictly increases with $\xi$, and so does $Y_l$. Consequently, the aggregate output $Y = Y_h + Y_l$ also monotonically increases with $\xi$. Q.E.D.

Proof of Proposition 3: According to the definition, the $\xi^*$ satisfies

$$\alpha (\xi^* K)^{\alpha-1} \frac{A_h}{\alpha} = \Gamma (z_1^*),$$

where $z_1^* = \arg \min \Gamma (z) = \hat{z}$. Let $z_2^*$ denote the equilibrium cutoff in Regime 2 when $\xi = \xi^*$, i.e., $\alpha (\xi^* K)^{\alpha-1} \frac{A_h}{\alpha} = \Phi (z_2^*)$. As discussed earlier $\Phi (z) < \Gamma (z)$ for any $z > z_{\text{min}}$ and $\Phi (z)$ is monotonically decreasing in $z$, we must have $z_2^* < z_1^*$. So the equilibrium cutoff $z^*$ is discontinuous at $\xi = \xi^*$.

Based on the above Lemma, we can further show that the aggregate output is discontinuous at the $\xi = \xi^*$. In particular, the aggregate output in Regime $j \in \{1, 2\}$ can be written as

$$Y_j = Y_{j,h} + Y_{j,l} = A_h \{K_{j,h} \mathbb{E} (z \mid z \geq z_j^*) \}^\alpha + A_l (\xi K - K_{j,h}) , \tag{A.20}$$

where $K_{j,h}$ is the capitals used in H sector in Regime $j$ satisfying

$$K_{1,h} = \xi K , \tag{A.21}$$
$$K_{2,h} = \xi K [1 - F (z_2^*)] . \tag{A.22}$$

Notice that since $K_{1,h} = \xi K$ is independent with $z_1^*$, we have

$$\frac{\partial Y_1}{\partial K_{h,1}} = A_h \alpha K_{1,h}^{\alpha-1} \mathbb{E} (z \mid z \geq z_1^*) \}^\alpha - A_l = \pi_{1,h} \mathbb{E} (z \mid z \geq z_1^*) - A_l > \pi_{1,h} \hat{z}_1 - A_l. \tag{A.23}$$

The second line is due to the definition of $\pi_{1,h}$ (see equation 9). It worth noting that the equilibrium in the Regime 1 implies the interbank market does not collapse, i.e., $\pi_{1,h} \hat{z}_1 = R' > A_l$. Thus, we
must have \( \frac{\partial Y_1}{\partial K_{h,1}} > 0 \). With this monotonicity property, we further have

\[
Y_1 = A_h [K_{1,h} \mathbf{E}(z | z \geq z_1^*)]^{\alpha} + A_l (\xi K - K_{1,h})
\]

\[
> A_h [K_{2,h} \mathbf{E}(z | z \geq z_1^*)]^{\alpha} + A_l (\xi K - K_{2,h})
\]

\[
> A_h [K_{2,h} \mathbf{E}(z | z \geq z_2^*)]^{\alpha} + A_l (\xi K - K_{2,h})
\]

\[
= Y_2. \tag{A.24}
\]

The second line is due to the fact \( K_{1,h} > K_{2,h} \) and \( \frac{\partial Y_1}{\partial K_{h,1}} > 0 \), while the third line just applies \( z_1^* > z_2^* \).

Finally, it remains for us to pin down the condition under which \( Y_1 (\xi = \xi^*) > Y_2 (\xi = 1) \), i.e., the output in Regime 2 is always below the level at threshold value \( \xi^* \) in Regime 1. First, note that according the definition of output, we have

\[
Y_1 (\xi = \xi^*) = A_h [\xi^* K \mathbf{E}(z | z \geq z_1^*)]^{\alpha}, \tag{A.25}
\]

\[
Y_2 (\xi = 1) = A_h [K_{2,h} \mathbf{E}(z | z \geq z_2^*)]^{\alpha} + A_l (\xi K - K_{2,h}), \tag{A.26}
\]

where we have already proved that

\[
\xi^* = \left( \frac{\alpha A_h}{\alpha A_l} \right)^{\frac{1}{\alpha+\eta-\alpha\eta}} \frac{1}{K}, \tag{A.27}
\]

\[
z_1^* = \arg \min_z \Gamma (z) = \hat{z} = \left( 1 + \frac{\alpha/\theta - \eta}{\eta - \alpha} \right)^{\frac{1}{\eta}} z_{\min}, \tag{A.28}
\]

\[
K_{2,h} = \xi K [1 - \mathbf{F}(z^*)] |_{\xi=1} = K [1 - \mathbf{F}(z^*)], \tag{A.29}
\]

\[
z_2^* = \left[ \frac{A_l}{\alpha A_h K^{\alpha-1} z_{\min}} \right]^{\frac{1}{\alpha+\eta-\alpha\eta}} z_{\min} |_{\xi=1} = \left[ \frac{A_l}{\alpha A_h K^{\alpha-1} z_{\min}} \right]^{\frac{1}{\alpha+\eta-\alpha\eta}} z_{\min}. \tag{A.30}
\]

We further have

\[
Y_1 (\xi = \xi^*) = A_h \left[ \left( \frac{\alpha A_h}{\alpha A_l} \right)^{\frac{1}{\alpha+\eta-\alpha\eta}} \left( 1 + \frac{\alpha/\theta - \eta}{\eta - \alpha} \right)^{\frac{1}{\eta}} \right]^{\alpha}, \tag{A.31}
\]
and

\[
Y_2 (\xi = 1) = A_h \left[ K (1 - F(z_2^*)) \mathbb{E} (z|z \geq z_2^*) \right]^\alpha + A_l K F(z_2^*) \\
= A_h \left( K \int_{z_2^*}^{z_{\text{max}}} zdF \right) \alpha + A_l K F(z_2^*) \\
= A_h \left( \frac{\alpha A_h}{A_l} z_2^* \right)^{\frac{\alpha}{1 - \alpha}} + A_l K \left[ 1 - \left( \frac{z_2^*}{z_{\text{min}}} \right)^{-\eta} \right] \\
= \left( \frac{\alpha z_{\text{min}} A_h}{A_l} \right)^{\frac{\alpha(n-1)}{\alpha + \eta - \alpha \eta}} \left( 1 - \alpha z_{\text{min}} \right) A_h K^{\frac{\alpha}{\alpha + \eta - \alpha \eta}} + A_l K.
\]  

(A.32)

Therefore, \( Y_1 (\xi = \xi^*) > Y_2 (\xi = 1) \) if and only if

\[
\left( \frac{A_h}{A_l} \right)^{\frac{1}{1 - \alpha}} \left( \frac{\alpha}{\Gamma (\hat{z})} \right)^{\frac{\alpha}{\eta - \alpha}} \left( 1 + \frac{\alpha/\theta - \eta}{\eta - \alpha} \right)^{\frac{\alpha}{\eta}} > \left( \frac{\alpha z_{\text{min}} A_h}{A_l} \right)^{\frac{\alpha(n-1)}{\alpha + \eta - \alpha \eta}} \left( 1 - \alpha z_{\text{min}} \right) K^{\frac{\alpha}{\alpha + \eta - \alpha \eta}} + K.
\]

(A.33)

First, note that \( \alpha < 1 \), and \( z_{\text{min}} < 1 \). Therefore \( 1 - \alpha z_{\text{min}} > 0 \). Then we know that RHS of the above inequality strictly increases with \( K \). Second, note that \( \frac{1}{1 - \alpha} > \frac{\eta}{\alpha + \eta - \alpha \eta} \). Consequently, we know that \( Y_1 (\xi = \xi^*) > Y_2 (\xi = 1) \) if and only if (1) \( K \) is small enough, or (2) \( \frac{A_h}{A_l} \) is large enough.

Q.E.D.

**Proof of Proposition 4:** We analyze the steady state in Regime 1 and that in Regime 2 sequentially.

**Regime 1:** i.e., \( R_f > A_l \), only the H sector prevails. So \( Y_l = 0 \), and \( Y = Y_h = A_h [\mathbb{E} (z|z \geq z^*) \xi K]^{\alpha} \).

Then \((K, z^*)\) are jointly determined by

\[
\alpha A_h \xi^{\alpha - 1} = \Gamma(z^*), \\
\alpha A_h \xi^{\alpha - 1} = \Gamma(z^*).
\]

(A.34)

Combining last two equations yields

\[
\frac{r}{A_l \xi} = \frac{1}{z^*} \left[ \theta \frac{\mathbb{F} (z^*)}{1 - \mathbb{F} (z^*)} + 1 \right] \mathbb{E} (z|z \geq z^*),
\]

(A.35)

from which we can further solve the cutoff value of productivity \( z^* \) as \( z^* \approx \left[ \frac{\gamma z_{\text{min}} - (1 - \theta)}{\gamma A_l \xi} \right]^{\frac{1}{\gamma}} z_{\text{min}} \).

Therefore, Regime 1 can be supported if and only if \( \xi < \xi_H = \frac{z_{\text{min}}}{A_l \xi} \). Notice that the value of \( z^* \) varies with the credit policy \( \xi \). So given that \( \xi < \frac{z_{\text{min}}}{A_l \xi} \), we need to further analyze whether it is a **Good** or **Bad** equilibrium in Regime 1.
In particular, if \( z^* > \hat{z} \), where \( \hat{z} = \arg \min \Gamma (z) \), then it is the **Good** equilibrium; if \( z^* \in (z_{\text{min}}, \hat{z}) \), then it is the **Bad** equilibrium. Notice that under the Assumption 2 that \( \frac{\eta(1-\theta)}{\eta-\alpha} > 1 \), the **Good** equilibrium can be supported if and only if \( \xi < \xi_L \equiv \frac{\eta-\alpha}{\eta(1-\theta)} \xi_H \). In turn, the **Bad** one can be supported if \( \xi \in (\xi_L, \xi_H) \).

**Regime 2**: i.e., \( R' = A_l \), then H sector and L sector coexist. We have \( Y_l = A_l \xi K F (z^*) \) and \( Y_h = A_h \{ E (z|z \geq z^*) \xi K \}^\alpha \). Then \((K,z^*) \) are jointly determined by

\[
\alpha \frac{A_h}{A_l} \xi K ^{\alpha-1} = \Phi (z^*) ,
\]

\[
rK = \alpha Y_h + Y_l .
\]

The last equation implies

\[
F (z^*) = \frac{\frac{r}{A_l} \xi ^{\alpha} - \frac{1}{z_{\text{min}}}}{1 - \frac{1}{z_{\text{min}}}} .
\]

To guarantee the equilibrium exist, we must have \( 0 < \frac{\frac{r}{A_l} \xi ^{\alpha} - \frac{1}{z_{\text{min}}}}{1 - \frac{1}{z_{\text{min}}}} < 1 \). Since \( \xi > \xi_H \) implies that \( \frac{\frac{r}{A_l} \xi ^{\alpha} - \frac{1}{z_{\text{min}}}}{1 - \frac{1}{z_{\text{min}}}} > 0 \) always holds, the above condition implies \( \xi \) must satisfy \( \xi < \xi_X \equiv \frac{r}{A_l} \). Since \( \xi_X \) is greater than \( \xi_H \), for the case of Regime 2, the \( \xi \) must satisfies \( \xi \in (\xi_H, \xi_X) \). **Q.E.D.**

### B Dynamics under Technology Shocks

We now document the dynamic impacts of TFP shocks on the aggregate economy. In particular, we will discuss a shock to TFP in the H and L sectors, separately. In light of the previous steady-state analysis, the change in the TFP in the H sector does not influence the critical values \( \xi_L \) and \( \xi_H \) (see the Proposition 4). Therefore, given a credit policy \( \xi \), a reduction in \( A_h \) may not change the steady-state regime. That is, if the the economy is initially in the **Good** steady state, the new steady state after a permanent reduction of \( A_h \) remains the **Good** regardless of the magnitude of the adverse shock. Thus, a negative shock to \( A_h \) does not lead to oscillation dynamics. We now consider a concrete quantitative example. We assume that in the initial period the economy is at the steady state. We specify the initial level of credit policy \( \xi_0 = 0.9 \times \xi_L \), so the initial steady state is the **Good** equilibrium. In period 1, the TFP in the H sector declines permanently by 1%, 2% and 3%, respectively. Figure B.1 presents the corresponding transition dynamics. It shows that an adverse TFP shock in the efficient H sector depresses the aggregate economy. For relatively small shocks (namely, 1% and 2%), the depression is proportional to the decline in the TFP, whereas a larger shock (namely, 3%) triggers a large-scale decline in the aggregate output and capital in the short run because of the temporal collapse of the interbank market. This pattern looks very similar to those in the credit expansion in which \( \xi_t < \xi_L \).
Figure B.1: Transition Dynamics under Negative $A_h$ shock: $\xi = 0.9 \times \xi_L$
Figure B.2: Transition Dynamics under Positive $A_t$ Shock: $\xi = 0.9 \times \xi_L$
We now turn to TFP shocks to the L sector. Since a rise in $A_l$ reduces the dispersion of TFP in the two sectors, to document the negative effect of $A_l$ shocks, we consider an increase in $A_l$. As has been shown in the steady-state analysis, unlike the $A_h$ shock, the $A_l$ shock matters for the threshold values $\xi_L$ and $\xi_H$; therefore, a sufficiently large $A_l$ shock may alter the regime of steady states, resulting in oscillation dynamics. To see this, we consider a quantitative example. Akin to the previous analysis, we assume the economy initially stays in the Good steady state, where $\xi_0 = 0.9 \times \xi_L$. In period 1, the TFP in L sector increases by 1%, 5% and 10%, respectively. Figure B.2 reports the transition dynamics. For a small increase in $A_l$ (namely, 1%), the aggregate output and capital experience moderately decline. If $A_l$ increases further (namely, 5%), the economy may temporally hit the crisis regime, causing a large drop in output and capital in the short run. Since the new steady state now is still a Good equilibrium, the transition path eventually converges. However, if the increase of $A_l$ is sufficiently large (e.g., 10%), the steady state may not be well defined (the policy function $g(K_t)$ does not cross the 45 degree line); consequently, the transition paths present oscillation dynamics (the blue lines in Figure B.2). The above analysis indicates that sectoral TFP shocks may have distinct effects on the real economy. Unlike standard business cycle theories, TFP shocks to the inefficient sector may lead to endogenous fluctuations.