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Univariate unobserved-component model with a nonrandom-walk permanent component

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In this article, we revisit the univariate unobserved-component (UC) model of the US GDP by relaxing the traditional random-walk assumption of the permanent component. Since our general UC model is unidentified, we investigate the upper bound of the contribution of the transitory component, and find the GDP fluctuation is dominated by the permanent component.

Keywords: unobserved-component model; random-walk assumption; permanent and transitory shocks

JEL Classification: C22; C49; E32

I. Introduction

Morley et al. (2003) study the equivalence of univariate unobserved-component (UC) model and the Beveridge–Nelson (BN) (1981) decomposition. They conclude that the permanent component of the US GDP extracted by the UC model is exactly the same as the BN trend. The innovations of the two (permanent and transitory) components are highly negatively correlated (further discussions about this point can be found in a recent paper by Oh et al. (2008)). The nonorthogonality of the two innovations is mainly caused by the random-walk assumption imposed on the permanent component, see Nagakura (2008) for the formal discussion. In this article, we relax the random-walk assumption by allowing the permanent component to follow a general unit root process. Under our assumption, the real GDP can be decomposed into two orthogonal parts so that the impulse responses to permanent and transitory shocks can be implemented. Since our generalization of the random-walk assumption increases the parameter set of the UC model, the model becomes unidentified. However, we can investigate the upper bound of the contribution of the transitory component to GDP and study the dynamics of this extreme case by implementing impulse response and variance decomposition. We find that the transitory component explains less than 35% of output volatility; therefore, the permanent component is the main source of the GDP fluctuation.

II. The UC Model

Our modified UC representation takes the form

\[ y_t = g_t + c_t \]

\[ g_t = \mu + g_{t-1} + \frac{\Theta_{g_t}(L)}{\Phi_{g_t}(L)} \eta_t, \eta \sim i.i.d \mathcal{N}(0, \sigma^2_{\eta}) \]  

(1)

\[ c_t = \frac{\Theta_{c_t}(L)}{\Phi_{c_t}(L)} \varepsilon_t, \varepsilon \sim i.i.d \mathcal{N}(0, \sigma^2_{\varepsilon}) \]
where \{y_t\} is the log real GDP, and \{g_t\} is an unobserved permanent component with a unit root (i.e., its first difference is an autoregressive-moving-average (ARMA) \((p_1, q_1)\) process with drift \(\mu\)). The unobserved transitory component \(\{c_t\}\) is a stationary ARMA \((p_2, q_2)\) process. Moreover, we assume the two innovations satisfy

\[
\text{cov} (\eta_t, \epsilon_{t+k}) = \begin{cases} 
\sigma_{\eta \epsilon} & \text{for } k = 0 \\
0 & \text{otherwise}
\end{cases}
\]

The parameters under interest include the mean growth rate \(\mu\), and the coefficients of the two ARMA process, \(\{\Phi_{p_1}(L), \Theta_{q_1}(L), \Phi_{p_2}(L), \Theta_{q_2}(L), \sigma_\eta, \sigma_\epsilon, \sigma_{\eta \epsilon}\}\).

Writing the model (1) more compactly gives the ARIMA representation of \(y_t\),

\[
\Phi_{p_1}(L)\Phi_{p_2}(L)\Delta y_t = \Phi_{p_1}(1)\Phi_{p_2}(1)\mu + \Phi_{p_1}(L)\Theta_{q_1}(L)\eta_t + (1-L)\Phi_{p_2}(L)\Theta_{q_2}(L)\epsilon_t
\]

This expression implies we can recover the parameters of the UC model by estimating the growth rate of GDP as a ARIMA process. Here, we follow the strategy of Morley et al. (2003) to estimate the GDP as an ARIMA (2,1,2) process:

\[
(1 - \phi_1 L - \phi_2 L^2)\Delta y_t = (1 - \phi_1 - \phi_2)\mu + (1 + \theta_1 L + \theta_2 L^2)\epsilon_t
\]

Table 1 reports the estimated results. Note that the \(\gamma_j\) are the j-th order auto-covariance of MA part of the ARIMA process, and the \(\sigma_{\eta \epsilon}\) and \(\gamma_j\) are in percentages. The data used are US quarterly real GDP from 1948:Q1 to 2008:Q4.

The absence of real roots in AR part indicates that the polynomial \((1 - \phi_1 L - \phi_2 L^2)\) cannot be factored further. This fact induces us to determine the form of \(\Phi_{p_1}(L)\) and \(\Phi_{p_2}(L)\) only in two alternative ways: \(\Phi_{p_1}(L) = 1,\)

<table>
<thead>
<tr>
<th>(\Phi_{p_2}(L))</th>
<th>(\Phi_{p_3}(L))</th>
<th>(\Phi_{p_4}(L))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Phi_{p_1}(L) = 1)</td>
<td>(\Phi_{p_2}(L) = 1)</td>
<td>(\Phi_{p_3}(L) = 1)</td>
</tr>
</tbody>
</table>

\(\Phi_{p_i}(L) = (1 - \phi_1 L - \phi_2 L^2)\) or \(\Phi_{p_i}(L) = \Phi_{p_2}(L) = (1 - \phi_1 L - \phi_2 L^2)\). Obviously, the first case is just the specification in Morley et al. (2003) in which permanent component \(g_t\) is a random walk. The second case is the one we want to discuss in which \(g_t\) is a general ARIMA (2,1,2) process.

Once \(\Phi_{p_1}(L)\) and \(\Phi_{p_2}(L)\) are determined, we can find the form of MA polynomials \(\Theta_{q_1}(L)\) and \(\Theta_{q_2}(L)\). In particular, to ensure the right-hand side RHS of Equation 2 a MA(2) process, \(\Theta_{q_1}(L)\) and \(\Theta_{q_2}(L)\) can at most take the form of \((1 + \psi_1 L + \psi_2 L^2)\) and \((1 + \theta L)\), respectively. Now the parameters of interest are \(\{\psi_1, \psi_2, \theta, \sigma_\eta, \sigma_\epsilon, \sigma_{\eta \epsilon}\}\), and the representation (2) is reduced to

\[
(1 - \phi_1 L - \phi_2 L^2)\Delta y_t = (1 - \phi_1 - \phi_2)\mu + (1 + \psi_1 L + \psi_2 L^2)\eta_t + (1 - L)(1 + \theta L)\epsilon_t
\]

Remember that we have estimated the auto-covariances of the RHS of the last equation from the data, see \(\{\gamma_0, \gamma_1, \gamma_2\}\) in Table 1. Equate these moments to their counterparts in Equation 4 and after some algebra, we get three equations for six parameters \(\{\psi_1, \psi_2, \theta, \sigma_\eta, \sigma_\epsilon, \sigma_{\eta \epsilon}\}\):

\[
\begin{align*}
\sigma_\eta^2 &= \gamma_0 + 2\gamma_1 + 2\gamma_2 \\
\sigma_\epsilon^2 &= -2(1 + \psi_1 \theta - \psi_1 - \psi_2 \theta)(\psi_2 - \psi_1 \sigma_\eta^2) - (\theta - \psi_2)(\gamma_0 - (1 + \psi_1^2 + \psi_2^2)\sigma_\eta^2) \\
\sigma_{\eta \epsilon} &= \theta\gamma_0 - (1 + \psi_1^2 + \psi_2^2)\sigma_\eta^2 + 2(1 - \theta + \theta^2)(\gamma_2 - \psi_2 \sigma_\eta^2) \\
\sigma_{\eta \epsilon} &= 2\theta(1 + \psi_1 \theta - \psi_1 - \psi_2 \theta) - 2(\theta - \psi_2)(1 - \theta + \theta^2)
\end{align*}
\]

1 Oh et al. (2008) also recommend this specification. They find that ARIMA (2,1,2) is preferred by the Akaike information criterion (AIC) and ARIMA (1,1,0) is preferred by the Bayesian information criterion (BIC). However, the latter specification is not able to capture the periodic behaviour of output due to its oversimplified structure.

2 The setting \(\Phi_{p_1}(L) = (1 - \phi_1 L - \phi_2 L^2)\), \(\Phi_{p_2}(L) = 1\) is infeasible, since this will make the order of MA part of \(\Delta y_t\) (the RHS of Equation 2) exceed 2.

3 The mean growth rate \(\mu\) is just the same as that in ARIMA representation.
The MA(2) process has only three auto-variances, but we have six unknown parameters. This implies our UC model is unidentified.

In order to obtain two structural (or orthogonal) shocks, we need to set \( \sigma_{\text{w}} \) to be zero. The reader may ask whether this restriction is feasible,\(^4\) since in Morley et al. (2003), when permanent component is a random walk, two innovations are always highly negatively correlated. In fact, as long as the long-run effect (see the last row in Table 1) in the ARIMA representation of GDP is larger than 1, the orthogonality restriction in our modified UC model is always feasible. A formal mathematical proof can be found in Corollary 1 of Nagukara (2008).

To learn the relationships of the unknown parameters, one method is to solve three of them as functions of the other two. Unfortunately, equation system 5 is nonlinear and fairly complicated, we cannot solve it in a closed form.

So we resort to numerical method. Figure 1 plots \( \psi_1, \sigma_\eta, \sigma_\varepsilon \) as functions of \( \psi_2 \) when \( \theta = 0 \). For other values of \( \theta \) in \((-1, 1)\), the pattern changes little. In addition, to ensure \( \Delta g_t \) be invertible and \( \sigma_\varepsilon^2 \) be always positive, \( \psi_2 \) must be in the range around 0.6–1.

One thing worth noting in Fig. 1 is that \( \{ \psi_1, \sigma_\eta, \sigma_\varepsilon \} \) are monotonic functions of \( \psi_2 \), and the monotonicity does not change for different \( \theta \). Furthermore, the SD of transitory shock \( \varepsilon_t \) reaches its maximum when \( \psi_2 \) approaches to 1. Since \( \sigma_\varepsilon \) is a continuous function of \( \psi_2 \) and \( \theta \), without loss of generality, we fix \( \psi_2 = 1 \) for different \( \theta \) to find the largest transitory component (in terms of variance) in our modified UC model. Figure 2 plots \( \sigma_\varepsilon \) against \( \theta \), when \( \psi_2 = 1 \). From the figure, we can see that \( \sigma_\varepsilon \) reaches its unique maximum of 0.4442 at \( \theta = -0.63 \).

The above analysis implies that our UC model can be just identified, if the transitory and permanent components are forced to be orthogonal and the volatility of transitory component reaches its upper bound. In Section III, we will study the dynamic features of the two components under the above identification method.

### III. Dynamics

The largest possible variance of the transitory component \( \{ \psi_1 \} \) has SD of 0.4442 when setting \( \theta = -0.63 \) and \( \psi_2 = 1 \). The remaining parameters \( \psi_1 \) and \( \sigma_\eta \) can be solved directly from equation system 5. In particular, we have \( \psi_1 = -1.2612 \) and \( \sigma_\eta = 0.6059 \).\(^5\) Since both BQ (1989)\(^6\) and our UC model implement orthogonal decomposition with a general unit root permanent component, we can use impulse responses and variance decomposition to compare our results with theirs. To ensure consistency (i.e., GDP in the bivariate BQ decomposition must also follow an ARIMA (2,1,2) process), we estimate a two-variable vector autoregression (VAR) system with GDP growth and unemployment rate as a vector ARMA (1,1) process. We use RATS 7.0 (Estima, Evanston, IL, USA) to conduct the estimation.

Figure 3 plots the impulse responses of GDP to a one SD permanent and transitory shock, respectively.\(^7\) In particular, under the permanent shock \( \eta_t \) (the left graph), output in our UC model has a larger and periodic response.

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\(^4\) Here, ‘feasible’ means that equation system 5 always has solution when \( \sigma_{\text{w}} = 0 \).

\(^5\) The parameters \( \{ \sigma_\eta, \sigma_\varepsilon, \psi_1 \} \) are statistically significant, we calculate their \( t \)-statistics by bootstrapping method, but not reported here.

\(^6\) In their paper, Blanchard–Quah (BQ) decompose GDP based on a structural bivariate VAR system of (\( \Delta \)GDP, unemployment rate). They just identify the model by imposing a long-run restriction on the transitory component.

\(^7\) The dashed lines are 95% bootstrapped confidence interval computed (200 replications) by Hall’s percentile interval.
compared with that obtained by the BQ method. The maximum response climbs to the peak after six quarters.

The long-run effect of the permanent shock is also significantly larger (about 1.1), while under the BQ decomposition this value is only around 0.6. Under the transitory shock $\epsilon_t$ (the right graph), output movement in our model dies out quickly, while under the BQ decomposition the response is much larger and more persistent.

To see the relative importance of two shocks to the GDP volatility, Table 2 reports the variance decomposition, i.e. the proportion of fluctuations due to transitory shock $\epsilon_t$ in different forecasting horizons.

The numbers in parentheses are 95% confidence intervals. Even though these error bands of the BQ decomposition are large, contribution of transitory shocks to GDP are significantly lower in our model even compared with the
lower bound of the BQ decomposition (except for the impact period). That is, our model attributes most fluctuations of output to permanent shock; the transitory component is less important.

To see what may have caused these discrepancies in the two different approaches, we compare the data-generating processes of output implied by these two estimations. Since we estimate the bivariate system of BQ decomposition as a vector autoregressive-moving-average (VARMA) (1,1) process, the growth rate of GDP can be recovered as an ARMA (2,2) process. Table 3 (in comparison with Table 1) lists the implied parameters under the VARMA. Clearly, these different values implied by the VARMA (1,1) and the univariate ARMA (2,2) induce a much smaller long-run effect. This explains why the permanent shock in the BQ decomposition has smaller long-run effect than what we obtain in the UC model.8

IV. Conclusions

This article re-examines the UC method of decomposition of GDP by relaxing the random-walk assumption made in the existing UC literature. Based on this generalization, we are able to decompose GDP into two orthogonal components: permanent and transitory. The orthogonality allows us to conduct impulse response analysis and variance decompositions. We find that the permanent component explains the bulk of GDP fluctuations, in sharp contrast to the conclusion reached by Blanchard and Quah (1989).

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References


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8 In fact, this point can be easily seen from a spectrum perspective: the spectrum of growth rate of GDP shares the same value with growth rate of permanent component at zero frequency, and this value is just the squared long-run effect multiplying the variance of innovation in ARIMA process.