1. The residents of a certain dormitory have collected the following data: People who live in the dorm can be classified as either involved in a relationship or uninvolved. Among involved people, 50 percent experience a breakup of their relationship every month. Among uninvolved people, 10 percent will enter into a relationship every month. What is the steady-state fraction of residents who are uninvolved?

Answer:
In this example, the break-up rate is similar as a job separation rate and the newly-engage rate is similar as a job finding rate. Hence the steady-state fraction of residents who are not involved is similar as the natural rate of unemployment that equals
\[
\frac{50\%}{50\% + 10\%} = 83.3\%
\]

2. In this chapter we saw that the steady-state rate of unemployment is \( U/L = s/(s + f) \). Suppose that the unemployment rate does not begin at this level. Show that unemployment will evolve over time and reach this steady state. (Hint: Express the change in the number of unemployed as a function of \( s, f, \) and \( U \). Then show that if unemployment is above the natural rate, unemployment falls, and if unemployment is below the natural rate, unemployment rises.)

Answer: Assuming the total labor force to be constant, then the change of the number of unemployed workers is
\[
\Delta U_{t+1} = s(L - U_t) - fU_t.
\]
Divide both sides with \( L \), we have
\[
\Delta U_{t+1}/L = s(1 - U_t/L) - fU_t/L,
\]
which leads to
\[
\frac{\Delta U_{t+1}}{L} = (s + f)\left[\frac{s}{s+f} - \frac{U_t}{L}\right].
\]
Therefore \( \frac{\Delta U_{t+1}}{L} > 0 \) when \( \frac{U_t}{L} < \frac{s}{s+f} \), and \( \frac{\Delta U_{t+1}}{L} < 0 \) when \( \frac{U_t}{L} > \frac{s}{s+f} \). That is to say when the unemployment rate is above the natural rate, unemployment falls. Otherwise, unemployment rises.

3. Consider how unemployment would affect the Solow growth model. Suppose that output is produced according to the production function
\[
Y = K^\alpha [(1 - u)L]^{1-\alpha},
\]
where \( K \) is capital, \( L \) is the labor force, and \( u \) is the natural rate of unemployment. The national saving rate is \( s \), the labor force grows at rate \( n \), and capital depreciates at rate \( d \).

1). Express output per worker \((y = Y/L)\) as a function of capital per worker \((k = K/L)\) and the natural rate of unemployment. Describe the steady state of this economy.
2). Suppose that some change in government policy reduces the natural rate of unemployment. Describe how this change affects output both immediately and over time. Is the steady-state effect on output larger or smaller than the immediate effect? Explain.

Answer:

1) Divide both sides with L and we have

$$\frac{Y}{L} = \left(\frac{K}{L}\right)^\alpha (1 - u)^{1-\alpha},$$

which leads to

$$y = f(k) = k^\alpha (1 - u)^{1-\alpha}.$$  

At the steady state, investment equals depreciation plus the capital that has to be supplied for the new workers:

$$sf(k^\ast) = (\delta + n)k^\ast$$

which leads to

$$sk^\ast\alpha (1 - u)^{1-\alpha} = (\delta + n)k^\ast.$$  

Hence the steady state capital stock per worker is

$$k^\ast = (1 - u)(\frac{s}{\delta + n})^{\frac{1}{1-\alpha}}.$$  

The corresponding steady state output per worker is

$$y^\ast = f(k^\ast) = (1 - u)(\frac{s}{\delta + n})^{\frac{\alpha}{1-\alpha}}.$$  

2) Figure 1 shows that when u is increased, we have a new steady state with lower capital stock per worker and output per worker. Now we are experiencing a reduction of u, we would expect to have a new steady state with higher capital stock per worker and output per worker. Figure 2 below shows the pattern of output over time. As soon as unemployment falls from u1 to u2, output jumps up from its initial steady-state value of y^*(u1). The economy has the same amount of capital (since it takes time to adjust the capital stock), but this capital is combined with more workers. At that moment the economy is out of steady state: it has less capital than it wants to match the increased number of workers in the economy. The economy begins its transition by accumulating more capital, raising output even further than the original jump. Eventually the capital stock and output converge to their new, higher steady-state levels.
4. The Solow model we have learned in class is for the closed economy. For an open economy, the national saving does not necessarily equal investment as international capital flow is possible. Let's assume that the country has an imperfect financial system so that only a $\lambda$ fraction of the national saving can be transformed as investment, that is

$$I = \lambda sY.$$

Assume the production function takes a standard form as $Y = K^\alpha L^{1-\alpha}$. 
1). Express output per worker \( y = Y/L \) as a function of capital per worker \( k = K/L \). Describe the steady state of this economy.

2). Suppose that some changes in government policy make the financial system more efficient and hence raise the saving to investment transformation rate \( \lambda \). Describe how this change affects output both immediately and over time. Is the steady-state effect on output larger or smaller than the immediate effect? Explain.

Answer:

1) The relationship between output per worker and capital per worker is unchanged by introducing this saving to investment transformation rate \( \lambda \): 
\[ y = f(k) = k^\alpha. \]

At the steady state we have that investment equals the depreciation (it is fine if you assume there is population growth as well):
\[ \lambda s k^\alpha = \delta k^\ast \]

Hence the steady state capital stock per worker is 
\[ k^\ast = \left( \frac{\lambda s}{\delta} \right)^{\frac{1}{1-\alpha}}. \]

The corresponding steady state output per worker is 
\[ y^\ast = f(k^\ast) = \left( \frac{\lambda s}{\delta} \right)^{\frac{\alpha}{1-\alpha}}. \]

As evident from this equation, the saving to investment transformation rate has a positive effect on the steady state capital stock and output.

2) Overtime, the economy will transit to a new steady state where the output per capita is higher. However, then immediate effect is zero as the capital stock and amount of labor cannot change immediately right after the change of \( \lambda \).