

第八讲 货币经济学经验研究方法

张国雄

上海交通大学安泰经管学院

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- Propensity Score Matching
- Difference in Difference
- Regression Discontinuity
- 单元时间序列方法
- 向量自回归方法

Propensity Score Matching (PSM)

- PSM的核心思想在于假设conditional on X_i , 个体的outcome与treatment无关

$$E(Y_{i0}|D_i = 1, X_i) = E(Y_{i0}|D_i = 0, X_i)$$

- 所以treatment effect可以表示为

$$ATT = E(Y_{i1}|D_i = 1, X_i) - E(Y_{i0}|D_i = 1, X_i) = E(Y_{i1}|D_i = 1, X_i) - E(Y_{i0}|D_i = 0, X_i)$$

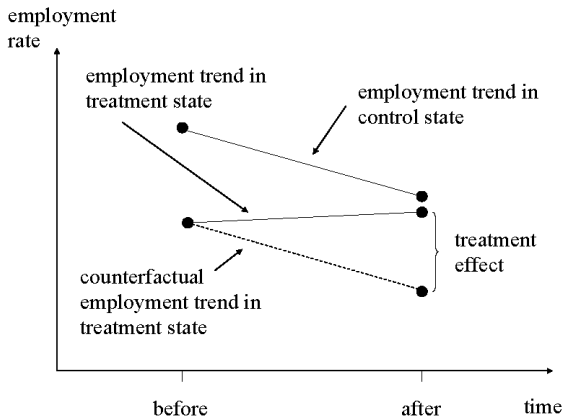
- 更practical地可以表示为

$$ATT = E(Y_{i1}|D_i = 1, p(X_i)) - E(Y_{i0}|D_i = 0, p(X_i))$$

Propensity Score Matching (PSM)

- 两个主要的问题
 - 如何选取 X_i 具有“随意性”
 - 如何决定两个个体的propensity score足够接近具有“随意性”

Difference in Difference (DID)



Difference in Difference (DID)

- DID的核心思想在于假设实验组($s=1$)和对照组($s=2$)有common trend:

$$Y_{ist} = \alpha_s + \lambda T + \beta D_{st} + \epsilon_{ist}$$

- 所以有

$$\beta = [E(Y_{i,s=1,t=1}) - E(Y_{i,s=1,t=0})] - [E(Y_{i,s=2,t=1}) - E(Y_{i,s=2,t=0})]$$

Difference in Difference (DID)

- 或者更方便地可以创建dummy(D_s 和 D_t)直接用回归的形式

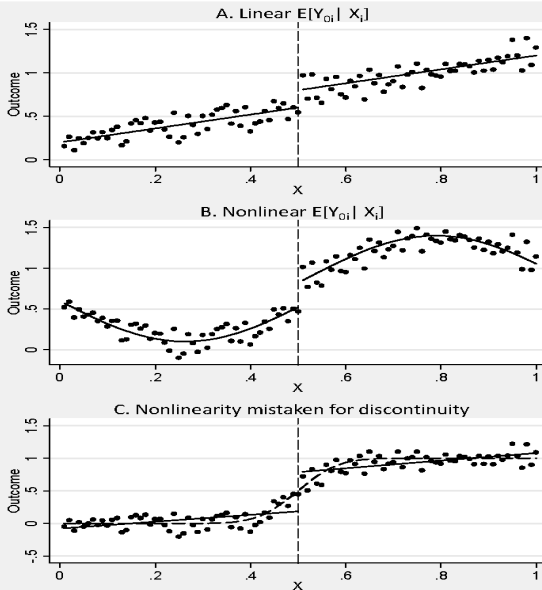
$$Y_{ist} = \alpha + \gamma D_s + \lambda D_t + \beta D_s * D_t + \epsilon_{ist}$$

- 如果两组明显没有common trend怎么办?

$$Y_{ist} = \alpha + \gamma D_s + \lambda D_t + \beta D_s * D_t + X_{ist} \delta + \epsilon_{ist}$$

- 如果没有现成地实验组和对照组怎么办?

Regression Discontinuity (RD)



Regression Discontinuity (RD)

- 假设 ρ 是我们关心的treatment effect

$$Y_i = f_0(x_i)(1 - D_i) + f_1(x_i)D_i + \rho D_i + \epsilon_i$$

- 我们用polynomial来拟合非线性方程 $f(x_i)$

$$E(Y_{0i}|x_i) = f_0(x_i) = \alpha + \sum_{j=1}^p \beta_{0j}(x_i - x_0)^j$$

$$E(Y_{1i}|x_i) = f_1(x_i) = \alpha + \sum_{j=1}^p \beta_{1j}(x_i - x_0)^j + \rho$$

Regression Discontinuity (RD)

- 写成回归方程的形式

$$Y_i = \alpha + \sum_{j=1}^p \beta_{0j}(x_i - x_0)^j + \rho D_i + \sum_{j=1}^p (\beta_{1j} - \beta_{0j}) D_i (x_i - x_0)^j + \epsilon_i$$

- 这种参数方法的风险在于我们的拟合方程可能不够准确, 这个时候可以采取非参数的方法

$$\begin{aligned} \lim_{\delta \rightarrow 0} [E(Y_i | x_0 < x_i < x_0 + \delta) - E(Y_i | x_0 - \delta < x_i < x_0)] \\ = E(Y_{1i} - Y_{0i} | x_i = x_0) = \rho \end{aligned}$$

Regression Discontinuity (RD)

- Discontinuity还可能以非deterministic的方式出现

$$P(D_i = 1|x_i) = \begin{cases} g_0(x_i), & x_i \leq x_0 \\ g_1(x_i), & x_i > x_0 \end{cases}, \quad g_1(x_i) > g_0(x_i)$$

- 这时我们可以用2SLS来进行估计

$$D_i = \gamma_0 + \sum_{j=1}^p \gamma_j x_i^j + \xi T_i + u_i, \quad T_i = 1(x_i = x_0) \quad \text{First Stage}$$

$$Y_i = \mu + \sum_{j=1}^p \beta_j x_i^j + \rho \hat{D}_i + \nu_i \quad \text{Second Stage}$$

Regression Discontinuity (RD)

- 当Discontinuity以非deterministic的方式出现时,我们也可以使用非参数的方法进行估计

$$\begin{aligned} \lim_{\delta \rightarrow 0} \frac{E(Y_i | x_0 < x_i < x_0 + \delta) - E(Y_i | x_0 - \delta < x_i < x_0)}{E(D_i | x_0 < x_i < x_0 + \delta) - E(D_i | x_0 - \delta < x_i < x_0)} \\ = E(Y_{1i} - Y_{0i} | x_i = x_0) = \rho \end{aligned}$$

时间序列数据(time series data)

- 无论用那种时间序列方法，第一步都是看时间序列数据；
- 单时间序列数据按照稳定性可以分为三种
 - 稳定的(stationary)
 - 带有非随机的趋势的(deterministic trends)
 - 带有随机趋势的(stochastic trends)=单位根(unit roots)
- 多个时间序列数据可能还存在协整(co-integration)的问题

Stationarity: example

$$y_t = at + u_t$$

$$u_t = \rho u_{t-1} + \epsilon_t, \quad 0 < \rho \leq 1$$

- $a = 0, \rho < 1$: stationary
- $a \neq 0, \rho < 1$: deterministic trends
- $a \neq 0, \rho = 1$: unit root

协整关系(cointegration)

$$x_t = x_{t-1} + \epsilon_t$$

$$z_t = z_{t-1} + \nu_t$$

- 如果存在 θ 使得 $x_t - \theta z_t$ 是stationary的, 那么我们就说 x_t 和 z_t 具有协整关系

非稳定性时间序列的处理

- deterministic trends: detrend
- unit root: first difference
- cointegrated: do nothing

自回归滑动平均模型(ARMA)

- 一个ARMA(p,q)模型可以表示为

$$z_t = \alpha_0 + \sum_{i=1}^p \alpha_i z_{t-i} + \sum_{j=1}^q \beta_j \epsilon_{t-j}.$$

- ARMA的优点在于**灵活性**: 基本上可以模拟(fit)所有的时间序列数据, 因此适合进行预测
- 但是ARMA模型在经济学分析中的用处非常有限

$$y_t = \gamma_0 + X_t\gamma + u_t.$$

其中 $\text{var}(u_t) = \sigma_t^2$.

- ARCH(p)模型

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^p \alpha_i u_{t-i}^2.$$

- GARCH(p,q)模型

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^p \alpha_i u_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2.$$

- 同样, ARCH和GARCH比较适合做数据拟合进而做forecast, 而进行经济学分析的作用不大.

向量自回归模型(Vector Autoregression)

- VAR分为structural form和reduced form;
- 前者包含经济学含义，用来进行经济学分析(but still not quite structural)
- 后者用来做计量估计(estimation)

从structural form到reduced form



$$y_t = \alpha_y + \alpha_{y,0}i_t + \sum_{j=1}^p \alpha_{y,j}^y y_{t-j} + \sum_{j=1}^p \alpha_{y,j}^i i_{t-j} + \epsilon_{y,t}$$

$$i_t = \alpha_i + \alpha_{i,0}y_t + \sum_{j=1}^p \alpha_{i,j}^y y_{t-j} + \sum_{j=1}^p \alpha_{i,j}^i i_{t-j} + \epsilon_{i,t}$$



$$E \begin{pmatrix} \epsilon_{y,t} \\ \epsilon_{i,t} \end{pmatrix} = 0, \quad \text{cov} \begin{pmatrix} \epsilon_{y,t} \\ \epsilon_{i,t} \end{pmatrix} = \begin{pmatrix} \sigma_y^2, 0 \\ 0, \sigma_i^2 \end{pmatrix}$$

- 因为这两个方程都有明显的内生性所以不能直接用OLS估计

- 矩阵的形式为

$$\begin{pmatrix} 1, -\alpha_{y,0} \\ -\alpha_{i,0}, 1 \end{pmatrix} \begin{pmatrix} y_t \\ i_t \end{pmatrix} = \begin{pmatrix} \alpha_{y,1}^y, \alpha_{y,1}^i \\ \alpha_{i,1}^y, \alpha_{i,1}^i \end{pmatrix} \begin{pmatrix} y_{t-1} \\ i_{t-1} \end{pmatrix} + \dots \\ + \begin{pmatrix} \alpha_{y,p}^y, \alpha_{y,p}^i \\ \alpha_{i,p}^y, \alpha_{i,p}^i \end{pmatrix} \begin{pmatrix} y_{t-p} \\ i_{t-p} \end{pmatrix} + \begin{pmatrix} \epsilon_{y,t} \\ \epsilon_{i,t} \end{pmatrix}$$

- 或者可以表述成

$$A_0 \begin{pmatrix} y_t \\ i_t \end{pmatrix} = A_1 \begin{pmatrix} y_{t-1} \\ i_{t-1} \end{pmatrix} + \dots + A_p \begin{pmatrix} y_{t-p} \\ i_{t-p} \end{pmatrix} + \begin{pmatrix} \epsilon_{y,t} \\ \epsilon_{i,t} \end{pmatrix}$$

从structural form到reduced form

- 两边同乘以 A_0^{-1} :

$$\begin{pmatrix} y_t \\ i_t \end{pmatrix} = A_0^{-1}A_1 \begin{pmatrix} y_{t-1} \\ i_{t-1} \end{pmatrix} + \dots + A_0^{-1}A_p \begin{pmatrix} y_{t-p} \\ i_{t-p} \end{pmatrix} + A_0^{-1} \begin{pmatrix} \epsilon_{y,t} \\ \epsilon_{i,t} \end{pmatrix}$$

- 假设 $\sigma_y^2 = \sigma_i^2 = 1$, 那么

$$u_t = A_0^{-1} \begin{pmatrix} \epsilon_{y,t} \\ \epsilon_{i,t} \end{pmatrix} \quad \Sigma_u = A_0^{-1}A_0^{-1}'.$$

从reduced form到structural form

- reduced form已经没有内生性，可以用OLS直接估计
- 估计出来的reduced form可以直接拿来forecast，但是如果要进行经济学分析还必须解出structural form
- 所以问题就是如何从 $\hat{\Sigma}_u$ 推出 \hat{A}_0^{-1}
- 所以在这里我们一般需要加入identification assumption (restriction)
- 最常见的restriction是 A_0^{-1} 中部分元素为0: **Choleski Decomposition**, 背后的经济学含义是某些变量不受当期内其他变量的影响(所以ordering很重要)

做structural var的几个步骤

- 选定变量: 关键是一定要有经济学含义
- 确定变量的形式(level, log level, log difference, HP filter):
用log level一般比较保险, 除非有明显的trend像中国的GDP就用HP filter
- 确定滞后项的个数: 一般以一年为单位, 所以季度数据就用四个, 当然也可以用BIC或者AIC来做选择
- 回归估计, 得到参数
- 做impulse response和variance decomposition
- 向量误差修正模型(vector error correction model)?

Structural VAR in Eviews

The screenshot displays the EViews interface. The main window shows the results of a Vector Autoregression (VAR) estimation. The title bar indicates the file is 'CHINA_DATA:China_data'. The menu bar includes File, Edit, Object, View, Proc, Quick, Options, Add-ins, Window, and Help. The toolbar contains View, Proc, Object, Print, Name, Freeze, Estimate, Forecast, Stats, Impulse, Resids, and Zoom. The main area is titled 'Vector Autoregression Estimates' and contains the following text:

Vector Autoregression Estimates
Date: 11/16/15 Time: 10:44
Sample (adjusted): 6/01/1998 6/01/2013
Included observations: 61 after adjustments
Standard errors in () & t-statistics in []

	OUTPUT_GAP	M2_GROWTH
OUTPUT_GAP(-1)	0.562926 (0.18115) [3.10749]	-0.065567 (0.03822) [-1.71536]
OUTPUT_GAP(-2)	0.167372 (0.20973) [0.79803]	0.038396 (0.04425) [0.86764]
OUTPUT_GAP(-3)	0.052122 (0.20349) [0.25613]	-0.023262 (0.04294) [-0.54177]
OUTPUT_GAP(-4)	0.011271 (0.17415) [0.06472]	0.033225 (0.03675) [0.90418]
M2_GROWTH(-1)	0.524678 (0.63922) [0.82081]	0.136903 (0.13488) [1.01502]
M2_GROWTH(-2)	-0.621418 (0.66515) [-0.93426]	-0.225204 (0.14035) [-1.60462]

An 'SVAR Options' dialog box is open, showing the 'Identifying Restrictions' tab. The 'Endogenous variable list' includes @e1 for OUTPUT_GAP residuals and @e2 for M2_GROWTH residuals. The 'Short-run example' shows @e1 = C(1)*@u1 and @e2 = C(2)*@e1 + C(3)*@u2. The 'Identifying Restrictions' section is set to 'Specify' with the equations @e1 = c(1)*@u1 and @e2 = -c(2)*@e1 + c(3)*@u2. The 'Text' radio button is selected. The 'Matrix' radio button is also present. The dialog box has '确定' (OK) and '取消' (Cancel) buttons.

Path = c:\users\situ\documents DB = season WF = china_data

Structural VAR in Eviews

Var: UNTITLED Workfile: CHINA_DATA::China_data\

View Proc Object Print Name Freeze Estimate Forecast Stats Impulse Resids Zoom

Structural VAR Estimates

Structural VAR Estimates
Date: 11/16/15 Time: 10:44
Sample (adjusted): 6/01/1998 6/01/2013
Included observations: 61 after adjustments
Estimation method: method of scoring (analytic derivatives)
Convergence achieved after 1 iterations
Structural VAR is just-identified

Model: $Ae = Bu$ where $E[uu'] = I$
Restriction Type: short-run text form
@e1 = c(1)*@u1
@e2 = -c(2)*@e1 + c(3)*@u2
where
@e1 represents OUTPUT_GAP residuals
@e2 represents M2_GROWTH residuals

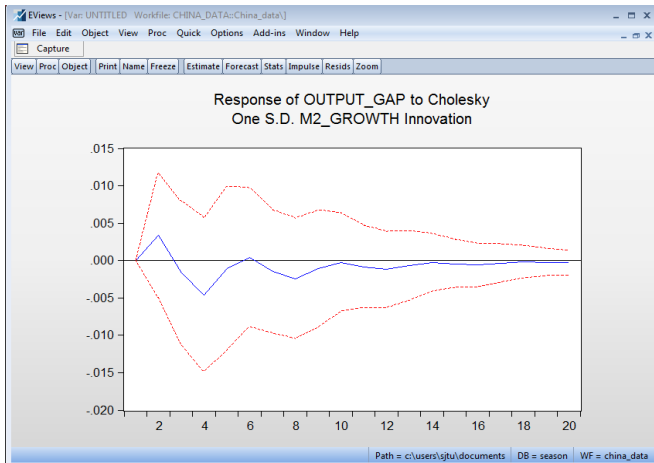
	Coefficient	Std. Error	z-Statistic	Prob.
C(2)	0.008572	0.026994	0.317562	0.7508
C(1)	0.031018	0.002808	11.04536	0.0000
C(3)	0.006540	0.000592	11.04536	0.0000

Log likelihood 345.5761

Estimated A matrix:
1.000000 0.000000
0.008572 1.000000

Estimated B matrix:
0.031018 0.000000
0.000000 0.006540

Structural VAR in EViews



Structural VAR in EViews

